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The Mathematical Apparatus of Compromise of Efficiency Estimation of Investment Projects

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Abstract: The problem of compromise efficiency estimation of investment projects is presented. Application of the aggregation theory and the fuzzy sets theory in an efficiency estimation of investment projects is proved. The mathematical apparatus of compromise efficiency estimation of investment projects is developed.

Key words: Investment project, estimation, fuzzy sets, desirability function, efficiency

INTRODUCTION

The problem essence of effectiveness estimation of investment projects: The problem is how to objectively and systematically evaluate (to measure) the effectiveness of the upcoming investment activity (project), including the opportunity to assess all significant and not significant, simple and complex, economic and non-economic (social, environmental, resource, technical, technological, etc.), cost and time, quantity and quality and other parameters.

Thus from our point of view, it should be reduced to a single criterion to give clear, objective and unambiguous estimation to the project efficiency. The total (final) indicator of efficiency has to meet to a number of requirements imposed to parameters of optimization (Adler *et al*., 1976):

- To be quantitative
- Uniform (expressed by one number)
- To be unambiguous, i.e., the set of values of the considered indicators in an estimation (private parameters of an estimation) corresponds to one value total
- To be universal, comprehensively to characterize object (efficiency of the investment project)
- To conform to the requirement of completeness, i.e., to be rather general not specific to characterize object as a unit

The problem of assessing the effectiveness of the upcoming investment enterprises' activity in modern conditions is multifactorial and compromise. It faces need of the big variety's account of parameters of various physical essence, dimension, the status and mutual "concessions" between opposite various parameters. This is especially true for engineering objects.

MATERIALS AND METHODS

Reason of application of the aggregation theory and the fuzzy set theory in an estimation of investment projects efficiency: The most convenient way of the solution of such compromise multiple-factor tasks is the generalization parameters procedure conducting to the uniform parameter of optimization (Adler *et al*., 1976). A number of problems are connected with such generalization.

Firstly, due to the fact that each private parameter of an estimation (optimization), any possible parameter of object which is exposed to an estimation and (or) optimization makes the physical sense and the dimension. It is necessary to enter some dimensionless scale which is uniform for all parameters for each of them. It allows to compare them. Secondly, difficulty arises in the rule selection of an initial private parameters' combination in the generalized indicator.

One way to construct a scale of preference is the Harrington's desirability function (Adler *et al*., 1976; Novik, 1979; Novik and Arsov, 1980; Puryaev, 2007b; Novik *et al*., 1974; Shtarkman *et al*., 1969a, b) which allows to simulate the process of consistent behavior of individual subsystems of a whole and to take into account the impact of communication between them in solving the problem of the choice from set of existing alternatives. Basis of construction and priority possibility of this generalized function is transformation of natural values of private parameters of various physical essence and dimension to a uniform dimensionless scale of desirability (preference). Purpose of a scale consists in establishment of compliance between physical and psychological parameters of optimization. The physical are understood as the various parameters characterizing functioning of the studied object. This may include economic, technical and economic, technical and technological, aesthetic, statistical and other parameters. Psychological parameters are understood as subjective evaluation of the researcher (desirability, preference). Psychological parameters are expressed through numerical system (points, marks) at a desirability scale.

For receiving a scale of desirability it is convenient to use the ready developed tables of correspondence between the preference relations in empirical and numerical (psychological) systems. The numerical system of preferences presented in Table 1 is also a dimensionless scale of the desirability developed by Harrington. Values of this scale have an interval from 0-1 and are designated (d).

The ith value of private parameter of optimization converted to a dimensionless scale of desirability, denoted by d_i is called as private desirability, i = 1,2,3...n current issue of parameter, n-number of private parameters.

Value $d_i = 0$ corresponds to absolutely unacceptable level of ith parameter optimization. Value $d_i = 1$ the best value of ith parameter.

The desirability function corresponding to the Harrington's desirability scale has the following form:

- For unilateral constraint:

$$d = e^{-e^{-y'}} \quad (1)$$

- For bilateral constraint:

$$d = e^{-|y'|^n} \quad (2)$$

Where:

y' = The coded value of private parameter y, i.e., its value in conditional scale
n = Exponent

The conversion curves expressed by the Eq. 1 and 2 aren't the only opportunity, however, these formulas appeared empirically as result of supervision over real decisions of researchers-experimenters (Adler *et al*., 1976). Curves have useful properties such as continuity, monotonicity, smoothness.

Table 1: The Harrington's desirability s cale

Empirical system of preferences (desirability)	Numerical system of preferences (system of psychological parameters)
Very good	1.00-0.80
Good	0.80-0.63
Satisfactory	0.63-0.37
Bad	0.37-0.20
Very bad	0.20-0.00

To use this method at an estimation and a selection of optimum version of the decision, it is originally necessary to establish (to set) the boundaries of admissible values for all private parameters of an estimation (optimization). Constraints can be unilateral (y_{min} or y_{max}) or bilateral (y_{min} and y_{max}). At unilateral constraint to $d_i = 0.37$ mark on a scale of desirability there correspond y_{min} or y_{max} (the lower or top limit respectively is set), at bilateral constraint y_{min} and y_{max}.

After all private parameters (y_i) are converted to desirabilities (d_i), it is necessary to start creation of the generalized parameter of estimation (optimization) called by Harrington the generalized function of Desirability (D). One of successful ways of solving the problem of a choice the optimum option is representation of the generalized desirability function as average geometrical private desirabilities:

$$D = \sqrt[n]{d_1 \times d_2 \times d_3 \cdot \ldots \cdot d_i \cdot \ldots \cdot d_n} \quad (3)$$

The generalized indicator of this type allows to use, first, the same scale of preference (Table 1); secondly "to reject" version of the decision from set considered if at least one its private parameter doesn't meet the strict requirement of the researcher ($d_i = 0$); thirdly, meets to a number of the requirements imposed to total criterion of optimization and stated above.

The question of an estimation the efficiency of enterprise activity (including investment) is many-sided, systemic, ambiguous and actual for all economic systems of managing at all times and for all subjects of an estimation. In the solution of real questions of efficiency estimation of the mathematical theories operating with the idealized, poorly connected with reality problems still prevail. New means and opportunities for processing of the qualitative verbal statements possessing high uncertainty is the Fuzzy-Set Theory (FST).

The fuzziness takes place in cases when numerical boundaries of concept or an event can not be specified unambiguously (Bellman *et al*., 1976). For this purpose the concept of fuzzy number (fuzzy set special form) is entered into FST. Fuzzy numbers are entered into FST as the tool for numerical representation of fuzzy values. Methods of fuzzy mathematics allow carry out all logical and arithmetic operations (intersection, union, inversion, addition, multiplication, composition) over fuzzy values by certain actions over their membership functions (Orlovskij, 1981). Fuzzy information (fuzzy input or an output) carries in itself more information than the "traditional" numerical system.

The variable which values can be words or phrases of some natural or artificial language (Shtovba, 2007) is called as linguistic variable. The concept of a Linguistic Variable (LV) is entered by Zade (1976). The concept of application the LV provides the approximate description of the difficult or inexact phenomena which difficult or can not be described in usual quantitative terms.

The bases of application the FST in the questions' solution of the system analysis, an estimation and modeling (Zade, 1976; Mardamshin, 2006):

- The FST operates with systems for which there is no clear boundary between the elements entering and not entering into this system. Uncertainty of FST depends on an outcome of any event and isn't connected with random quality. Uncertainty in traditional mathematics associated with belonging or not belonging to fuzzy set
- The FST is based on the degree of the true result of belonging to a certain category and not on the degree of probability that the result will be obtained
- The FST admits that the element can be simultaneously belong along with positive degree of truth to some set and with other value of positive degree of truth not to belong to this set
- The FST admits that the element can be simultaneously with positive degree of truth belong to two or more fuzzy sets
- The FST operates with the concepts which aren't connected with a statistically precise data (samples) and is based on logical judgments and conclusions. Thus qualitative and (or) asymmetric information, uncertain (fuzzy) quantitative data are exposed to the analysis
- The used tools of traditional mathematics for modern conditions of activity are linear in parameters (are too simplified concerning the used parameters and the accounting of factors), abstracted, inconvenient for use, "are sometimes twirled" in mathematical sense (aren't clear for users)

RESULTS AND DISCUSSION

The mathematical apparatus of efficiency estimation of investment projects: In this technique of an estimation, the mathematical apparatus of the conversion parameters values set in the form of a fuzzy set in values of the desirability function developed and offered by us is necessary (the mechanism for transforming the values of linguistic variable in the value of the desirability function).

So far, the generalization model in the form of the Harrington's desirability function was applied only to the restrictions set in the form of crisp sets, i.e., value of parameter still belonged to one interval or to another but not to two at the same time in any way that according to the theory of fuzzy sets perhaps, it is necessary and is more real.

In case of the constraint task and (or) desirable level in parameters set in the form of linguistic variables even with the simplest membership functions (Eq. 4-7), two estimations carried to different values of the LV (terms, admissibility intervals) turns out. These two estimates need to be converted to the value of the desirability function so that really to reflect sense of illegibility in the formalized estimation of IP efficiency.

This conversion mechanism of fuzzy sets membership functions in the values of the Harrington's desirability function better represented graphically in the form of a nomogram with explanations. In Fig. 1, membership function graphs on the following formulas are shown:

$$\mu_{\text{"unacceptable"}}(u) = \begin{cases} 1, \text{if } u \le 20 \\ -0.05u + 2, \text{if } u \in (20;40) \end{cases} \quad (4)$$

$$\mu_{\text{"satisfactory"}}(u) = \begin{cases} 1, \text{if } u = 40 \\ 0.05u - 1, \text{if } u \in (20;40) \\ -0.05u + 3, \text{if } u \in (40,60) \end{cases} \quad (5)$$

$$\mu_{\text{"good"}}(u) = \begin{cases} 1, \text{if } u = 60 \\ 0.05u - 2, \text{if } u \in (40;60) \\ -0.05u + 4, \text{if } u \in (60,80) \end{cases} \quad (6)$$

$$\mu_{\text{"excellent"}}(u) = \begin{cases} 1, \text{if } u \ge 80 \\ 0.05u - 3, \text{if } u \in (60;80) \end{cases} \quad (7)$$

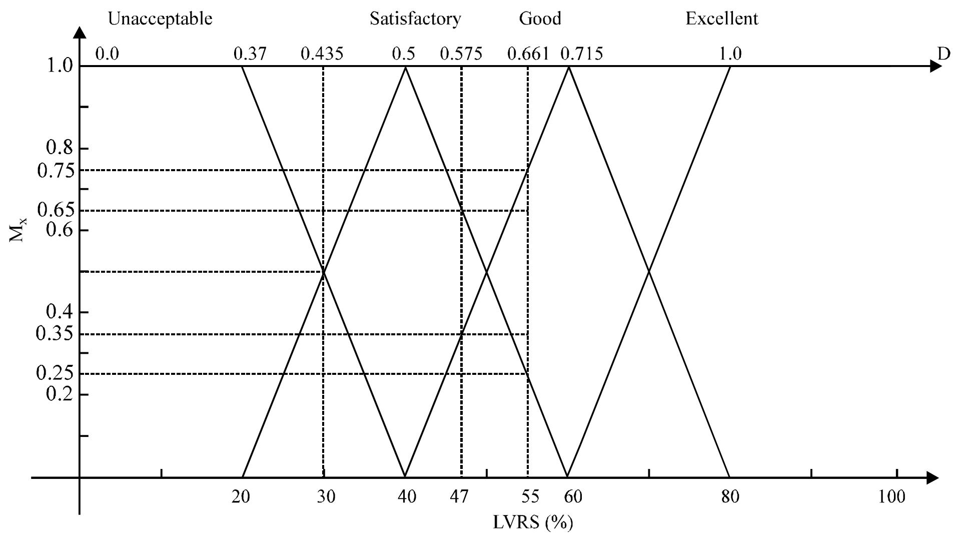

Fig. 1: The graphic mechanism of the conversion the value of the linguistic variable "resource security" (LV "resource security") in value of the desirability function

But intervals scales of the Harrington's desirability function are imposed on them graphically: unacceptable, satisfactory, good and excellent according to terms of membership function of a fuzzy set (unacceptable, satisfactory, good, excellent).

This is done so as to take into account the properties of the two functions (the fuzzy set membership function and the desirability function). The mechanism of the conversion of a fuzzy set scale to a scale of desirability function is offered to be constructed as follows. Originally for "peak" and "critical" values of each term LV is necessary to define values of desirability functions.

It is obvious that median values (medians) of the corresponding intervals of desirability scale most are suitable for terms "satisfactory" and "good" (Table 1), i.e., median values of desirability scale for all 100% belong to the corresponding term (to μ_x for them equally 1). For example for value of LV RS = 40% and LV RS = 60% (Fig. 1, RS-resource security) need to be established from a logic position the following values of desirability function, respectively:

$$d = \frac{0.37 + 0.63}{2} = 0.5; \ d = \frac{0.63 + 0.8}{2} = 0.715$$

The sum of boundary values of intervals "satisfactory" and "good", "good" and "excellent" desirability scales respectively is presented in numerator of each fraction. Somewhat different is the case of the boundary intervals ("unacceptable" and "excellent"). In an interval "unacceptable" for "critical" value of a universal set LV RS = 20% it is recommended to establish desirability, equal 0.37 (d = 0.37).

For all values of a horizontal site of membership function of a term "unacceptable" (i.e. for LV RS<20%) it is necessary to establish desirability, equal 0 (d = 0), i.e., all values of LV RS<20% are unacceptable. In an interval "excellent" all values of LV RS≥80% in accordance with the membership function (Fig. 1) have to have the desirability equal 1 (d = 1).

All desirabilities of intermediate values of LV RS can be defined graphically (less accurately), drawing the perpendicular line passing through the corresponding value before crossing with the received desirability scale (Fig. 1: values 30, 47 and 55%), or to calculate on the following formula:

$$d_X = d_{tc1} \times \mu_{X1} + d_{tc2} \times \mu_{X2} \qquad (8)$$

Where:

d_X = The desirability of the calculated value from the universal set of LV

d_{tc1}, d_{tc2} = The desirability of "threshold", "critical" value according to the 1st and 2nd terms which belongs to (with a certain value of membership function) value of the estimated parameter X

μ_{X1}, μ_{X2} = Values of membership functions respectively 1st and 2nd terms for the value X

For example, the LV RS = 30% (X = 30%) with the same degree of membership ($\mu_{X1} = \mu_{X2} = 0.5$) refers to the term "unacceptable" (d_{tc1} = 0.37) and to the term "satisfactory" (d_{tc2} = 0.5). Then $d_{30} = 0.37\times0.5 + 0.5\times0.5 = 0.435$.

For understanding other example is shown (Fig. 1). Value of LV RS = 47% (X = 47%) belongs to a term "satisfactory" (d_{tc1} = 0.5) with degree of membership $\mu_{X1} = 0.65$ and to a term "good" (d_{tc2} = 0.715) with degree of membership $\mu_{X1} = 0.35$. Then the desirability of X = 47% value is defined so $d_{47} = 0.5\times0.65 + 0.715\times0.35 = 0.575$. It's similarly also for other values.

Thus, the monotony condition of desirability function (the desirability of consistently located values of LV monotonously grows) is observed and property of different degree of membership the desirability's values and values of LV simultaneously two terms specified in the form of a fuzzy set is observed.

CONCLUSION

The Eq. 8 is developed on the basis of the deffazifikation principle of fuzzy sets (conversion of fuzzy set to clear number) but used thus values of desirability scale but not value of a universal set of LV. In Puryaev (2007a), the technique of an efficiency estimation of investment projects is offered (on a conditional example of the investment project of modernization of a melting site of foundry). The developed mathematical apparatus in principle differs from the existing mechanism for the conversion of parameter values given in the form of crisp sets in values of desirability scale and allows to consider nonuniqueness and carelessness in a task of restrictions in the parameters set in the form of linguistic variables.

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