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Dynamic Portfolio Selection: A Literature Revisit

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Abstract: Dynamic portfolio selection optimization is essential and critical objective for any investment strategy by individuals and institutional investors. The purpose of this study is to verify the various factors effect on selecting dynamic portfolio and spot a light on opportunities for future research. This study revisits existence literature of the factors that affect in selecting the optimal portfolio in multi-stages. Dynamic portfolio selection optimization suffers the ancient problem in estimation the risk measures and lack of information in uncertain markets and economies with demanded investors to achieve their growing objectives. Many researches conducting this problems offered new models, searching new constraints, relaxing existence constraints, adding many objectives, decreasing computation time, increasing portfolio size, reducing trading cost, finding new sources of information, new funding sources and maintaining the objectives of investors of risk return and liquidity. These researches success in providing mathematical and theoretical models that enriched the finance literature but few of it satisfies the market application. This study provides a historical background and future insight for future researches. As a revisit, this work derived from secondary sources. This revisit provides researchers and practitioners with the latest improvements of approaches by minimizing their effort in collecting the relevant material and selecting the suitable model that solve the problem of selecting their portfolio, a better understanding of each factor the limitations of their portfolios, draw attention to specific areas for further research.

Key words: Dynamic portfolio, portfolio selection, transaction cost, liquidity, information, agent action, short sell, portfolio size

INTRODUCTION

Portfolio selection is search for the best allocation of financial resources among a basket of financial assets. The mean-variance MV articulation by Markowitz (1959) offers an essential basis for portfolio selection in a single-period (Li and Ng, 2000). The seminal contribution in dynamic portfolio referred back to Merton (1972) as other researcher followed the pioneering works of Markowitz, he aims to find the optimal portfolio by extending his model to more than single period considering more investor risk preferences (Liu and Brige, 2012) and at the same time overcome the obstacles that faced Markowitz static model such as a high complexity in computing quadratic programming, the input problem in estimating the required parameters of variance and covariance (Mansini and Speranza, 1999) and its lack of dynamic adaptability to the variability in market environment (Alvord, 1981).

Multi-period proposed to take the advantages of attaining information about expected future returns needed in rebalancing the portfolio. Also, it takes in its consideration the objectives and liabilities that may face the long-term investors at a determined date in the future (Valian, 2009).

Therefore, optimizing a multi-period portfolio benefits from dynamic programming which found by Richard Bellman in 1940s in solving problems of selecting best decisions sequentially (one after another). They concluded that multi-period portfolio model performs better than single-period portfolio model for the investor in the long-run. The most important advantage from several of the multi-period model is improving the performance of the investment portfolio through the fixed mixed rule (Mulvey *et al*., 2003). Topcu *et al*. (2008) report in his study that nearly all of literatures actually dealing with not more than two periods' model and few assets when conducting multi-period model. While, Liu (2006) specified that the presence of a dynamic constraint in a continuous-time model will direct the averse investor to eliminate the exposure to risky assets more than it would have been chosen in case of its absence. Also, Skaf and Boyd (2008) comment that published papers deal frequently with portfolios consist of only two assets

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(one risky-stock and one risk free-bond) such as Liu (2006) and some of them employed multiple risky assets for example Cai *et al*. (2013). In addition, the choice of objective varies in these literatures between, maximizing utility is the typical choice (Akian *et al*., 2001) continuous versus discrete time (Davis and Norman, 1990; Ziemba and Vickson, 2006) and finite versus infinite horizon (Cai *et al*., 2013).

The decision of rebalancing the investment portfolio, generally based on three types of costs: the tracking error related to the deviation any portfolio from the optimal portfolio, the trading costs related to the transactions (buying or selling) of the assets while rebalancing the portfolio and the expected future cost from next month onwards given our actions in the current month. The optimal strategy dynamically minimizes the total cost which is the sum of these three costs (Sun *et al*., 2010). In dynamic portfolio optimization, optimal investment strategies are influenced significantly by the drift (variation) in the price process of the underlying asset. Conversely it is known the difficulty of estimating drift parameters from historical asset price data which is the prediction process of the asset price depends on receiving new information about the determined asset in this case, historical data of the asset price is information already reflected in the market. Hence, it is normal to comprise expert opinions or investors' views as additional stream of information in the calculation of optimal portfolios (Frey *et al*., 2014).

Multi-period models considered more complex comparing with single-period models, therefore some researchers attempted to introduce a solution for large-scale multi-period portfolio selection problem in the beginning years of Modern Portfolio Theory (MPT) as a progression of one period models for instance (Merton, 1972; Valian, 2009).

There is a large body of work on dynamic portfolio optimization with constraints (Skaf and Boyed, 2008). This study has an interest in the effects of these constraints on dynamic portfolio selection. This study attempts to review literatures related to dynamic portfolio selection and factors that affect dynamic portfolio selection.

DYNAMIC PORTFOLIO

The term of dynamic portfolio comprise from applying the characteristics of dynamic programming into portfolio management. Dynamic portfolio defined as a portfolio management strategy involves in rebalancing a portfolio in order to bring the asset mix back to its long-term target. Generally, these rebalancing involved reducing positions in the best-performing asset class while adding to positions in underperforming assets. The general premise of dynamic asset allocation is to reduce the fluctuation risks and achieve returns that exceed the target benchmark (www.investopedia). While, Wikipedia defined dynamic asset allocation as a strategy employed by structured investment products (mutual fund and index fund, etc.) to achieve exposure to various investment opportunities and provide 100% principal protection. It includes Constant Proportion Portfolio Insurance (CPPI) related to its components of a zero-coupon bond and an underlying investment where the assets in the portfolio shifted between them based on their performance. Even a borrowing facility can be used if the underlying investment products experience strong return to improve the exposure, otherwise, the CPPI automatically deleverages, reducing exposure in falling markets (en.wikipedia.org).

Valian (2009) identified the term of dynamic portfolio as the Dynamic Portfolio Model looks at the portfolio as a moving object to achieve a maximal expected utility for a given risk level and time horizon. Karamanis (2013) defined Multi-stage Stochastic Programming (MSP) for investor who considers the effects of the investment strategies for both short and long-term. In a situation of discrete time horizon, these strategies are achieved by accounting for an investment planning horizon where the investor has to make sequential decisions in order to achieve his goals at some date in the future.

Based on the previous definitions the researcher can give her definition as it is an investment strategy that aiming to achieve high total return while reducing the volatility risk via reallocate the components of the portfolio by holding the promising or underpriced assets by using the proceeds from and dispose the overpriced assets during a time horizon.

FACTORS AFFECTING DYNAMIC PORTFOLIO SELECTION

Several factors mentioned in the literatures of selecting dynamic optimal portfolio rather than the most attractive one of return and risk. The following this study will review the most important studies examine these factors:

Number of assets: Jobst *et al.* (2001) investigated the impact of applying transaction roundlot (discrete numbers of assets which are taken as the basic unit of investment), buying in threshold (minimum level below which an asset is not purchased) and cardinality (number of assets in the portfolio) constraints restrictions to the portfolio selection problem regarding Markowitz Model (MV) and the risk-return efficient frontier construction. Practically, these discrete constraints are important but cause a discontinuity for the efficient frontier. Researchers proposed an alternative model Quadratic Mixed-Integer Programming (QMIP) instead of quadratic programming which makes the estimating of efficient frontier is NP-hard computationally challenging. The limitation of the model is it's difficultly to handle trading constraints because they introduce discontinuities in the space of feasible portfolios. Difficulty arises when introducing additional classes of constraints (e.g., transaction costs) or new features in the model.

Liu (2006) shows that imposing a dynamic constraint in a continuous-time model leads an agent to select a smaller exposure to risky stocks than it would have been chosen in case of its absence.

Stoyan (2009) presents three mathematical algorithms to three well-know portfolio problem referred to risk-return portfolios, index tracking portfolios and an integrated stock-bond portfolio selection model. These approaches answers and spots the light on the portfolio selection question of money management problems where the developments in finance are carefully researched and examined by the investors especially after the financial crises 2007 that affected the global. Different sources of uncertainty are portrayed in a Stochastic Programming framework and Goal Programming techniques are used to facilitate various portfolio goals. The algorithms are tailored to each portfolio design and involve decompositions and heuristics that improve solution speed and quality. The first problem investigated limiting the number of shares in Markowitz risk verses return portfolio which is the most interesting and difficult constraint to include in the model. The resulted model compared with of Jobst *et al*. (2001) after its implement an algorithm proved its ability to handle problems larger than 65 times and produce respective efficient frontiers. The second problem presented a stochastic programming index tracking portfolio. The design of the model included a set of constraints such as minimize a tracking error, hold a small number of securities, minimize transaction cost, include uncertainty in the value of future security and possess a portfolio managing or rebalancing strategy at future time decision distribution that used a stochastic mix integer programming SMIP modeling structure to facilitate future uncertainties related to security prices and index values. The algorithm created to solve the model was specifically aimed at model structure and satisfying the names-to-hold constraint. Finally, the third problem concerned with portfolio design that involve rebalancing strategy over risky assets and risk free asset where portfolio goals (few number) set to match target value.

This kind of portfolio integrates uncertainty in stock price and portfolio in stochastic goal mix integer programming SGMIP. The large mixed integer programming MIP possessed a number of sub-problems that were sequentially solved then combined in an algorithm that produced competitive results with respect to various benchmarks. The advantage of this dissertation that each model mentioned is in the first of their nature and novel to the field.

Crama and Schyns (2001) investigate the optimization difficulty that rose when realistic side-constaints added to Markowitz Model. The portfolio describe the application of a simulated annealing approach to the solution of a complex portfolio selection model. The model is a mixed integer quadratic programming problem. The main objective of this study is to clarify the adequacy of simulated annealing for providing a solution of a portfolio optimization model difficultly. In this study, researchers investigated the ability of the Simulated Annealing metaheuristic (SA) to deliver high-quality solutions for the mean-variance model enriched by additional constraints. They made an attempt to emphasize the difficulties encountered designing the SA metaheuristic to the particular problem. Notice, in particular that his model involves continuous as well as discrete variables, contrary to most applications of simulated annealing. Also, the constraints are of various types and cannot be handled in a uniform way. The resulting algorithm allowed us to approximate the mean-variance frontier for medium-size problems within acceptable computing times. The algorithm has an ability to deal with more classes of constraints than most other algorithms found in the literature. Although, there is a clear trade-off between the quality of the solutions and the time required to compute them, the algorithm can be said to be quite versatile since it does not rely on any restrictive properties of the model. On the negative side, it must be noticed that the tailoring work required to account for different classes of constraints and to ne-tune the parameters of the algorithm was rather delicate. The trading constraints, in particular are especially difficult to handle because of the discontinuities they introduce in the space of feasible portfolios. Introducing additional classes of constraints or new features in the model (e.g., transaction costs) would certainly prove quite difficult again.

Transaction cost: Skaf and Boyd (2008) consider the problem of multi-asset multi-stage portfolio optimization problem over a discrete time as a stochastic control problem with minimizing the mean-square deviation to achieve a desired wealth. With the assumption of self-financing trade, no transaction cost and the trading policy is affine (total revenue from sale equals total cost of purchase) which can be solved by using standard dynamic programming.

When assuming suboptimal policy which holds additional constraints such as the presence of transaction costs or no-shorting constraint, the optimal policy becomes hard to compute and required a convex quadratic program in each stage using Bellman function to approximate the value of future portfolios. This study provides an example showing two suboptimal policies with and without transaction costs and/or no shorting constraints. This suboptimal policy is often achieve an objective value close to that for the associated problem without constraints and is therefore nearly optimal. In particular, suboptimal trading policy with transaction costs is perform almost as well as when there are no transaction costs. Simulations revealed that the more complicated suboptimal policy performs very well.

Brown and Smith (2011) studied dynamic portfolio optimization problem in a discrete-time, finite-horizon setting. This study attempted to propose a heuristic model with respect of risk aversion, return predictability, transaction cost and without shortening. A stochastic dynamic program formulated with non-zero transactions cost, number of the assets nearly equals the dimension of the state space. Number of easy to compute heuristics considered as a trading strategies, dual approach with upper bounds based on Brown and Smith (2011) was developed for investigating the superiority of these heuristics. In these bounds are given after considering the trader who has the ability to access perfect information on other hand penalized him for using these information. A variety of utility functions, transaction cost forms, constraint sets and different models for returns considered in this general approach. Monte Carlo simulation was used to evaluate the strategies and bounds by using numerical example with a risk free asset and three to ten risky assets. The results of the numerical experiments revealed that the heuristic strategy is extremely near the upper bound indicating that it is closely optimal. Furthermore, the experiments are capable in running time, even without software optimization and reasonable. At a high level, the key issue is to manage the trade-off between improving asset positions and minimizing transaction costs. This study received a critizem from Cai *et al.* (2013) that their method cannot give the optimal portfolios.

Moallemi and Saglam (2013) the central concern of this study is to provide a tractable framework for determining rebalancing rules in multi-period portfolio optimization problem. These linear rebalancing rule of past return for predicting factors can be used in a wide range of portfolio selection models with practical considerations for return predictability, transaction costs, trading constraints (short-sale constraints, leverage constraints or restrictions requiring market neutrality (or specific industry neutrality) and risk aversion. Also, it considers an optimal implementation problem where the liquidation of the investor position allowed over a fixed time horizon, under the transaction costs and a prediction of returns. An efficient computational procedure was made to measure and compare the performance of the various rules. The resulting optimal implementation problem does not permit an exact solution. Hence, a comparison made between the best linear policy and other tractable approximate policies including a deterministic policy, model predictive control and a projected variation of the linear quadratic control formulation of Garleanu and Pedersen (2009). By predicting of price movement of various policies a significant differences found. The TWAP policy, manage to minimize the transaction cost, fail in predicting the price movement then achieves the worst performance. Other policies acquire higher transaction costs than TWAP but not more than the gain from the opportunity of timing the liquidation relative to predictable price movements. Of the remaining policies, the projected LQC and optimal linear policies achieve the highest performance. The underlying dynamic portfolio optimization problem is a convex (globally optimal policy) programming problem and tractable numerically such flexibility can offer significant practical benefits. Linear rebalancing polices involved Single-period and deterministic policies and outperforms them in optimal rebalancing. The research concludes that the best linear policy attains superior performance to the alternatives. Furthermore, a number of upper bounds on the performance were computed for each policy in the problem, revealed that the best linear policy is near optimal with a gap of at most 5%. Moreover, this optimality gap is a factor of two better than the next closest policy.

This study implement numerical dynamic programming to optimize the problem of multi-asset dynamic portfolio involved small transaction cost (they considered bid-ask spread transaction, even it is small cost but with iterating rebalancing it becomes, theoretically, high in addition to the transaction fees representing by brokerage fees) (Cai *et al.*, 2013). Numerical example includes one riskless asset "bank account" plus two to six risky assets "stocks" traded during the period 7-360 traded periods in a finite horizon problem. The results show that the iteration of numerical value function able to solve multi-asset dynamic portfolio optimization problem with transaction cost in an efficient and precise way. The trading strategies were illustrated describing the no-trade regions for various alternatives of asset returns and transaction costs. May the numerical DP algorithms require intensive computation for large portfolio optimization problems but this problem solved by modern hardware? The advantage of their study is the number of risky asset is more than three and rebalancing period is greater than six.

Palczewski *et al*. (2014) introduced an efficient numerical approach to recognize the optimal portfolio strategies with state-depend-drift, long-time horizon and proportional transaction cost. Number of scenarios arise to explore investor behavioral biases reacting to the drift in three cases: drift unknown; trend-follower to the stock price movements; naive and ignore the available information. The numerical algorithm explains dynamic optimal portfolio strategies for time-horizons of up to 40 years. It is useful to measure the value of information and the loss from transaction costs using the indifference principle. The mathematical results revealed that forecasting behavior has a strong influence on trading in the existence of transactions costs. The benefits effect can be quantify of transaction costs. Transaction costs are most harmful to naive investors. The total loss in utility from proportional transaction costs is approximately twice times as direct cost incurred. In fact, learning decreases the losses in utility resulted from the uncertain drift and transaction costs, particularly for long horizons.

Information: According to Frost and Savarino (1986), the main object of this study is to reduce estimation error and maximize the portfolio return in optimizing the portfolio. Depending on the Bawa (1979) result saying: when portfolio optimization is executed by historical characteristics of stock returns, estimation error can corrupt the desirable properties of the selected investment portfolio. The problem raised in estimation the risk; expected return, variance and covariance with other security returns are required for each security, these measures are unknown and should be estimated either from available historical data or depending on subjective information. In this situation, it is accepted to use Bayesian framework when initiating portfolio selection rules that based on maximization of expected return conditioned on the predictive security returns distribution.

In order to reduce estimation error a non-informative diffuse prior has been addressed from most researchers however, it does not minimize the estimation error directly but minimizes the detrimental effect of estimation risk. A proposed informative prior derived from prior knowledge which is if the deviation between samples estimate of a parameter for a specified stock and the mean of that parameter increased, the possibility of estimating the sample estimate with error will increase for all stocks. The informative prior used to enhance the performance of the portfolio at the same time eliminate estimation error. This prior knowledge insures that all stocks have identical expected returns, variances and pairwise correlation coefficients. The posterior estimates that drawn for each stock's characteristics (expected return, variance and pairwise correlation coefficients) toward the mean of these characteristics reduces the estimation errors of all stocks in the population by a Bayesian adjustment factor.

Elton and Gruber (1987) questioned and want to know if all investors in the market are only informed about the grouping of the stock plus average characteristics of the stocks in a group at best, what knowledge one can gain from portfolio theory about optimal decisions? Other previous literatures in selecting optimal portfolio assumed that an agent received all required information to make his estimation of the expected return for each security, the variance and covariance matrix between securities to make his investment decision. This is a theoretical assumption and differs from actual practice where the investor just receives a list of discrete ranked securities (from one to five) and maybe some risk information.

Given this data how can an investor make optimal portfolio decisions? In their attempt to find the theory that maintain the process of decision making to be more useful, they explored a number of rules and reached to the following results. For example, if the grouping is based on expected return and no additional information is used:

- The whole groups are selected or rejected
- Select the whole group when its expected return exceeds the risk free rate
- Invest equal amount in each security
- Each security proportion to be invest is related to the excess return of a group

They set different rules for different alternatives result from different beliefs used to form the groups. In his study, he derived appropriate rules for alternative sets of grouping criteria that are used in the financial community without complex computation.

Nuaimi (2004) categorize the knowledge in two types explicit knowledge represented by the quantitative techniques and software and the implicit knowledge represented by the experience and skillful of the manager which cannot be participate or communicate. The portfolio manager as a decision maker is welling to acquire both types to trade of between risk and return to optimize his portfolio. Many models built-called as explicit financial knowledge to solve the portfolio models problems, the searching continued in this sector to develop these model through studying the model's variable representing in risk and return which one can express them by quantitative values not by guessing and hushing but the problem still unsolved. Here, the role of implicit knowledge appears to capture the portfolio features in term of return and risk.

Mansour *et al*. (2007) employed the imprecise GP to develop model for portfolio selection problem within a decision-making environment characterized by imperfection of the information. They built the most satisfactory portfolio seeking to integrate explicitly the portfolio manager's intuition, experience and judgment. The concept of satisfaction functions will be utilized to integrate explicitly the preferences of the portfolio's manager. The considered criteria in their model are as follows: the return, the risk and the liquidity of portfolios. In order to deal with the imprecision related to the model parameters, they express the goals as intervals. The developed model has been applied to portfolio selection within the Tunisian stock exchange market. They concluded that the model can be applied for cases with large size portfolio selection problems.

Valian (2009) in her dissertation introduced a model to get the optimal sequence of actions of a dynamic portfolio under uncertainty. These actions were measured based on estimated drifts. Uncertainty, parameters were taken into account because the price of the assets at specific moment is only accessible information for agents at this time since, the underlying Brownian Motion and the drift process of the asset prices are not directly observable (incomplete information). These parameters are measured as stochastic variables. Filtering theory was applied to obtain the unobservable rate of returns as an optimal estimator.

In order to set up the results, the optimal sequence structure of actions in the presence of incomplete information was formally defined in mathematical terms. Then, a comparison made between the optimal actions of the investor considering the error of predicted drift (uncertain parameters) of asset prices and the optimal actions of an investor considering fixed parameters. That means, the impact of the uncertainty of parameters was identified by comparing the solution of fixed parameters with the solution of uncertain parameters. This comparison revealed that the risk-averse agents usually forced by uncertainty of parameters to select a higher trading volume. However, these trading volumes may be lower in cases where the agent's assessment of drift is lower than the mean of drift.

Liu (2009) examine the impact of using high-frequency data on the portfolio optimization decision. Trying to locate a best estimator to reduce tracking errors and propose a solution for benefit of the professional investment manager who is looking for pursuing the S&P 500 with the 30 Dow Jones Industrial Average stocks (30 DJIA), a framework was built by constructing several covariance matrix estimators depending on the daily return an intraday return to execute the optimal portfolio. A significant result was found in which the type of data to be used is depending upon two factors the rebalancing frequency and estimation horizon. If the investor rebalanced his portfolio monthly for a 12 months daily return data, the daily return will act as the high-frequency data (intraday data) performed, potentially. The investor needs to make a substantial improvements to his portfolio by switching to high-frequency data if he rebalancing his portfolio daily or his estimation horizon <6 months.

The strength of this study is based on its application in real world. Although, prior literatures such as Merton (1972) valued estimating volatility based on high frequency return, this research constructed the volatility based on daily return rather than intraday return to avoid the problem representing with leptokurtosis in high-frequency data characteristics, autocorrelation in the returns, deterministic patterns and volatility clustering in intraday variance. At the same time researcher considered that the accurate estimation of the drift generally requires long spans of data, regardless of the frequency with which returns are sampled. The weaknesses of this study that researcher used the traditional risk measure variance to measure the return volatility when it is valid if return distribution achieve the normality. Researcher may use other measures of risk such as VAR it is more suitable with quadratic utility. Other thing he didn't take the transaction cost in his when it become necessity due to rebalancing.

Frey *et al*. (2012) investigated the strategies of optimal portfolio regarding expert's opinions as partial information and modeled the asset price as diffusion and derives its drift by Markov chain Y as hidden finite-state (continuous-time). In addition to the observation of the stock price made by investors at a marked point process and depending on current state of expert opinion Y, they can decide the Jump-size distribution. According to this process, Frey *et al*. (2012) develop a finite-dimensional filter p_t under complete information, more than that they formulate the equation of the value function V for dynamic programming problem and proposing a solution for it assuming that the equation accepts a classical solution. The results shows that the myopic strategy is very close to first approximation on the policy improvement then conclude that additional information permits for precise estimates for the drift which are close to the actual values. As a result, the investor has nearly full information on the drift and both, the optimal strategy under incomplete information and the myopic strategy are close to the optimal strategy under full information. The advantage of this study that researchers used external source of information to estimate drifts even they employed the continuous-time because drifts tend to fluctuate over time and still it is need a long time series to be estimated precisely. The model is theoretical and solved by example not applied to real world portfolio it may considered as disadvantage.

Frey *et al.* (2013) inspect the effect of drift of the market determined by unobserved Markov chain on the optimal portfolio strategies. The information was obtained on Markov chain from stock prices and expert opinions as signals at arbitrary discrete time points. By using stochastic filtering under full information as by Frey *et al.* (2012), the original problem transformed to optimization one and they used a state variable as the filter for the Markov chain. This problem was investigated with dynamic programming techniques and regularization arguments in order to overcome the main difficulty accompanied the classical solution of the dynamic programming represented by the shape of the equation where it is not strictly elliptic if the number of states of Y is larger than the number of assets.

In order to complete what they began in their previous study, this study grows from unpublished paper of Frey *et al.* (2013). Frey *et al.* (2014) addressed the same problem with dynamic programming equation but investigate it in addition to regularization argument a viscosity solution technique. In analyzing the dynamic programming equation there is a major challenge related to number of states of Y, the challenge began when number of states of Y exceeds the number of assets affecting the elliptic shape of the equation. Due to this difficulty, there is no possibility to reach the classical solutions to this equation by employing any of the known results. Therefore, they studied two ways to deal with this problem. First, by following the results revealed that the value function of the related dynamic programming equation is a viscosity solution. However, the methodology of the viscosity solution does not offer any information on the (nearly) optimal strategies. The second approach conducted using regularization arguments additional noise term. The results show an existence of classical solution for the dynamic programming equation accompanied with the regularized optimization problem. Finally, the optimal strategy for the regularized problem can be characterized as solution of a quadratic optimization problem that involves Vm and its first derivatives. But, it is not sufficient to proof the corollary.

Palczewski *et al.* (2014) offer an efficient numerical algorithm to solve the optimal portfolio problems under certain condition of state-depend-drift, long-time horizon and proportional transaction cost. This scenario develops when investors have a bias behavior or drift is known to him. The numerical algorithm solved dynamic optimal portfolio problems for time-horizons of up to 40 years. The mathematical results revealed that forecasting behavior has a strong influence on trading in the existence of transactions costs. Using the indifference principle, the value of information can be measured of transaction cost. Information is most valuable to the least risk-averse investor.

Liquidity: Parra *et al.* (2001) studied a multi-objective portfolio problem consists of three criteria: risk (variance), return (expected return) and liquidity for a private investor and discussed how to develop technical decision rules for buying, holding and selling both risky and riskless assets over time in consistency with investor's preferences. Researchers assumed no considerable loss in converting the investment into cash and the investor desires greater liquidity. A capable model assists the private investor to locate his efficient portfolio that verifies his preferences resulting from their study. Also, they determine the target value of the three criteria for each type of investor. The risk averse investor should exposure to small target value of risk; on the contrast, the risk seeker investor demand higher level of risk and profitability target at the same time lower target of liquidity. Furthermore, the model able to find out infeasible combination and helps the investor in improving his expectations.

Agent action: Valian (2009) developed a model to comprehend the optimal action of non-price taking agent with and without debts. These agents regard how to select their trading strategies considering their price impact to maximize their objective function (expected utility), depending on their own action in addition to other traders' actions. The agent is full of aware of both facts that its action has an impact on its payoff at a given period and the available opportunity in the future. A game Theory was applied to figure out the optimal action of the large agent during infinite time period. Uncertainty is considered because the return of most assets is uncertain and the agent has no information about the probability of the future return, besides the unavailability of information about other agents' action within the environment. Each large agent considers some constraints forced by its information because of the limitation on information that alters the agent's behavior. The availability of information classified into two categories; imperfect information game and a perfect information game. The results of the study revealed that higher level of debts obliged the agent i to sell more regardless information from his competitor agent j as an optimal action. An agent turn into bankrupt when he cannot settle his debt obligations, it occurred when the marginal returns much lower than extra output.

Short sell: Elton and Gruber (1987) developed a multi-grouped model and index model employing quadratic programming and compared between both models in presence of short selling and when it is not allowed. He attempted to solve the problem comes up from the complexity of producing inputs to the portfolio model in its general form, resulting in the difficulty in training portfolio manager to manage risk-return trade off time and cost accompanying solving quadratic programming. The research has been applied in the real world choosing a sample consists of chemical and steel stocks. The research got the following results about the characteristics of stock to be included in optimal portfolio: in case of multi group short sell is allowed: if the excess return over standard deviation is greater than the group constant the stock should be bought, if not sold short. The optimum amount to invest M_0 in each security is determined easily by scaling the objective function $(Z_i/\Sigma N_p)$.

In case of multi-group short sell is not allowed: all securities with higher excess return to standard deviation gained a positive amount Z_i will be in the optimal portfolio, otherwise not included. The optimal amount should be invest in each security is determined by dividing each objective function of each group by the sum of objective function for all groups ($Z_{iz}/\Sigma Z_i$'s). In case of multi-index short sell is allowed: researcher recommended that this case is negotiable.

In case of multi-index short sell is not allowed: determined that:

- Include all securities with positive β_i and ratio of excess return to beta i greater than the cut-off point
- Exclude all securities with positive β_i and ratio of excess return to beta i less than the cut-off point
- Include all securities with negative β_i and ratio of excess return to beta i less than the cut-off point
- Exclude all securities with negative β_i and ratio of excess return to beta i greater than the cut-off point

In considering new security to include in optimal portfolio the cut-off point can determine quickly the effect of new security on the optimal portfolio by following:

- If the value of excess return over beta $[(R_i-R_f)/\beta_i]$ for new security is less than the constant value of the group, it can be safely discarded
- If the value of excess return over beta $[(R_i-R_f)/\beta_i]$ for new security is more than the constant value of the group, it must be included and optimum recalculated. Yet, new calculation may need but the amount of computation is very small

This study derives its strength from the developed decision rules that permits the investor to find the optimum solution for his portfolio without any complicated mathematics in addition to the implicit and facile calculation of security characteristics that made it desirable. This study offers and formally validates a simple ranking approach for short selling in optimal portfolio selection under institutional procedures (Kwan, 1995). It also grants economic insights of the clear solution of the portfolio problem. This approach is appropriate to different treatments of the short-sale incomes and any margin deposits. This approach does not involve the assumption that exaggerate short-sell gains to maintain the analytical tractability like previous approaches and it has the ability to identify and filter out stocks that are unattractive for investing or for short selling. Thus, this study can improve the effectiveness of portfolio modeling for supporting practical investment decisions.

Jacobs *et al*. (2005) presents a fast model for tracing out efficient sets of portfolio which consists of large number of components, when a factor, scenario or particular historical model of risk (covariance) is assumed at what time investor allowed selling his stocks short. Currently, with controlled other conditions. This algorithm is valued because other models such as Monte Carlo algorithm needs many reoptimization and may simulation run when the investor have no time to do this duty to take his decision of rebalancing his portfolio.

The results show according to the "Property P" is the sufficient condition that assures the existence of (originally long-only) factor or scenario code will compute the correct answer to the long-short problem. The long-short model is achieved when risk algorithm (factor or scenario) is assumed and Property P is satisfied. Here, no needs for new programming. The long-only program generates the right answer to the 2n-variable long-short problem, in spite of the "error" in assumption. Also, the fast algorithm for historical covariance matrices (when the number of stocks greatly exceeds the number of observations) generate right answers to the 2n-variable long-short problem whether or not property P holds.

The strengths of this study are the fast model includes more than one constraint referring to the real world circumstances, the hypothetical assumption of the economy models representing in "one can sell stock short without limit and use the proceeds to buy stocks long" is unrealistic. Also, fast algorithm modeled the constraints faced from brokers, clients, regulations and investor himself related to the long-short portfolios variation over time and at a given moment, On the contrary of other literature such as Elton and Gruber (1987). The results of this study generalized attributable to the results of Kwan (1995). But, their results still restricted since the model not applied in real world condition.

CONCLUSION

Various researchers have been conducting studies of dynamic portfolio selection to solve the problems faced by the market traders such as the size of the portfolio and the proportion invested in each asset, the investment amount and the ability to leverage or borrow it, availability of information to estimate the required investment measures, the accuracy and speed of the used model, the cost of the investment process which increase with rebalancing, predictability of other agent actions and portfolio liquidity. This study reached to the following results and conclusions the investors become more aware about the investment process and the innovations in modeling the problem and factors including in it, especially after financial crises of 2007 for that researcher proposed a flexible and robust model in dealing with uncertainty comparing with othe models and limiting the size of portfolio (Stoyan, 2009), some proposed a model but not tolerance additional constraints (Jobst *et al*., 2001), the effect of portfolio size on the dynamic portfolio selection represents by if the portfolio size increased the friction will increase in the presence of multi-period which will affect the profit at the same time increase the complexity of the model in turn will affect the CPU even with the existence of software. Therefore, there is a direction to limit the portfolio size.

The effect of transaction cost on dynamic portfolio selection: the more the constraint in the model the more complexity and difficulty of solving this model, the tendency to decrease number of constraint. While, researchers build mathematical models they aimed to relieve the effect of increasing number of constraint, especially transaction const. Some of researcher success in that for example, Skaf and Boyd (2008) proposed a model and examine it, results that the model act well in presence of transaction cost as absence comparing with the determined upper bound. Brown and Smith (2011) introduce model with large number of assets, capable in running-time software not required but this model not reach optimal portfolio. The gap in the model of Moallemi and Saglam (2013) in the presence of transaction cost is 5% comparing with upper bound. Cai *et al.* (2013) in his model fail to manage the number of risky asset is more than three and rebalancing period is greater than six without modern hardware. Transaction costs are most harmful to naive investors (Palczewski *et al*., 2014). The total loss in utility from proportional transaction costs is approximately twice times as direct cost incurred. In fact, learning decreases the losses in utility resulted from the uncertain drift and transaction costs, particularly for long horizons.

The effect of information on dynamic portfolio selection: some of the article concludes that risk-averse agents usually forced by uncertainty of parameters to select a higher trading volume. Other report, the least risk-averse investor is most valuable from information. Frey *et al.* (2012) and Mansour *et al.* (2007) valued the subjective information as alternative source of information. Elton and Gruber (1987) derived appropriate rules for alternative sets of grouping criteria that are used in the financial community without complex computation. Liu (2009) advice to use high frequency data when the investor rebalance its portfolio daily or the estimation length <6 months.

The effect of liquidity on dynamic portfolio selection: the risk averse investor should exposure to small target value of risk; on the contrast, the risk seeker investor demand higher level of risk and profitability target at the same time lower target of liquidity. The effect of short sale on dynamic portfolio selection: the investor deal with this condition depending on his expectation that the stock price will decrease in future. Therefore, he borrow it now from his agent and return it back when price decrease but the agent has the right to call his stock at any time, the risk is appear in this situation. Elton and Gruber (1987) model do not need complex calculation, example if the excess return over standard deviation of one group is greater than the group constant the stock should be bought. Jacobs *et al.* (2005) presents a fast model to trace the optimal portfolio without further computations.

This study concludes that all researchers do their efforts to facilitate the investors' mission by introducing the different mathematical approaches in different conditions and different point views three of the reviewed studies applied on the real portfolios which form a small percentage while the other stands as a theoretical models. This gives the researcher of this study an insight to propose future implementations for these approaches to prove their validity. This effort may help the aware investor to choose and benefits from a collection of relevant information and models. This study provides a broad view of historical and modern studies in this topic which save his time and efforts, at the same time open the door for further research such as use other risk measure than variance such as MAV and apply the previous models in real markets.

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