

## Theoretical-Methodological and Economical-Mathematical Approaches to Building Model of Non-Observed Economy

Veleruya L. Vorontsova, Yelena L. Fesina and Diana I. Makhmutova  
Economical-Mathematical Modeling of Institute of Management,  
Economy and Finances of Kazan Federal University, Kazan, Russia

---

**Abstract:** In this study, we have presented theoretical-methodological aspects of essential and conceptual approach to building economical-mathematical model of non-observed economy and selection of its advanced model. For solving the task of detection of connections and dependencies between indicators of non-observed economy, we have analyzed it with correlation and regression analysis at micro-level with detaching main components that define development of latent processes in the sphere of commercial production. Structural elements in economical-mathematical models of non-observed economy are regression equations and time series which allow describing scales and dynamics of its development as well as to detach prevailing factors. In spite of the fact that instruments of correlation and regressive analysis do not give univocal conclusions regarding the reasons of occurrence if latent processes, it allows following the development of non-observed economy as a system, to define and forecast main proportions in its sectors. Correlation and regression analysis may be successfully used for definition of influence level of introduced economy policy on the processes that take place in shadow and informal sectors of economy as well as on variation of productivity of their activity. Building model of non-observed economy is based on investigations in the sphere of economic theory, mathematical-statistical and econometric instruments.

**Key words:** Non-observed economy, latent processes, economical-mathematical modeling, correlation and regression analysis, time series

---

### INTRODUCTION

Evolution of methods, used for assessment of non-observed economy led to co-integrational development of economical-mathematical and econometrical approaches which are used for quality assessment of various alternative assessments of latent processes (Grigoreva and Fesina, 2014). Market economy requires advancements in information operation which characterizes scales and dynamics of latent processes' development. Dynamic regression plays an important part in solving of this task (Vetoshkina and Tukhvatullin, 2015).

Questions of building dynamic models are reflected in the works of both domestic and Foreign researchers (Fedoseeva, 1999; Foerster and Rentz, 1982). In particular, Haavelmo (1994) points at necessity of development of reasonable model with the help of which one could explain and forecast economical phenomena. Time series of observations are one of the data sources.

Economic researches prove that Russian economy is far from market balance (Grigoreva, 2015). Market misbalance remarkably complicates tasks of applied modeling of non-observed economy (Fesina and Savdur, 2014).

Often it is difficult to estimate the results of calculations, obtained with the help of mathematic and econometric methods of analysis (Kadochnikova *et al.*, 2015; Popkova *et al.*, 2013). This is connected with multiple-valued character of the goals set, incorrect selection of mathematical and statistical instruments for obtaining estimations.

Meeting the principles of mathematic and econometric approach allows approaching the issue of selection of its advanced model and search of their new kinds (Vorontsova and Gorskaya, 2015a, b).

**Problem setting:** At implementation if dynamic regression model of non-observed economy, the following problems were set:

- To detect the type of dependency of endogenic variable from time

- In various points of factor space, some experiments were taken, as a result of which we defined values of observed endogenic variable
- To assess model's parameters using experimental data
- To assess the level of adequacy of obtained dependency
- To get predictive estimates of endogenic variable from time

Construction of dynamic model's algorithm is conformed to its following characteristics: determinacy, massiveness and effectiveness.

**Subject of the research:** The main task of economical and mathematical calculations at analysis of non-observed economy is obtaining more definitive and reliable assessments of it. For solving the task of finding connections and dependencies between indicators of non-observed economy, we have analyzed it with fragmental correlation-regression method at micro-level, in the sphere of commercial production with the application of dynamic model.

**MATERIALS AND METHODS**

For analysis of non-observed economy and tendencies of its development at micro-level, we have used materials, taken from the following sources: Federal State Statistics Service, the Bank of Russia, International Monetary Fund, Deutsche Bank Research, British Petroleum, Bureau of Economic Analysis, World Trade Organization, Forest and Agriculture Organization (FAO) of the United Nation, calculations of the Center of Financial and Bank investigations of Institute of Economics of the Russian Academy of Sciences as well as the results of scientific researches of domestic and Foreign researchers.

Consideration of probability parameters of non-observed economy and their control is connected with tracking hidden and artificial economic activity (Vetoshkina and Tukhvatullin, 2014). There is a large group of indices of non-observed economy, many of which are interacting with each other with a certain delay or lag (Kadochnikova, 2013).

Regression and dynamic models allow investigating interconnections of non-observed economy's phenomena, help reproducing the image of possible condition of shade markets and their trade flows as well as to detect real economic condition of business entities at taking shade economic activities.

With the purpose of demonstrative presentation of indices that characterize dynamics of changes of hidden returns in time, we shall build analytical function that characterizes dependency of row levels from time or trend.

Table 1: Dependency of hidden returns from time, normal units

Years	Hidden returns (nu)
2006	0.511
2007	0.497
2008	0.478
2009	0.516
2010	0.524
2011	0.474
2012	0.515
2013	0.526
2014	0.522

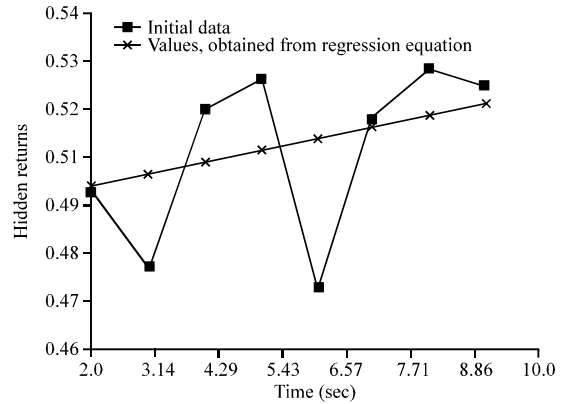


Fig. 1: Dependency of hidden returns from time, normal units

Values of hidden returns are obtained on the base of official and expert assessments for an aggregate of 20 enterprises in the sphere of commercial production at an example of the Republic of Tatarstan for the period of 2006-2014 (Abalkin, 1994; Bogomolov, 2006; Senchagov, 2002; Tretyakova, 2013; Ranjan *et al.*, 2013; Safiullin *et al.*, 2013). With the purpose of keeping non-observed phenomena closed, empirical data is normalized in correspondence with available procedure of building algorithms.

Calculations were performed in Mathcad package, like in many other works, where possibilities of specialized mathematic packages are taken place. In this package, we have built linear equation of trend (1) which defined dependency of hidden returns V from time t:

$$V(t) = at + b \tag{1}$$

Where:

V (t) = Endogenic variable (hidden returns)

a =  $2.716 \times 10^{-3}$

b = 0.494

Dependency of hidden returns from time is presented in Table 1. Graphical representation of dependency of hidden returns from time is presented at Fig. 1.

**RESULTS AND DISCUSSION**

The procedure of smoothing of empirical data at equation of a straight line allowed continuing the value of independent variable of time and define predicted values of hidden returns for 2015-2020 (Table 2).

While the information is processed at computer, selection of equation type is usually made experimentally, i.e., by comparing values of correlation ratio, calculated at various models. In case, there is a linear or non-linear dependence between two attributes, correlation ratio is evaluated through correlation of external variance to total variance and it characterizes dispersion of points of correlational field with reference to regression line  $\hat{y}(t)$  :

$$\eta = \sqrt{\frac{\sigma_y^2 - \sigma_{\hat{y}_t}^2}{\sigma_y^2}}$$

Where:

$\sigma_y^2$  = A dispersion of empirical (actual) values of resultative attribute

$\sigma_{\hat{y}_t}^2 = \frac{1}{n} \sum_{t=1}^n (y_t - \hat{y}(t))^2$  is a residual dispersion of empirical values of resultative attribute

It reflects influence caused on variation for each other factors except x. Comparison of correlation for linear ( $\eta = 0.38$ ), exponential ( $\eta = 0.38$ ) and hyperbolic trends ( $\eta = 0.12$ ) showed moderate linear or exponential connection; linear trend equation was selected, since it was easier to be analyzed and perceived.

Further analysis calculation of autocorrelation coefficient of the first order  $r_1 = 0.087$  speaks of absence of linear tendency. Hypothesis regarding absence or presence of the tendency may be checked with the help of Abbe criterion. Empirical value of Abbe criterion is calculated according to the following formula:

$$q = 0.5 \cdot \frac{\sum_{t=1}^{n-1} (y_{t+1} - y_t)^2}{\sum_{t=1}^n (y_t - \bar{y})^2}$$

Where:

$\bar{y}$  = An arithmetical mean from the sample set

n = A number of values in the sample set

Table 2: Dynamics of predictive information about hidden returns for 2015-2020, normal units

Years	Hidden returns (nu)
2015	0.521
2016	0.523
2017	0.526
2018	0.529
2019	0.532
2020	0.534

Let us find out whether there is a tendency for hidden returns to grow which is evidenced by obtained linear trend equation. Null hypothesis is the one regarding the absence of tendency, i.e., regarding the equality of hidden returns' levels.

$$q_{\text{emp}} = 1.04$$

In the table of Abbe criterion, we find the information that at n = 9 and significance level of 0.05 critical value  $q_{\text{kprr}} = 0.512$  which is less than the one we received (1.04), thus we may admit the hypothesis about the absence of tendency for hidden returns to grow.

Consequently, linear trend is not a good model for describing tendencies of hidden returns' behavior. In case there is a set of variables, among which a polynomial dependency is implies of a higher order than of the first one and in case of necessity of more precise approximation of their values to theoretical ones, one may apply various methods (Azimov *et al.*, 2014).

One may use the function that creates various polynoms of the second order depending on disposition of the curve and making local approximations, it smoothens the curve. The function allows doing it by investigating empirical data in small neighborhood of the point that is of the most interest. For this one need to select the size of the neighborhood which, for example equals 0.75 and obtain interpolation for  $f_1$  function for initial data of vector V. After that one may select the neighborhood which equals 2 and obtain interpolation of  $f_2$  vector.

Correlation, calculated per vector  $f_1$ ,  $\eta = 0.816$ , speaks of the presence of strong connection. Correlation, calculated per vector  $f_2$ ,  $\eta = 0.479$ , speaks of the presence of moderate connection only. Fluctuations of data of vector V and calculated values of vector  $f_1$  are presented at Fig. 2.

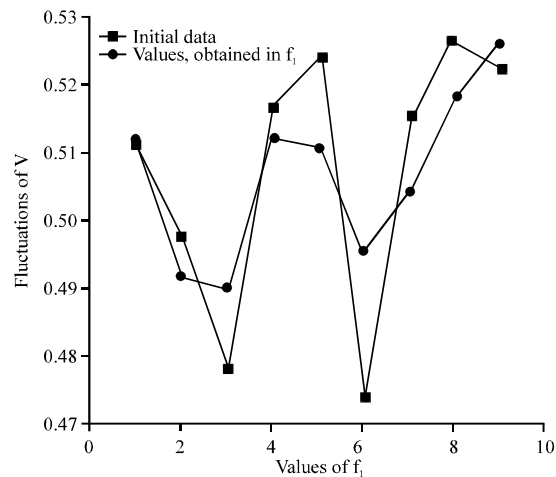


Fig. 2: Fluctuations of data of vector V and obtained value of vector  $f_1$

Table 3: Comparative analysis of actual values of hidden revenues and functions  $f_1$  and  $f_2$  (initial data), normal units

Hidden returns (nu)	$f_1$	$f_2$
0.511	0.512	0.505
0.497	0.492	0.501
0.478	0.490	0.499
0.516	0.512	0.500
0.524	0.510	0.502
0.474	0.495	0.506
0.515	0.504	0.511
0.526	0.518	0.517
0.522	0.526	0.525

Table 4: Calculated values of hidden returns per third-order polynom

Years	$f_1$
2006	0.509
2007	0.496
2008	0.495
2009	0.500
2010	0.505
2011	0.508
2012	0.510
2013	0.514
2014	0.527

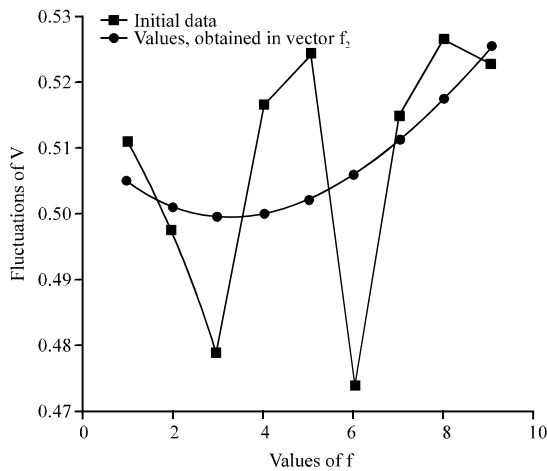


Fig. 3: Fluctuations of data of vector V and obtained values of vector  $f_2$

As we may see from Fig. 2, values of vector  $f_1$  are close to initial data. They smoothen their fluctuations well and create more smooth approximating function in comparison with vector  $f_2$  (Table 3). Fluctuations of data of vector V and obtained values of vector  $f_2$  are presented by Fig. 3. For approximation of model's data one may use single polynom, in particular, polynom of the third order. Obtained data are brought out in vector  $f_3$  (Table 4). Correlation, calculated according to vector  $f_3$ ,  $\eta = 0.512$  demonstrates the presence of medium connection.

Fluctuations of data of vector V and obtained values of vector  $f_3$  are presented by Fig. 4. Parabolas of higher orders are rarely used for expression of dynamic's tendency and are too complex for obtainment of reliable assessments at limited length of times series.

Sometimes, it is necessary to estimate the values of functions in points that are beyond initial data. Mathcad is supplied with an option that allows doing that. This function uses linear algorithm of prediction, which is useful when extrapolated function is smooth and oscillating. This linear prediction may be considered as a variety of extrapolation; however, it should not be mixed up with linear or polynomial extrapolation.

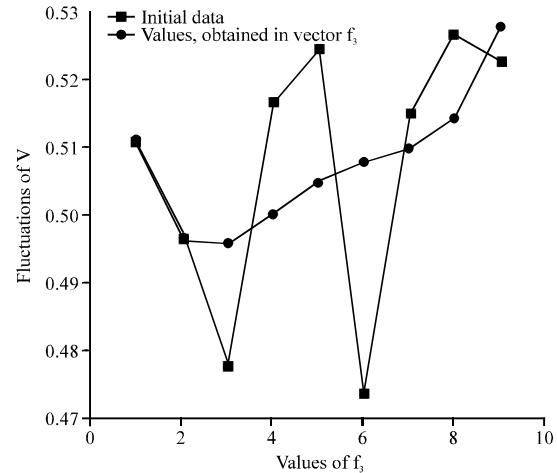


Fig. 4: Fluctuations of data of vector V and obtained values of vector  $f_3$

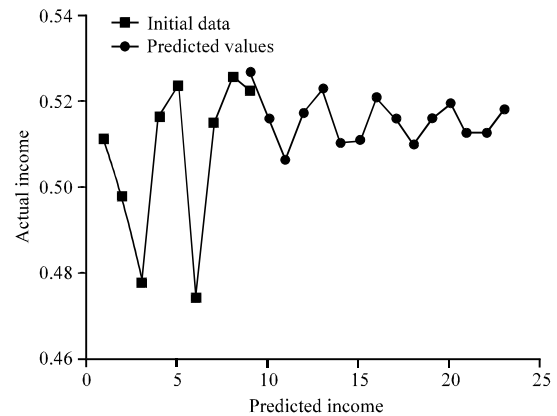


Fig. 5: Extrapolation of actual and predicted values of incomes

Figure 5 and Table 4 demonstrate results of the functions which allows appraise its values in the point that are located beyond initial data and obtain extrapolated results.

Comparative analysis of obtained functions on the base of correlation's value indicates that the best approximation is provided by function  $f_1$ , it is the best for smoothing of data fluctuations and creation of smooth approximating function.

Thus, the values of initial function are smoothed best of all by polynomial of second order, which performs local approximations, at this, we investigate the data that is located in minor neighborhood of the point that is of the greatest interest for us.

**Summary:** Any economic policy involves regulation of economic variables and it should be based on the knowledge how these variables are connected with each other. We have analyzed three variants of dependence of hidden returns from time.

Currently, there are no officially approved recommendations regarding methods of forming of predicted estimations of non-observed economy. This leads to necessity of search of effective prediction methods for managing non-observed economy.

In case of pair regression, the model is usually selected according to type of dislocation of observed points at correlational field. However, not rare are situations where location of points approximately corresponds to several functions and one needs to define the best of them all.

The researchers performed assessment of different types of dependencies, using experimental data. The choice of linear form of trend equation was conditioned by the fact that linear pair regression is the easiest for perception and interpretation.

For detection of endogenic variable's behavior depending on time, we advanced a hypothesis about the presence of both linear and polynomial dependence. For two types of dependencies, we built curves that reflect tendencies of changes in endogenic variable in time. Endogenic variable was represented by revenues from hidden and fictive economic activity (hidden returns). For the second type of dependency, we used various polynomials of the second order with the aim of obtaining local approximations of empirical data to theoretical one. With this purpose, empirical data were investigated in minor neighborhood of the point of the most interest. As a result, we obtained function's interpolation for empirical data of the vector of endogenic variable. Comparative analysis on the base of calculated correlation ratio value of two types of dependencies allowed selecting the most preferable function of approximation of empirical data to theoretical ones. This allowed assessing function's values in points that are located beyond the area of initial data and obtain extrapolated indices, which may be used for understanding the realistic scales of transferring resources into shadow turn-around.

## CONCLUSION

The task of this scientific research is maximal approximation of variables' values which describe hidden

values with the help of trend equation. It was necessary to define the equation of the line that would concord best with all values from Table 1. Analysis of obtained curves was performed. Comparison of correlation ratio values and test of hypothesis about constantly growing amount of hidden returns showed inadequacy in application of linear trend. The best approximation of the function of hidden revenues  $V(t)$  was obtained at the usage of function  $f_1$  which creates polynomials of the second order and making local approximations, smoothens initial curve. We have also obtained predicted values of hidden returns for 2015-2020.

## ACKNOWLEDGEMENT

The research is performed according to the Russian Government Program of Competitive Growth of Kazan Federal University.

## REFERENCES

- Azimov, Yu.I., S.N. Savdur and E.L. Fesina, 2014. Ensuring environmental safety based on the modeling of biological process of oily SEWAGE. *Mediterranean J. Soc. Sci.*, 5 (24): 372-377.
- Abalkin, L. 1994. Economical security of Russia: threats and their reflection. *Issues of Economy*, No. 12.
- Bogomolov, V.A., 2006. *Economical security*, V.A. Bogomolov. Moscow: YUNITI-DANA.
- Fedoseeva, V.V., 1999. *Economical and mathematical methods and applied models*. Under the editorship of M.: YUNITI.
- Foerster, E. and B. Rentz, 1982. *Methods of correlational and regression analysis*: Translated from Germany. E. Foerster, B. Rentz. *Finances and statistics*.
- Fesina, E.L. and S.N. Savdur, 2014. Modeling of Sewage Bioremediation as a Modified Petri Net. *World Applied Sci. J.*, 31 (6): 1191-1197.
- Grigoreva, E.A. and E.L. Fesina, 2014. Economic Security as a Condition of Institutional Support of Economic Modernization. *World Applied Sci. J.*, 31 (5): 940-948.
- Grigoreva, E.A., 2015. Problems of Institutional Provision of the State's Economic Security. *Mediterranean J. Soc. Sci.* MCSER Publishing, Rome-Italy, May, Vol. 6, No. 3.
- Haavelmo, T., 1994. The probability approach in econometrics. *Econometrica*, 12 (1): 115.
- Kadochnikova, E.I., E.A. Polovkina, E.A. Grigoreva and L.D. Badrieva, 2015. Use of Binary Dependent Variables for Modelling Borrowers' Creditworthiness. *Asian Soc. Sci.*, Vol. 11, No. 11.

- Kadochnikova, E.I., 2013. Methodological issues of macroeconomic accounting. *World Applied Sci. J.*, 24 (6): 719-723.
- Popkova, E.G., E.S. Akopova, E.S. Alekhina, Yu.I. Dubova, J.N. Popova, I.A. Avdeeva and I.Yu. Proskurina, 2013. Methodology of Development of Strategy of Development of Economic Systems. *World Applied Sci. J.*, 26 (4): 489-493.
- Ranjan, R., W.C. Ngai and R. Ruslan, 2013. Development of Indicators for Sustainable Rice Farming in Bangladesh: A Case Study with Participative Multi-Stakeholder Involvement. *World Applied Sci. J.*, 22 (5): 672-682.
- Senchagov, V.K., 2002. Economic security: Geopolitics, globalization, self-preservation and growth. V.K. Senchagov. Moscow: Finstatinform.
- Safiullin, M.R., L.A. Elshin, A.I. Shakirova, P.O. Ermolaeva and M.I. Prygunova, 2013. Influence of Territorial Ecological Load Factors on Soc. Eco. Well-Being of Population: Methodology Development and Econometric Model Construction. *World Applied Sci. J.*, 25 (7): 1057-1061.
- Tretyakova, E.A., 2013. Evolution of Research and Evaluation Methodology of Sustainable Development of Soc. and Eco. Systems. *World Applied Sci. J.*, 25 (5): 756-759.
- Vorontsova, V.L. and T.Uy. Gorskaya, 2015a. Numerical methods of the decision differential the equations for continuous models of economy. *Mediterranean J. Soc. Sci.*, S3, 6 (1): 198-203.
- Vorontsova, V.L. and T.Uy. Gorskaya, 2015b. Approximate Methods of the Decision Differential the Equations for Continuous Models of Economy. *Asian Soc. Sci.*, 11 (11): 214-220.
- Vetoshkina, E.Yu. and R.Sh. Tukhvatullin, 2014. The Problem of Accounting for the Costs Incurred after the Initial Recognition of an Intangible Asset. Vetoshkina E.Yu., Tukhvatullin R.Sh., *Mediterranean J. Soc. Sci.*, 5 (24): 52-56.
- Vetoshkina, E.Yu. and R.Sh. Tukhvatullin, 2015. Economic Efficiency Estimation of Intangible Assets Use/E. Yu. Vetoshkina, R.Sh. Tukhvatullin. *Mediterranean J. Soc. Sci.*, S3, 6 (1): 440-443.