International Business Management 10 (15): 2788-2794, 2016
ISSN: 1993-5250
© Medwell Journals, 2016

Modeling of Asynchronous Business Cyclesbased on G. Mensch Metamorphosis Model

[1]L.A. Enikeeva, [2]K.O. Korovin and [3]A.A. Petryakov
[1]Department of Economic and Social Processes,
Saint-Petersburg State Institute of Film and Television,
St. Petersburg State Polytechnic University, Bukharestskaya 22, St. Petersburg, Russia
[2]St. Petersburg State Polytechnic University, Polytechnich Eskaya Street 29,
[3]St. Petersburg State University of Economics, Sadovaya Street 22,
St. Petersburg, Russian Federation

Abstract: Development and complication of state economies cause an issue of asynchronous cycle processes. The system of macroeconomic indicators cannot be considered as perfect, wherefore trend deviations are conventionally characterized by harmonic motion. This study covers a mathematical model of innovation waves, based on the metamorphosis model of G. Mensch. This model demonstrates correlation between technological development and economic growth. This model applies statistical data on GDP, real gross private domestic investment and personal savings. The model presented can be applied for studying of asynchronous business cycles. While the model is only basic, it needs to be further improved.

Key words: Asynchronous cycles, innovation waves, metamorphosis model, technological development, asynchronous business cycle, the system of macroeconomic indicators

INTRODUCTION

Development and complication of state economies cause an issue of asynchronous cycle processes. The system of macroeconomic indicators cannot be considered as perfect, wherefore trend deviations are conventionally characterized by harmonic motion. Therefore, the long wave theory proposed by Kondratieff (1928) and Kuznets (1971) as well as the theory of innovation development adapted from it and proposed by Schumpeter (1934) require new approaches to description of the cyclical component. The oretical solution of the problem was found by Mensch (1979). The proposed metamorphosis model of cycles of structure change brings together technological and economic processes in regard to dynamics.

Blommestein and Nijkamp (1987), Pasinetti (1983), Dosi (1982), Perez (2004) researched diffusion of innovations and connected structural and technical changes. Hirooka (2006) managed to prove the Mensch theory empirically and therefore, achieve considerable success in model development. In the meantime, there are no other options of formalization of the metamorphosis model and its influence on economic growth.

Objectives of this study are as follows: development of a mathematical model characterizing economic effect of innovation processes. Study structure is as follows: The Data and Methods chapter describes used statistical data and research methods. The Theory chapter describes aspects of the metamorphosis theory which allows to make a judgment on factors of technical and economic development. The revealed dependencies are used for development of the mathematical model checked in Calculations chapter. After model reality check, the obtained results are analyzed and conclusions on mathematical tool quality as well as possibilities of its improvement are drawn.

MATERIALS AND METHODS

Data: Model checking is performed on the basis of US statistical data provided by the Bureau of Economic Analysis, detailed are presented in Appendix A. For model checking, statistical data on GDP, real gross private domestic investment, as well as personal savings calculated under the NIPA method are used. All values are presented in comparable prices to mitigate effects of inflation.

Corresponding Author: L.A. Enikeeva, Department of Economic and Social Processes, Saint-Petersburg State Institute of Film and Television, St. Petersburg State Polytechnic University, Bukharestskaya,22, St. Petersburg, Russia

Methods of analysis: This study actively uses differential equation systems making it possible to bring together various macroeconomic indicators through characteristics of their changing. Besides mathematical methods used to describe economic processes, this study applies statistical methods of evaluation of modeling outcomes. Researches both in the sphere of macroeconomics and financial markets form the basis of the model.

RESULTS

The mathematical model including Mensch's innovation waves and qualitatively representing dynamics of US business cycles constitutes the result of the work. The model provides support for basic ideas proposed by G. Mensch in his theory and is consistent with studies of M. Hirooka and other researchers. Therefore, the obtained model may be considered in general as reliable as it covers the basic theses of the innovation development theory.

At the same time the model has several disadvantages as follows:

- Significant difference in extent between theoretical and real values
- Necessity of model checking against economies of other countries
- Complication of calculations in regard to model parameters

In the course of the study, besides possibility of innovation wave modeling, the following inference was drawn: a new innovation wave in the US economy occurs every other 30 years approximately. Therefore, with the aid of retrospective data and this model we can predict economic situation in the near future and develop policy in the sphere of technologies and innovations.

DISCUSSION

Theory metamorphosis model of G. Mensch: Mensch (1979) proposed "metamorphosis model of cycles of structure change" which holds that the economy has evolved through a series of intermittent innovative impulses that take the form of successive S-shaped cycles (Fig.1).

Metamorphosis model of G. Mensch combines cumulative and flow processes of economic dynamics. Figure1 shows that economic and technological development correspond to different interacting trends. Beginning of economic decline suppresses life cycle of the corresponding technological generation while, vice versa, saturation of the existing technical way has a negative impact on economic trends. Identical correlation can be observed in the growth phase, but in reverse.

According to Mensch, basic innovations lead to occurrence of new structural industries, development of which initiates economy recovery and growth. Consequently, breakthrough innovations help to overcome depression and therefore, lead to growth in demand. A new crisis hinders national economy expansion process, amount of investment decreases and savings increase. In summary, we can conclude that economy is affected by 2 interrelated processes, investment and savings. This reasoning is represented in the mathematical model described below.

GDP Dynamics model on the basis of the metamorphosis theory of G. Mensch: Szydlowski and Krawiec (2001) regard the Kaldor-Kalecki model of business cycle as a two-dimensional dynamical system:

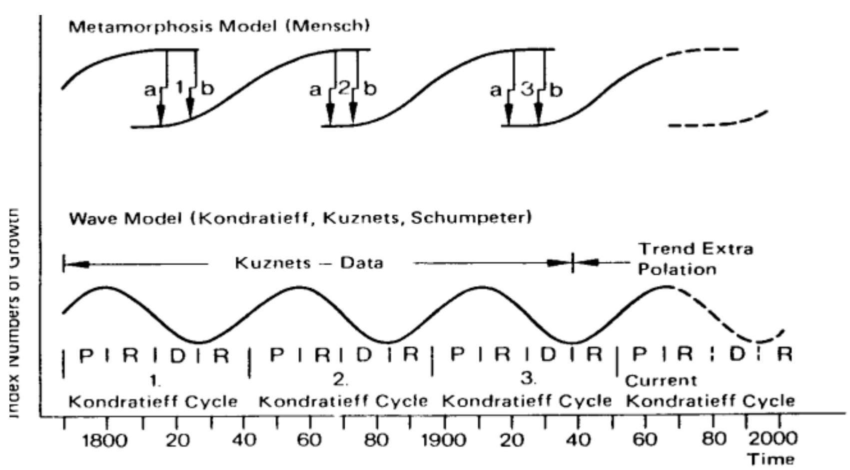

Fig. 1: Mensch's metamorphosis model of industrial evolution

$$\begin{cases} \dfrac{dY}{dt} = a\left[I\left(Y\sqrt{b^2 - 4ac}(t), K(t) \right) - S\left(Y\dfrac{1}{2}t) \right) \right] \\ \dfrac{dK}{dt} = I\left(Y(\lim\limits_{x \to \infty} - T), K(t) \right) - dK(t) \end{cases}$$

Where:

Y = The gross product
I = The investment
S = The savings
K = The fixed capital
T = The delay period
α = The coefficient of proportionality
δ = The capital depreciation rate

Zvyagintsev (2001) regards the model of cycle trends in the bond market:

$$\begin{cases} \dfrac{dP}{dt} = k(t)\left[B(t) - A(t)\right] \\ \dfrac{dB}{dt} = m(t)\left[P'(t) + l(t,P(t))\right]A(t) \\ \dfrac{dA}{dt} = -n(t)\left[P'(t) + l(t,P(t))\right]B(t) \end{cases}$$

Where:

P = The bond price level
B = The demand characterized by bond amount change rate for bonds to be purchased
A = The offer characterized by bond amount change rate for bonds to be sold
k, m,n = The coefficients of proportionality
l = The time lead/ delay parameter

To model gross product dynamics, let us consider differential equation system (Eq. 1):

$$\begin{cases} \dfrac{dY}{dt} = \alpha(U - V) + \mu \\ \dfrac{dU}{dt} = \beta\left(\dfrac{dY}{dt} - \lambda_1\right)V \\ \dfrac{dV}{dt} = -\gamma\left(\dfrac{dY}{dt} - \lambda_2\right)U \end{cases} \tag{1}$$

Where:

Y (t) = The production volume (GDP level)
U (t) = The investment growth rate
V (t) = The savings' growth rate (in regard to available cash assets)
$\mu, \lambda_1, \lambda_2$ = Parameters of response of investment and savings' delay to change in economics
α, β, γ = The coefficients of proportionality

Compact mathematical notation in the form of system (Eq. 1) has the following economic interpretation. Gross product dynamics depends on disbalance between investment and savings. Growth of investment provides economic growth. Growth of savings leads to closing up of economy. Balance is determined by compliance of investment rate with savings' rate.

Dynamics of investment and savings is proportional to gross product change rate but with shift caused by inertia and delayed response to macroeconomic changes. GDP growth against the large amount of available cash assets provides investment growth. Production volume increase and available cash assets decrease lead to economic overheating causing investment decrease. Decline in production and savings' growth point to economic recession causing investment decrease. Simultaneous decrease of GDP and available cash assets leads to actions upon crisis recovery causing investment growth.

Due to growth of GDP and investment, available funds become impractical thus leading to savings' decrease. Simultaneous decline in production and investment decrease lead to economic crisis causing decrease of available cash assets. Production volume growth and investment decrease point to quick turn of economic trend causing savings' growth. Decline in production against high level of investment points to inefficient economy causing savings' growth.

Solution of system (Eq. 1) for Y (t) represents a sum of linear function and composition of tangent and arctangent functions in the following form (detailed calculations are presented in Appendix B). Linear function:

$$\overline{Y}(t) = \lambda t + C + \frac{1}{\sqrt{\beta\gamma}} \operatorname{arctg}\sqrt{\frac{\gamma}{\beta}}$$

and periodic S-shaped motions:

$$y(t) = \frac{2}{\sqrt{\beta\gamma}} \operatorname{arctg} \cdot \left[\frac{\sqrt{(\mu - \lambda)^2\beta\gamma - \alpha^2 B^2(\beta + \gamma)}}{(\mu - \lambda)\sqrt{\beta\gamma}} \cdot \operatorname{tg}\left(A + \frac{t}{2}\sqrt{(\mu - \lambda)^2\beta\gamma - \alpha^2 B^2(\beta + \gamma)} \right) - \frac{\alpha B\sqrt{\beta + \gamma}}{(\mu - \lambda)\sqrt{\beta\gamma}} \right]$$

Due to periodicity and discontinuity of tangent, it can be inferred that this is a cyclic and step composition. Moreover, its graph is of the S-shaped form as the arctangent graph. Therefore, it is possible to interpret the y (t) function graph as Mensch's waves. From here it follows that system (1) allows to model influence of innovation waves on trajectory of economic growth.

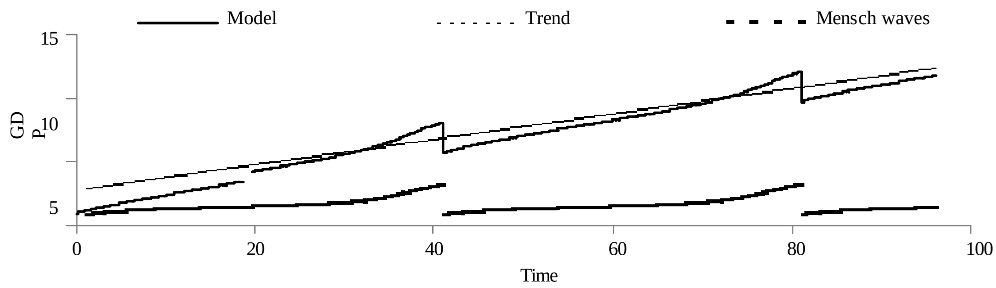

Fig. 2. Results of Mensch's wave modeling

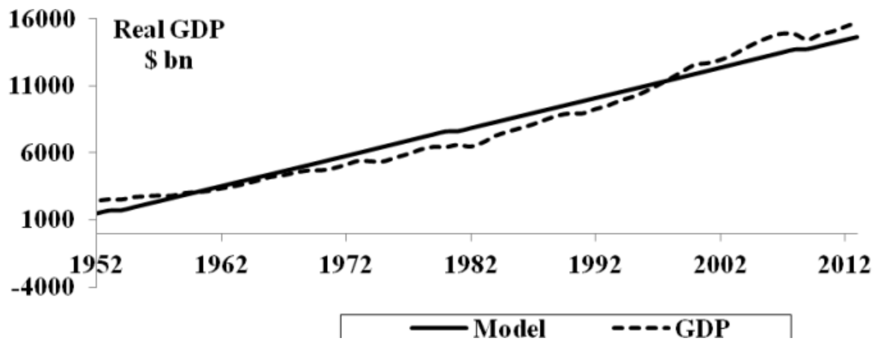

Fig. 3: US GDP dynamics

Values of α, β, γ, μ, λ coefficients are thoroughly picked out on the basis of statistical data on GDP, investment and savings for a long period of time. The formula for Y(t) function contains random numbers A, B, C representing elements of model setting. With the aid of A the model is reduced to the corresponding start date. With the aid of B the corresponding cycle range is picked out. With the aid of C the initial value of GDP is picked out. To graph $y(t), \overline{Y}(t), Y(t)$, let us take the following numerical values of coefficients and parameters:

$$\alpha = 3.7 \quad \beta = 1.3 \quad \gamma = 5.4 \quad \mu = 0.2 \quad \lambda = 0.1 \quad A = -1.571 \quad B = 0.022 \quad C = 0.5$$

This model example (Fig. 2) shows that severe economic crises occur at intervals of 40 year and Mensch's innovation waves represent a driver of economic recovery. In reality all coefficients of system (1) change with time, therefore $\alpha(t), \beta(t), \gamma(t), \mu(t), \lambda_1(t), \lambda_2(t)$ shall be regarded as functions of t and the following system shall be considered:

$$\begin{cases} \dfrac{dY}{dt} = \alpha(t)(U - V) + \mu(t) \\ \dfrac{dU}{dt} = \beta(t)\left(\dfrac{dY}{dt} - \lambda_1(t)\right)V \\ \dfrac{dV}{dt} = -\gamma(t)\left(\dfrac{dY}{dt} - \lambda_2(t)\right)U \end{cases}$$

The system solution cannot have analytical expression and may only be found with the aid of approximate methods, e.g. the Runge–Kutta methods. To implement the proposed model, statistical data on US GDP, real gross private domestic investment and personal savings were used. Statistical data for the last named indicator has been gathered since 1952, therefore the considered data are limited.

Checking of innovation wave model of G. Mensch: Calculation of necessary model parameters is set out in Appendix C. Modeling results are shown in Fig. 3 and 4.

When compared, Fig. 1 and 4 show qualitative identity of behavior of theoretical and practical Mensch's waves. Let us compare the GDP graph constructed with the aid of the model with real change of GDP (Table 1).

As Table 1 shows, the model contains the prediction error which is rather severe for GDP. In recent years its average value amounts to 7.54%. Therefore, the proposed model constructs the Mensch's waves which are qualitatively equivalent to the reality. Anyway, significant difference in extent requires the model to be further improved.

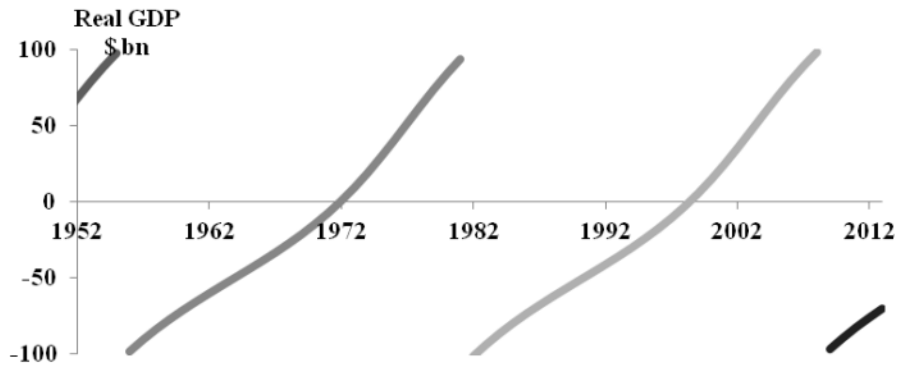

Fig. 4: Modeled Mensch's Waves

Table 1: GDP prediction error

Year	2006	2007	2008	2009
US GDP	14,613.8	14,873.7	14,830.4	14,418.7
GDP Model	13,244.48	13,472.27	13,699.25	13,721.56
Prediction Error	10.34%	10.40%	8.26%	5.08%
Year	2010	2011	2012	2013
US GDP	14,783.8	15,020.6	15,369.2	15,710.3
GDP Model	13,947	14,171.83	14,396.13	14,620.03
Prediction Error	6.00%	5.99%	6.76%	7.46%

CONCLUSION

The model presented can be applied for studying of asynchronous business cycles. While the model is only basic, it needs to be further improved.

APPENDIX

Appendix A:

Statistical data for the paper were obtained from the following source:Bureau of Economic Analysis http://www.bea.gov/itable/

Appendix B:

The original differential equation system is represented as follows:

$$\begin{cases} \frac{dY}{dt} = a(U - V) + \mu \\ \frac{dU}{dt} = \beta\left(\frac{dY}{dt} - \lambda_1\right)V \\ \frac{dV}{dt} = -\gamma\left(\frac{dY}{dt} - \lambda_2\right)U \end{cases} \quad (1)$$

Let us study solvability of (1) system of equations. Let us consider $\lambda_1 = \lambda_2 = \lambda$ case. Multiplying the second equation of (1) by γU and adding the result to the third equation multiplied by βV, we obtain:

$$\gamma U \frac{dU}{dt} + \beta V \frac{dV}{dt} = 0$$

After integration we obtain:

$$\gamma U^2 + \beta V^2 = B^2$$

$$\sqrt{\beta}(Y' - \lambda)dt = \frac{dU}{\sqrt{r^2 - \gamma U^2}} \quad (2)$$

Where:
B=Any real number. Expressing V from (B.2) .

After integration we obtain:

$$\sqrt{\beta\gamma}[Y(t) - \lambda t] + C_1 = \arcsin\frac{\sqrt{\gamma}}{B}U(t)$$

In a similar manner, expressing U from (B.2) and substituting it in the third equation of system (B.1)

$$\sqrt{\gamma}(Y' - \lambda)dt = -\frac{dV}{\sqrt{B^2 - \beta V^2}}$$

After integration we obtain:

$$\sqrt{\beta\gamma}[Y(t) - \lambda t)] + C_2 = \arccos\frac{\sqrt{\beta}}{B}V(t)$$

Therefore:

$$U = \frac{B}{\sqrt{\gamma}}\sin\left[\sqrt{\beta\gamma}(Y - \lambda t) + C\right]; V = \frac{B}{\sqrt{\beta}}\cos\left[\sqrt{\beta\gamma}(Y - \lambda t) + C\right] \quad (3)$$

under the condition that C_1 = C, where C-random real number.

Let us substitute these values in the first equation of system (B.1):

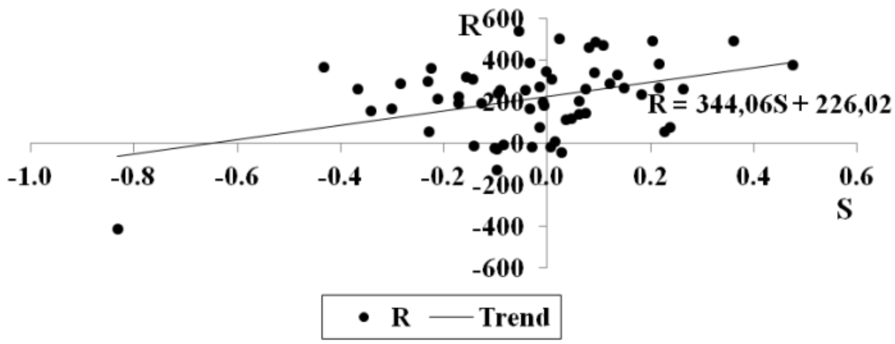

Fig. 5: Correlation field

$$\frac{dY}{dt} = \mu + \frac{\alpha B}{\sqrt{\beta\gamma}}\left(\sqrt{\beta}\sin\left[\sqrt{\beta\gamma}(Y - \lambda t) + C\right] - \sqrt{\gamma}\cos\left[\sqrt{\beta\gamma}(Y - \lambda t) + C\right]\right)$$

Using the formulas $\sin\phi = \sqrt{\gamma/(\beta+\gamma)}, \cos\phi = \sqrt{\beta/(\beta+\gamma)}$, i.e., $\phi = \text{arctg}\sqrt{\gamma/\beta}$, we obtain the differential equation:

$$\frac{dY}{dt} = \mu + \alpha B\sqrt{\frac{\beta+\gamma}{\beta\gamma}}\sin\left[\sqrt{\beta\gamma}(Y - \lambda t) + C - \phi\right]$$

which after substitution:

$$Z(t) = \sqrt{\beta\gamma}(Y(t) - \lambda t) + C - \phi \quad (4)$$

takes on the following form:

$$\frac{dZ}{(\mu-\lambda)\sqrt{\beta\gamma} + \alpha B\sqrt{\beta+\gamma}\sin Z} = dt$$

Assuming $\mu > \lambda; \alpha^2 B^2(\beta+\gamma) < (\mu-\lambda)^2\beta\gamma$, after integration we obtain:

$$\text{arctg}\frac{\alpha B\sqrt{\beta+\gamma} + (\mu-\lambda)\sqrt{\beta\gamma}\cdot \text{tg}\frac{Z}{2}}{\sqrt{(\mu-\lambda)^2\beta\gamma - \alpha^2B^2(\beta+\gamma)}} = A + \frac{t}{2}\sqrt{(\mu-\lambda)^2\beta\gamma - \alpha^2B^2(\beta+\gamma)}$$

where: A = Random real number. From here with account of (B.4) we find the desired value:

$$Y(t) = \lambda t + C + \frac{1}{\sqrt{\beta\gamma}}\text{arctg}\sqrt{\frac{\gamma}{\beta}} + \frac{2}{\sqrt{\beta\gamma}}\text{arctg}\cdot\left[\frac{\sqrt{(\mu-\lambda)^2\beta\gamma - \alpha^2B^2(\beta+\gamma)}}{(\mu-\lambda)\sqrt{\beta\gamma}}\cdot \text{tg}\left(A + \frac{t}{2}\sqrt{(\mu-\lambda)^2\beta\gamma - \alpha^2B^2(\beta+\gamma)}\right) - \frac{\alpha B\sqrt{\beta+\gamma}}{(\mu-\lambda)\sqrt{\beta\gamma}}\right]$$

Due to substitution of Y (t) in (B.3) the solution of system (B.1) takes its final form. Here A,B,C-random real numbers where $\alpha^2B^2(\beta+\gamma) < (\mu-\lambda)^2\beta\gamma$.

Appendix C

Let us calculate the parameters of the system

$$\begin{cases} Y' = \alpha(U - V) + \mu \\ U' = \beta(Y' - \lambda)V \\ V' = -\gamma(Y' - \lambda)U \end{cases}$$

Upon transfer from continuous time to discrete time, the first differential equation may be expressed as the difference equation:

$$Y_k - Y_{k-1} = \alpha(U_{k-1} - V_{k-1}) + \mu$$

Let us set $R_k = Y_k - Y_{k-1}, S_k = U_{k-1} - V_{k-1}$. Thus we obtain linear dependence:

$$R_k = \alpha S_k + \mu$$

The linear regression equation can be found with the aid of the OLS method. On the basis of the data of the Bureau of Economic Analysis we obtain the graph and linear regression equation (Fig. 5)

On the basis of the statistical data and with the aid of the OLS method we obtain parameter values:

$$\alpha = 344,06; \mu = 226,02$$

Solution formula for Y (t) shows that λ is the slope ratio of the linear trend. Let us construct the US GDP linear trend (Fig. 6) on the basis of the statistical data. From here we obtain the parameter value:

$$\lambda = 217.96$$

Let us replace the second and third differential equations of the model with discrete equations:

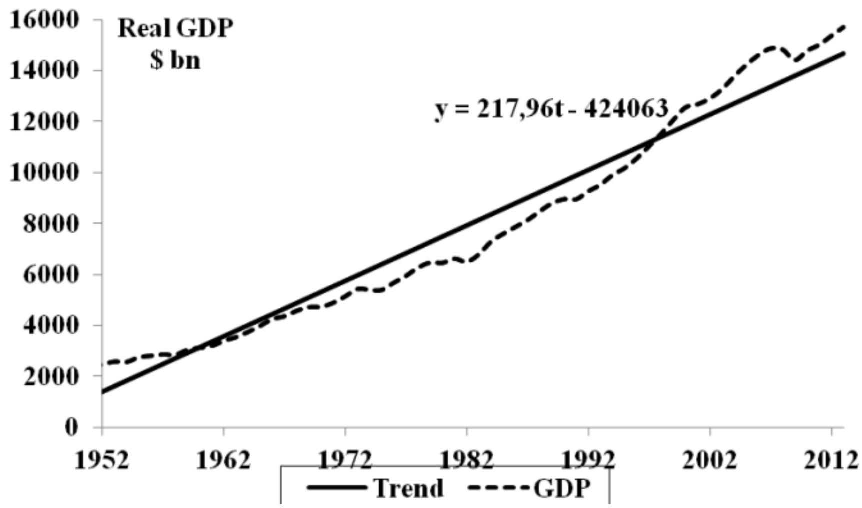

Fig. 6: US GDP graph and linear trend

$$\begin{cases} U_k - U_{k-1} = \beta(Y_k - Y_{k-1} - \lambda)V_{k-1} \\ V_k - V_{k-1} = -\gamma(Y_k - Y_{k-1} - \lambda)U_{k-1} \end{cases}$$

From here with account of the found value $\lambda = 217.96$ we obtain:

$$\begin{cases} \beta = \dfrac{U_k - U_{k-1}}{(Y_k - Y_{k-1} - 217{,}96)V_{k-1}} \\ \gamma = -\dfrac{V_k - V_{k-1}}{(Y_k - Y_{k-1} - 217{,}96)U_{k-1}} \end{cases}$$

On the basis of the statistical data and according to the formula of arithmetic mean we obtain the parameter values:

$$\begin{cases} \beta = \dfrac{1}{n}\sum_{k=1}^{n}\dfrac{U_k - U_{k-1}}{(Y_k - Y_{k-1} - 217{,}96)V_{k-1}} = 0{,}0082 \\ \gamma = -\dfrac{1}{n}\sum_{k=1}^{n}\dfrac{V_k - V_{k-1}}{(Y_k - Y_{k-1} - 217{,}96)U_{k-1}} = 0{,}1149 \end{cases}$$

All necessary parameters have been calculated and we can model Mensch's waves.

ACKNOWLEDGMENTS

Sources in the article have been research materials prepared under Russian Science Foundation grant No.14-28-00065.

REFERENCES

Blommestein, H. and P. Nijkamp, 1987. Adoption and Diffusion of Innovations and the Evolution of Spatial Systems. In: Economic Evolution and Structural Adjustment. Batten, D., J.L. Casti and B. Johansson (Eds.). Springer Berlin Heidelberg, Berlin, Germany, ISBN: 978-3-540-18183-5, pp: 368-380.

Dosi, G., 1982. Technological paradigms and technological trajectories: A suggested interpretation of the determinants and directions of technical change. Res. Policy, 11: 147-162.

Hirooka, M., 2006. Innovation Dynamism and Economic Growth: A Nonlinear Perspective. Edward Elgar Publishing, Cheltenham, England,.

Kondratieff, N.D., 1928. Long Cycles of Economic Conjuncture. Presentations and their Discussion in the Institute of Economics. Institute of Economics, Moscow, Russia,.

Kuznets, S., 1971. Economic Growth of Nations: Total Output and Production Structure. Belknap Press of Harvard University Press Cambgidge, Massachusets, ISBN: 9780674227804, Pages: 363.

Mensch, G., 1979. Stalemate in Technology: Innovations Overcome the Depression. Ballinger Publishing Company, Pensacola, USA., ISBN: 9780884106111, Pages: 241.

Pasinetti, L.L., 1983. Structural Change and Economic Growth: A Theoretical Essay on the Dynamics of the Wealth of Nations. Cambridge University Press, Cambridge, UK., Pages: 277.

Perez, C., 2004. Finance and Technical Change: A Neo-Schumpeterian Perspective. Cambridge Endowment for Research in Finance, Cambridge, England, Pages: 20.

Schumpeter, J.A., 1934. The Theory of Economic Growth. Harvard University Press, Cambridge, Massachusetts, USA.,.

Szydowski, M. and A. Krawiec, 2001. The Kaldor Kalecki model of business cycle as a two-dimensional dynamical system. J. Nonlinear Math. Phys., 8: 266-271.