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Optimized Design of Highways Network Taking into Account Land Relief, on the Basis of Steiner's Problem

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Abstract: Expansion of road networks is a perspective direction of development of economic capacity of a country. Application of new methods for construction optimization positively affects increase in a road network, thus the volume of resources decreases. Possibility of application of Steine's problem during construction and reconstruction of highways, taking into account topographical features and cost of construction is studied in the article. Practical application of method on the basis some settlements of the Republic of Tatarstan of the Russian Federation is considered.

Key words: Road networks, roads engineering, relief, steiner problem, steiner point

INTRODUCTION

The question of optimization of expenses at construction of highways is quite often discussed in practice of road design. And not only reduction of resource costs of construction is considered but also improvement of traffic management without harm for its participants. Possibility of application of various methods allowing to carry out objectives and at the same time optimizing expenses is the priority direction of development of road-building branch.

All constructions can differ seriously from each other depending on intensity of traffic, relief, hydro-geological and climatic conditions of the territory. It significantly limits possibility of application of standard projects, demands their careful binding to specific conditions. All this leads to high resource expenses: material, labor and in a consequence, monetary. There is a need of application of new methods of design of the road sections considering specific conditions and features of the district.

We will consider use of Steiner's problem (Bern and Graham, 1989) for design of roads. The problem consists in connection of some set points into the shortest network and if necessary possibility of addition of additional points-Steine's points differing from initial ones. We will design a new road site, using Steiner's problem taking land relief into account.

MATERIALS AND METHODS

We investigate possibility of application of Steiner's problem in road construction, we will make calculations in spherical coordinates system. Various systems are used for definition of the district of objects on a terrestrial surface: spatial rectangular coordinates, geodetic coordinates, flat rectangular geodetic coordinates (Maling, 2013). The surface of Earth has very difficult form, depending on the solved tasks, it is approximated by some figure which is easier to describe mathematically for example, a sphere or an ellipsoid. For the solution of many problems of various fields of knowledge (geodesy, navigation, cartography) the form of Earth is taken for a sphere (Dragomir *et al*., 2013). A coordinates of points on the sphere are defined by geographical coordinates in our algorithm: longitude and latitude. Spherical latitude (φ) is a corner between the plane of the equator and the direction from the center of the terrestrial sphere on the point. Spherical longitude (λ) is a dihedral angle between the plane of an initial meridian and the plane of the meridian passing through the point (Ran *et al*., 2007). It is possible to calculate distance between two points on the sphere which is determined by a formula according to laws of spherical geometry (Eq. 1):

$$\cos(d) = \sin(\varphi A)\sin(\varphi B) + \cos(\varphi A)\cos(\varphi B)\cos(\lambda A - \lambda B) \quad (1)$$

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Where:

φA and φB = Latitudes
λA and λB = Longitudes of these settlements
d = The distance between points measured in radians

The distance between points L (km) in formula L = d×R where R = 6381 km the average radius of the globe. It is necessary to consider that the signs "plus" and "minus" at the corresponding latitudes or longitudes have to be different when calculating distance between points located in different hemispheres (Southern or Northern, Eastern/Western).

We will consider algorithm of application of Steiner's problem in spherical coordinates system. We will find Steiner's point using modification of aspiration method (Orlov, 2009). Geographical coordinates of points are given. All Steiner's points are located on the short arch tracing radius circles:

$$R' = L \times \frac{\sin(\frac{\pi}{3})}{3} \quad (2)$$

where, L is the distance between two points on the sphere. Thus, the center of a circle is distanced R'/2 from the middle of a chord of a circumscribed circle. Aspiration point is a point of intersection of a long arch of a circle from the straight line passing through the center of this circle and perpendicular to a piece between two tops (Orlov, 2005). Analog of a straight line on the plane is the geodetic line on a surface. The geodetic line is a shortest curve which can be found on a surface between two points (Liu *et al*., 2015). Its coordinate functions $u^i = u^i(t)$ satisfy to Eq. 3 (Golovanov, 2002):

$$\frac{d^2u^m}{dt^2} + \Gamma_{ij}^{m}\frac{du^i}{dt}\frac{du^j}{dt} = 0 \quad (3)$$

where, Γ_{ij}^{m} Christoffel's symbols of the 2nd sort are in Eq. 4 (D'haeseleer *et al*., 2012; Brull, 2012):

$$\Gamma_{ij}^{m} = \frac{1}{2}g^{mk}\left(\frac{\partial g_{kj}}{\partial u^i} + \frac{\partial g_{ik}}{\partial u^j} - \frac{\partial g_{ij}}{\partial u^k}\right) \quad (4)$$

Geodetic line on the sphere is an arch of a big circle but only those sites which don't surpass its semi-circle will be shortest (Leonhardt and Tyc, 2009). The shortest is the arch which will be <180°. If the arch is >180°, it will be more than semi-circle. We will consider calculation of Steiner's point for three points on the sphere. We will bring known algorithm of Steiner's problem for

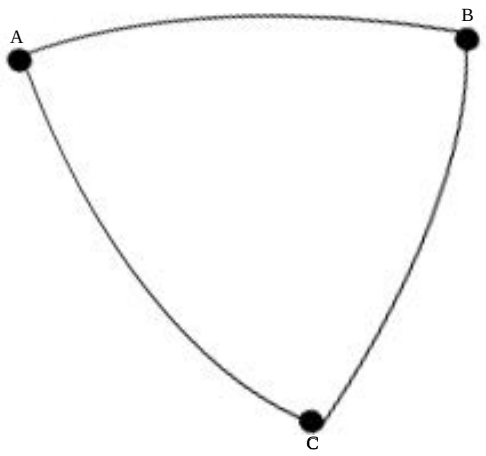

Fig. 1: Spherical ABC triangle

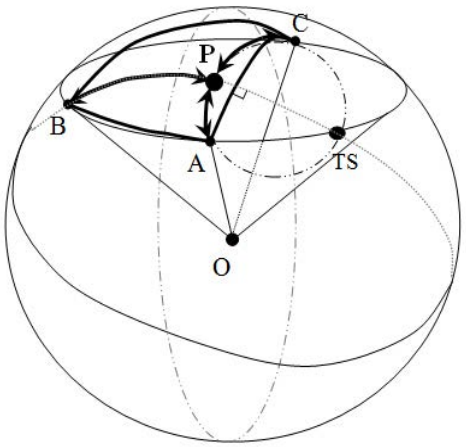

Fig. 2: Steiner's P point for three points on the sphere

3 points (Yang *et al*., 2013; Marcelo, 2012) into spherical coordinates system. There are spherical coordinates of 3 points A-C. A spherical triangle is formed in the point of connection of these points (Fig. 1) (Donnay, 2013). These points are connected by arches of a big circle-geodetic lines on the sphere.

Algorithm of finding of Steine's point for three points on the sphere is based on finding points of aspiration on a circumscribed circle. Considering the fact that it is possible to circumscribe a cone on the sphere round any trihedral corner, the point of aspiration will be on a big arch of its basis. Thus, finding Steiner's P point (taking into account all conditions of existence of this point) is reduced to search of a point of intersection of a small arch of a circumscribed circle with the geodetic line, the connecting point of aspiration of TS and one of coordinates, for example, B points (Fig. 2).

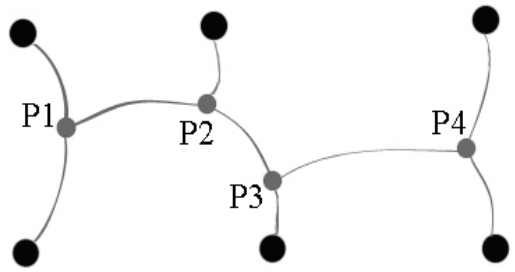

Fig. 3: Steiner's points P1-4

We find distance to Steiner's point on the Eq. 1 given above. Total value of distances of AP+BP+CP is the shortest way connecting the initial points. It is also possible to calculate position of points for bigger quantity of points. But thus graphic display of the solution of Steiner's problem will be complicated. So, no >4 Steiner's points P1-4 (Fig. 3) are the decision for 6 tops on the sphere (conditions of quantity of Steiner's points (n-2) (Du *et al*., 2013).

Depending on a relative positioning of the connected initial points, Steiner's point can be one or be absent (Protasov, 2005). Calculation of distance by this method is more effectively and in many cases is more exact, than its calculation for the designed coordinates (in rectangular systems of coordinates) as first for this purpose it isn't necessary to transfer geographical coordinates to rectangular system of coordinates (to carry out projective transformations). Secondly, many projections, if they are incorrectly chosen can lead to considerable distortions of lengths.

RESULTS AND DISCUSSION

It is known that the price of construction varies depending on the cost of construction materials, labor costs of workers, operation of construction cars and mechanisms and other factors. Undoubtedly, land relief on which road construction is planned has a great influence on the price. We will design a new road site, using Steiner's problem taking the land relief into account.

There is an active development of low construction in the territory of the Republic of Tatarstan, cottage settlements of various type are built. Each of new settlements needs to be connected with road network.

We will consider application of Steiner's problem taking into account features of the relief on the example of several cottage settlements of Verkhneuslonsky municipal area near Kazan. The following settlements were chosen: Solnechnaja Dolina (No. 1) and Novyj (No. 3). These settlements have no paving at the moment. It is necessary to connect them with network of roads, thus considering possibility of arrival to the village named after Kirov (No. 4) and the village Verkhny Uslon (No. 2) with infrastructure objects such as medical aid station, bank offices, mail, shops. The Verkhny Uslon village has an asphalt-concrete surfacing.

The decision to make an intersection-an arrival place to the new settlement was made when constructing the road to the village Verkhny Uslon and considering the forest close to the settlement Solnechnaja Dolina. In this regard, one more point N-the intersection connecting to the road to Verkhny Uslon (Fig. 4) is defined.

We will use algorithm of finding the shortest network of roads taking into account a land relief. Use of additional points-Steiner's points depends on a number of conditions (Isavnin and Sharipov, 2015). It is also necessary to consider and expediency of their addition at minimization of distances between objects, therefore there can possibly be no Steiner's point.

Costs of construction of highways depending on character and category of a land relief vary. For example, the cost of a road in the district with difficult natural (hydro-geological) conditions is higher, than a road of the same category on the flat district. Considering direct dependence of cost of construction on the extent of a site and topographical features of the district, we will count the weight of edges between the initial points from influence of the specified values, to choose a way with the minimum cost. We will carry out the solution of Steiner problem, using known algorithm (Orlov, 2009) with the modification of scales of edges offered by us. We will find Steiner's points for the points chosen by us and we will lay a new way. The following route (influence of the district on the construction price wasn't considered) was laid when using classical algorithm of finding Steiner's point (Fig. 5). Total length of construction of the road on sites (Table 1) was calculated.

Applying the algorithm of finding solution Steiner's problem presented by us taking into account the district, volumes of construction can be reduced when using already available way paved to the village Verkhny Uslon (N-M) and characteristics of relief of estimated construction site (Fig. 6).

Thus, the presented algorithm bypassed expensive construction through the wood, using an already constructed N-M Road instead. Construction volumes will be reduced by 1,59 km and will make 3,35 km, the cost of construction will decrease by 366,958,56 rub. (Table 2 and 3).

Table 1: Length of a new site

Area	Length (km)	Relief	Construction cost (rub.)
Solnechnaja Dolina-N	1,09	Flat	31006 576
N-P	1,6	Gentle terrain	45 514 240
P-Kirova	0,64	Flat	18 205 696
P-Novyj	0,73	Flat	20 765872
Novyj-Verkhny Uslon	0,58	Gentle terrain	16498 912
Total	4,64	-	131991 296

Table 2: Site length taking into account the available road

Area	Length (km)	Relief	Construction cost (rub.)
Solnechnaja Dolina-N	1,09	Flat	31006 576
N-M	1,56	Gentle terrain	45 229776
M-P	0,31	Gentle terrain	8 818384
P-Kirova	0,64	Flat	18 205 696
P-Novyj	0,73	Flat	20 765872
Novyj-Verkhny Uslon	0,58	Gentle terrain	16498 912
Total	4,64	-	131991 296

Table 3:Length of a new site

Area	Length (km)	Relief	Construction cost (rub.)
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P-Novyj	0,73	Flat	20 765872
Novyj-Verkhny Uslon	0,58	Gentle terrain	16498 912
Total	4,64	-	95 295 440

Fig. 4: Position of settlements on the Google Earth map

The cost of construction of the road of the 4th category (intensity of traffic about 200-1000 cars day^{-1}) on the specified site taking into account a relief (flat and gentle terrain district) will make 95,295,440 rub. (the price is calculated according to the state integrated standards of construction in the Russian Federation on 2016).

Fig. 5: Position of Steiner's point P and new road network on the Google Earth map

Fig. 6: Position of a new road network taking into account the available roads on the Google Earth map

CONCLUSION

So, use of this modification of Steiner's problem allows to consider not only length of a route but also the district where construction of the highway is planned during construction of the shortest network. The specified algorithm is based on the function influencing the weight of edges between points depending on crossing of an edge of certain sites of the district demanding various resource expenses. There is a calculation of distances between points already taking into account the additional weight of edges and the way with minimum weight, possibly bypassing difficult sites. For example, if the highway passes through the mountain district or bogs demanding big resource expenses at construction on a certain site of a construction object, it would be expedient to consider possibility of their round, having laid a way on less resource-intensive site. Thus, the shortest route won't always be optimum, criterion of an assessment of the received result will be the total cost of the whole site. It allows to define an arrangement of the new road taking into account economic rationality of construction more precisely.

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