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Power Transmission Improvement in Distribution Feeders By Shunt Capacitor Banks

[1]Abdellatif Hamouda and [2]Khaled Zehar
[1]Department of Mechanics, [2]Department of Electrical Engineering,
QU.E.R.E Laboratory, University Ferhat Abbas, 19000 Setif Algeria

Abstract: The aim of this study is the presentation of a new analytical formulation of the reactive energy compensation on distribution Feeders, which are characterised by their radial configuration. This study will be devoted to the determination of the optimal batteries size and location of a non-homogeneous main feeder without laterals. Due to the line resistance and reactance, the voltage is non-constant and then, for the network solution, it is required to know the voltage at each node and at the batteries location. On the basis of what has been just said and due to the radial type of the line, an iterative method based on the backward and forward sweeps is applied for the voltage calculation. The voltage rms values and phase-angles at all the nodes and on the capacitor banks are calculated. The mathematical models of the current distributions are made considering the line active and reactive power losses. New expressions for the power and energy loss reductions are used in the reactive energy optimisation process. The method is implemented and the results obtained then, are compared to those of authors having previously worked on the subject.

Key words: Shunt battery, voltage drop, radial line, capacitor size, capacitor location

INTRODUCTION

The transit of a strong reactive component of the current in an electrical line causes power losses, voltage drop and thus a reduction of the line power transmission. Compared to transmission lines, the distribution ones have a low voltage and high current. The loss in distribution systems is than significantly high. To improve the line power transmission and to avoid turning to investments in new distribution lines, the power utilities are firstly forced to reduce the losses in distribution systems. To achieve power and energy loss reductions as well as voltage correction, shunt capacitor banks are widely used.

There are various techniques to optimally install shunt batteries among of which we quote; the analytical methods[1-7] which are easy to understand and implement. The numerical methods[8-11] which are iterative methods with or without constraints. The heuristic methods[12-14], developed through intuition and experience. The latter, are a fast practical way which leads to a near optimal solution and reduce the search space. Artificial intelligence[15-20] based on natural evolution and the annealing of solids. These methods can also be a combination of a set of methods (hybrid methods). In this class of methods we note: Genetic algorithms[15-17]; fuzzy logic and genetic algorithms[18]; simulated annealing[19-20].

Our interest in this study relates to the analytical methods. Several methods have been used to conduct the reactive energy compensation in an optimal way i.e., to have less power and energy losses. If the early analytical works[1-4] constituted an important step in the modelling of the optimisation of the reactive energy compensation, they remain however non-realistic because of the number of assumptions that they considered. Some of these assumptions are uniform line, uniform load and constant voltage along the line. Later works based on heuristic search[12-14], although they do not make any simplifying assumptions, tackle the problem by initially identifying the possible nodes candidate to carry batteries, then determining their optimal sizes. The problem is thus reduced to a battery sizes optimisation. The advantage of this method lies in the fact of being easy and very practical especially for the feeders with laterals. It makes possible to determine the number of the batteries as well as their locations. The latter are the more indicated but not the optimal ones. Moreover, for more than one battery, the effect of the batteries the ones on the others does not appear clearly in the objective function for a possible mathematical analysis of the problem. It appears implicitly in the reactive current updating when we perform the load

Corresponding Author: Abdellatif Hamouda, Department of Mechanics, QU.E.R.E Laboratory, University Ferhat Abbas, 19000 Setif Algeria

flow. References[5-7], in our point of view, present best and complete mathematical models for leading the reactive energy optimisation for radial distribution feeder without laterals. Indeed, this method allows the electrical energy suppliers to choose either to optimise only one parameter (batteries location or size) or both of them at the same time. Nevertheless and because of the differences in the definition of the power and energy loss reductions[5-7] and in the formulation of the battery current[7], we present in this study the equations whose results conduct, according to studies, to the batteries optimal size and the batteries optimal location. However, objective function as well as a simple manner to calculate the nodes voltage and the voltage on the batteries are firstly given.

OBJECTIVE FUNCTION

To optimise the reactive energy compensation, the definition of an objective function is essential. This function called economic savings and noted S, is defined by Grainger[5-7]:

$$S = k_p \Delta P + k_e \Delta E - k_{cf} \sum_{k=1}^{n} Q_{ck} \tag{1}$$

By using the battery voltage and courant, expression (1) becomes:

$$S = k_p \Delta P + k_e \Delta E - k_{cf} \sum_{k=1}^{n} \frac{V_{ck} I_{cqk}}{\cos \varphi_{ck}} \tag{2}$$

Equation 2 shows that " S " is dependent on the batteries current and voltage. This function depends also on the locations of the batteries which appear explicitly in the expression of the power and energy loss reductions given below. These expressions are for a radial main feeder on which n batteries numbered, for the calculation suitability, from the end of the line to the substation end Fig. 1, as it follows:
For the power loss reduction.

$$\Delta P = 3 \sum_{i=1}^{n} \Delta P_i$$

where ΔP_i is given by:

$$\Delta P_i = 2 R I_{cqi} \int_0^{h_i} \left(I_s(t) F_q(x) - \sum_{k=1}^{i-1} I_{cqk} \right) dx - 2 R I_{cqi} \sum_{k=i+1}^{n} I_{cqk} h_k - R h_i I_{cqi}^2 \tag{3}$$

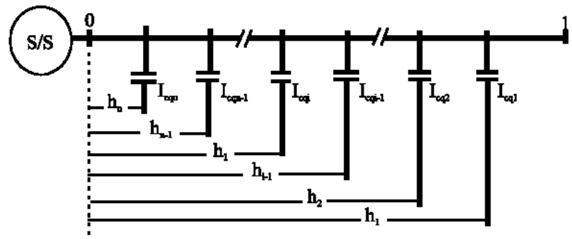

Fig. 1: Batteries current and location numbring

For the energy loss reduction.

$$\Delta E = 3 \sum_{i=1}^{n} \Delta E_i$$

ΔE_i being equal to the integral between 0 and T of ΔP_i, admits the following expression:

$$\Delta E_i = 2 R I_{cqi} T L_f \int_0^{h_i} I_s F_q(x) dx - 2 R I_{cqi} h_i T \sum_{k=1}^{i-1} I_{cqk} - 2 R T I_{cqi} \sum_{k=i+1}^{n} I_{cqk} . h_k - R T h_i I_{cqi}^2 \tag{4}$$

VOLTAGES CALCULATIONS

The load and battery currents are voltage-dependent thus, the calculation of the complex voltages is necessary. However, distribution networks being characterized by a high ratio R/X and a radial configuration, it is not recommended to use conventional load flow methods such as Gauss-Seidel or Newton decoupled which were essentially developed for transmission or strongly meshed networks. Applied for distribution networks, they can encounter convergence problems. In this study and for the voltage calculation, we suggest a method which we call voltage drop method. This method is accomplished in two steps, a backward sweep step for the branch current calculation and a forward sweep to determine the voltage. The complex voltage at the bus "i" is defined equal to that of the preceding bus i-1 decreased by the branch voltage drop located between the two considered nodes. For the node « i » Fig. 2, the complex voltage is written as it follows:

$$\bar{V}_i = \bar{V}_{i-1} - (r_i + j x_i) \left[F_{di} - j(F_{qi} - F_{ci}) \right]$$

Where the current distributions are given by:

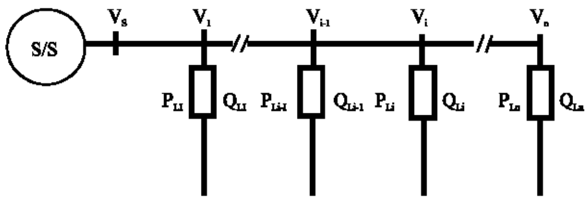

Fig. 2: One-line diagram of a n-node feeder

$$\begin{cases} F_{di} = \dfrac{P_i \cos\varphi_i + Q_i \sin\varphi_i}{V_i} \\ F_{qi} = \dfrac{Q_i \cos\varphi_i - P_i \sin\varphi_i}{V_i} \\ F_{ci} = \sum_{k=1}^{i} Icqk \end{cases} \tag{5}$$

The d and q component of the current due to a battery connected to the bus k are given by:

$$\begin{cases} I_{cdk} = -\dfrac{Q_{ck}}{V_{ck}} \sin\varphi_{ck} \\ I_{cdk} = \dfrac{Q_{ck}}{V_{ck}} \cos\varphi_{ck} \end{cases} \tag{6}$$

According to the one-line diagram of a line section between two consecutive nodes Fig. 3, we can write for the active and reactive powers injected into the ith node:

$$\begin{cases} P_i = P_{i+1} + P_{Li} + P_{lossi+1} \\ Q_i = Q_{i+1} + Q_{Li} + Q_{lossi+1} \end{cases}$$

The line active and reactive power losses of the branch i+1 are:

$$\begin{cases} P_{lossi+1} = r_{i+1} \dfrac{P_{i+1}^2 + Q_{i+1}^2}{V_{i+1}^2} \\ Q_{lossi+1} = x_{i+1} \dfrac{P_{i+1}^2 + Q_{i+1}^2}{V_{i+1}^2} \end{cases}$$

The d and q voltage-components, using the uniform normalised line concept used in Gainger[5-7], are:

$$\begin{cases} V_{di} = V_{di-1} - RL_{uni} F_{di} - x_{ni} L_{uni} F_{qi} + x_{ni} L_{uni} F_{ci} \\ V_{qi} = V_{qi-1} - x_{ni} L_{uni} F_{di} + RL_{uni} F_{qi} - RL_{uni} F_{ci} \end{cases} \tag{7}$$

Once the d and q components calculated, the voltage rms value and phase-angle of node i are obtained by:

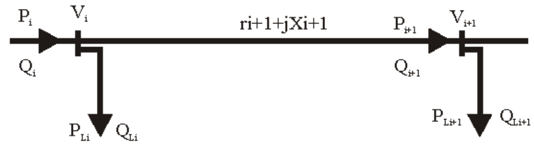

Fig. 3: Branch one-line diagram

$$\begin{aligned} V_i &= \sqrt{V_{di}^2 + V_{qi}^2} \\ \varphi_i &= \mathrm{artang} \frac{V_{qi}}{V_{di}} \end{aligned} \tag{8}$$

The kth battery complex voltage is equal to that of bus i if it is located on it. If the battery location is between the buses i-1 and i, its d and q components are given by:

$$\begin{cases} V_{cdk} = V_{di-1} - R(L_{nni} - h_k)F_{di} + X_{ni} \\ (L_{nni} - h_k)F_{qi} + X_{ni}(L_{nni} - h_k)F_{ci} \\ V_{cqk} = V_{qi-1} - X_{ni}(L_{nni} - h_k)F_{di} + R \\ (L_{nni} - h_k)F_{qi} + R(L_{nni} - h_k)F_{ci} \end{cases} \tag{9}$$

From d and q components we get:

$$\begin{aligned} V_{ck} &= \sqrt{V_{cdk}^2 + V_{cqk}^2} \\ \varphi_{ck} &= \mathrm{artang} \frac{V_{cqk}}{V_{cdk}} \end{aligned} \tag{10}$$

To determine both voltage magnitude and phase-angle, one will initialise all the complex voltages to that existing at the substation end (bus reference) and calculate initially the current distributions according to (5) by going up the line (backward sweep). Then d and q voltage components, in agreement with the expressions (7) for nodes and (9) for batteries, are calculated by going down the line (forward sweep). The method being an iterative one, the computing process will be stopped only if the results converge. As a convergence test, we have adopted a per-unit difference of voltages of two successive iterations equal or less than 0.0001.

OPTIMISATION OF THE REACTIVE ENERGY

Making the objective function maximum is equivalent to finding the batteries size and location which satisfy the following system:

$$\begin{cases} \partial S / \partial I_{cqi} = 0 \\ \partial S / \partial h_i = 0 \end{cases} \tag{11}$$

The solution strategy suggested is an iterative procedure being based on the solution of each Eq. of the system (11) for the two following major reasons:

- The solution facility which the iterative method offers in this study.
- Each Eq. of the system (11) taken separately, constitutes alone, a problem the importance of which is proven. Indeed, it happens that the interest of the electrical energy suppliers, for considerations which are peculiar to them, relates only to one of the two parameters independently of the other. Owing to the fact that one solves each equation separately, the access to the solution of only one of the two problems is than possible.

Optimisation of the sizes: The substitution of « S » by its expression (2) in the first Eq. of the system (11) and after reorganizing the equation, we end up with the following contracted matrix expression (appendix for more details).

$$[H]\left[I_{cq}\right]=[B] \tag{12}$$

Where:
[H]: is a nxn matrix called matrix locations and the elements of which are such that:

$$h_{ij}=\begin{cases} h_i & \text{if } i=j \\ 2h_j & \text{if } i \prec j \\ 2h_i & \text{if } i \succ j \end{cases} \tag{13}$$

[Iqc]: is a 1xn matrix called batteries size matrix which transposed ones is:

$$\left[I_{cq}\right]^{t}=\left[I_{cq1}, I_{cq2}, \dots, I_{cqn}\right]$$

[B]: is a 1xn matrix the elements of which are such that:

$$B_i=\frac{k_p+k_eTL_f}{k_p+k_eT}\int_0^{h_i} I_sF_q(x)dx-\frac{k_{cf}V_{ci}}{2R(k_p+k_eT)\cos\varphi_{ci}}$$

Obtaining the optimal sizes passes by the resolution of the matrix Eq. 12 which gives the battery currents I_{qck}. The reduced optimal sizes of the batteries (Q_{ck}) are then deduced from:

$$Q_{ck}=\frac{V_{ck}I_{cqk}}{\cos\varphi_{ck}} \tag{14}$$

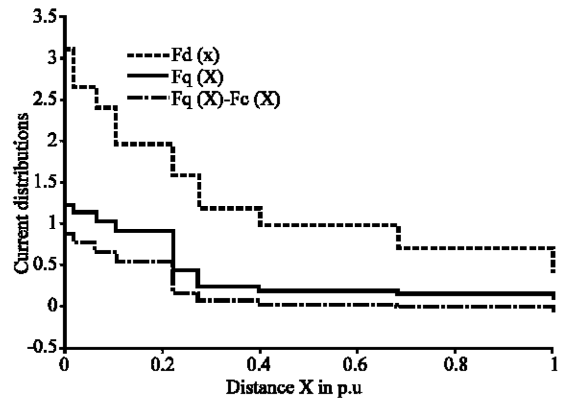

Fig. 4: D-current and q-current before and after compansation

Optimisation of the locations: Just like for the sizes of the batteries, the resolution of the second Eq. of the system (10) and after the equation reorganisation, leads to (appendix for details):

$$F_q(h_i)=\frac{2(k_p+k_eT)}{I_s(k_p+k_eTL_f)}\sum_{k=1}^{i-1}I_{cqk}+\frac{k_p+k_eT}{2I_s(k_p+k_eTL_f)}I_{cqi} \tag{15}$$

Knowing Fq(hi), the per-unit optimal locations and consequently the real ones, are then deduced from the graph of the reactive current distribution function $F_q(x)$ shown in Fig. 4. However and to make the locations determination automatic, a program was envisaged for this purpose.

Optimisation of two parameters: To optimise the locations and the sizes of the batteries at the same time, an iterative procedure is planned. It calls upon the programs developed to determine each of the two parameters separately. The execution of this iterative method requires the knowledge of an initial solution. Arbitrary values are then assigned to the two required parameters. The determination of the optimal locations and sizes will be done, according to the following flow chart:

Step 1: Read the line data.

Step 2: Read the arbitrary sizes and locations of the batteries.

Step 3: Initialise the tensions of the various bus bars and on the batteries.

Table 1: Batteries size and location optimisation results

N° of the battery	1	2	3	ΔP (kW)	ΔP in[7]	ΔE (kWh)	S ($)	number iterations
Initial sizes (kvar)	300	600	800					
Initial locations (mile)	3.27	07.32	3.02					
Optimal locations (p.u)	1.0000	0.2248	0.1074					
Locations in [7] (p.u)	1.0000	0.2248	0.1074					
Optimal locations (mile)	16.27	06.32	4.02	145.29	111.00	101240	17961	3
Optimal sizes (p.u)	0.1563	0.0987	0.0842					
Optimal sizes (kvar)	654.31	413.33	358.47					
Batteries currents (p.u)	0.1694	0.1046	0.0882					
Batteries sizes in [7] (kvar)	464	1070	2961					
$Q_{cq[7]}/Q_{cq}$ ratios	0.7091	2.5887	8.2601					

Step 4: Uniform and normalise the line and the loads.

Step 5: Calculate the normalised currents I_{cqk} due to the batteries from (6).

Step 6: While the convergence is not reached perform the following steps:

- Calculate the current distribution functions according to first and second expressions of (5).
- Calculate the bus voltages and the batteries voltages bus from (8) to (10).
- While the batteries locations and sizes are not identical to the precedent ones, carry out the following steps:
- Calculate the optimal locations according to (15).
- Calculate the optimal currents due to the batteries according to (12).
- Calculate the relative powers of the batteries according to (14).
- Calculate the cost reductions according to (1).
- Else continue.

Step 7: Else continue.

Step 8: Return to the real dimensions.

Step 9: Write the results.

APPLICATION

As an application example and in order to be able to undertake a comparative study, we have considered the line model whose data is detailed in Gainger[5-7]. It's a non-homogeneous medium voltage line, of nine sections of five wire-sizes, non-uniform loads concentrated at the end of each section. As a basic voltage we have adopted the voltage at the sub-station end (23 kV) which is also regarded as the angles origin. As a basic power we have considered the total reactive power at the substation end which in this example is equal to 4186 kvar.

Table 2: Voltage rms values and phase-angles after optimal batteries installation

Bus	V (p.u)	φ (rd)	$V_{[7]}$	$\varphi_{[7]}$
0	1.0000	0.0000	1.0000	0.0000
1	0.9924	-0.0088	0.9967	-0.0102
2	0.9831	-0.0200	0.9917	-0.0232
3	0.9691	-0.0415	0.9790	-0.0481
4	0.9534	-0.0481	0.9695	-0.0600
5	0.9419	-0.0704	0.9441	-0.0831
6	0.9340	-0.0783	0.9350	-0.0906
7	0.9240	-0.0884	0.9183	-0.1001
8	0.9183	-0.1048	0.8912	-0.1169
9	0.9166	-0.1168	0.8732	-0.1311

Table 3: Effect of the current battery definition

Battery	I_{cq} (p.u)	Q_{cq} (p.u)	$Q_{cq[7]}$ (p.u)	$Q_{cq[7]}/Q_{cq}$	Deviation (%)
1	0.1694	0.1563	0.1861	1.19	19
2	0.1046	0.0988	0.1113	1.13	13
3	0.0882	0.0842	0.0926	1.10	10

The three batteries size and location optimisation problem is then considered where: L_f= 0.45; k_p=168$/kW; K_e=0.015 $/kWh and K_{cf} = 4.9 $/3 phase kvar. A 14.3% annual fixed charge rate is applied for capacitor cost. The obtained results for the optimal sizes and locations are consigned in Table 1. Some others interesting results are also given. The voltage rms values and phase-angle after the optimisation of the reactive energy are consigned in Table 2 and those of the effect of the shunt battery current definition in Table 3.

CONCLUSION

- Unlike references[5-7], in the current distributions calculation, we have taken into account the effect of the line active and reactive power losses. This fact does not modify the mathematical formulation of the problem. If difference there is, it will appear in the numerical values only.
- Compared to the expression of ΔP_i given by Gainger[5-7], the expression
- Is increased by the term $(-2RI_{cqi}\sum_{k=i+1}^{n} h_k I_{cqk})$. This term

  is due to the effect of the batteries i+1 to nlocated upstream of the ith one, regarded as without effect in

Gainger[5-7]. The same term multiplied by T is found in the expression of ΔE_i

- These differences will undoubtedly lead to different results with respect to those of Gainger[5-7] in the reactive energy optimisation process.
- The comparison of the battery current
- With that given by Grainger[7], allows us to note a clear difference in the d and q current components. The determination of the units of the right hand side and the left hand side of the current formula given in[7] allows us to see that it is not homogeneous. The shunt battery current then will conduct to different values of the batteries power size.
- As consequences of the changes in the expressions of ΔP_i (3) and ΔE_i (4), we note:

1. If the batteries size optimisation is of interest, the non-diagonal terms of the matrix locations are multiplied by two, Eq. 13.
2. During the batteries location optimisation, the first term of the right-hand side of the Eq. 15 giving F_q(hi) is given F_q (hi) is multiplied by two.

- From the results point of view, if the optimal locations (nodes: 9, 5 and 4) of the batteries are identical to those given in Grainger[7], their sizes are completely different, Table 1. The ratios of the size batteries given in Grainger[7] to those which we have obtained, vary from 0.7091 to 8.2601 Table 1. The optimal choice of batteries location and size, taking into account the line power losses, conduct to a power loss reduction equal to 145.29 kW, more improvement in the voltage profile Table 2 and a decreasing in the reactive current distribution $F_q(x)$ Fig. 4. The cost reduction is then equal to 17961 $ and would be better in our study. Indeed, if we count the total number of kvar installed, it is equal to 1426.11 kvar in our study and 4495 kvar in Grainger[7]. Reported to the total reactive power (4186 kvar), the ratio is of 34.07% in our study and 107.4% in Grainger[7]. This last value i.e. 107.4% means that the total requested reactive energy is satisfied by an external contribution and violates the maximum limit of the reactive energy to be compensated.
- Note that the optimal sizes of the batteries obtained in our study are not standard ones. To overcome this, we suggest moving each non-standard size to that of smaller standard size or larger standard one and then choose those whose economic saving is the best.
- If we consider that the optimal reactive currents are the same in both their and our studies Table 3, the difference in the battery power definition (14) leads to a deviation between [10, 19%]. This deviation grows as one move away from the sub-station end (as voltage magnitude decreases).

NOMENCLATURE

$k_p(k_e)$	:	is the annual unit price of the kW (kWh).
K_{cf}	:	is the unit price of the three phase kVAr installed.
$V_i(\varphi_i)$	:	is the voltage rms value (phase-angle) at bus i.
$V_{ck}(\varphi_{ck})$	:	is the kth battery voltage rms value (phase-angle).
X_{ni}	:	is the per-unit normalised reactance of the ith line section.
R	:	is the per unit resistance of the normalised uniform line.
h_k	:	is the normalised uniform location of the kth battery.
Q_{ck}	:	is the kth battery size in kVAr.
$P_{i+1}(Q_{i+1})$	:	is the active (reactive) power fed into the node i+1. At the end bus of the line, these powers are equal to that of the load connected to it.
$P_{Li}(Q_{Li})$	:	is the active (reactive) power fee of the load connected to the the ithbus.
$I_s(t)$	:	is the time-dependent reactive current at the substation end.
T	:	is the in service duration of the batteries which are of fixed type. Its per-unit value is equal to 1.
Luni	:	is the uniform normalised length of the branch " i ".
Luni	:	is the uniform normalised total length from the reference node to the node " i ".

APPENDIX

Optimisation of the sizes: The optimal sizes of the batteries are obtained by the resolution of the system of equations $\partial S/\partial I_{qcj}$. To do it, the derivatives of the power and energy loss reductions are required. We then obtain:

$$\partial \Delta P/\partial I_{cqj} = \sum_{i=1}^{n} \partial \Delta P_i/\partial I_{cqj}$$

Where $\partial \Delta P_i/\partial I_{qcj}$ is equal to:

$$\frac{\partial \Delta P_i}{\partial I_{cqj}} = \begin{cases} -2RI_{cqi}h_i & \text{if } i \succ j \\ 2R\int_0^{h_i} I_s F_q(x)dx - 2Rh_i \sum_{k=1}^{i-1} I_{cqk} - 2R\sum_{k=i}^{n} h_k I_{cqk} & \text{if } i=j \\ -2RI_{cqi}h_i & \text{if } i \prec j \end{cases} \qquad \text{(A1)}$$

From where the expression of $\partial \Delta P/\partial I_{cqj}$:

$$\frac{\partial \Delta P}{\partial I_{cqj}} = 3\,(\,2R\int_0^{h_j} I_s\,F_q(x)dx - 4Rh_j\sum_{k=1}^{j-1} I_{cqk} - 4R\sum_{k=j+1}^{n} h_k I_{cqk} - 2R\,h_j I_{cqj})$$

Making the same reasoning we obtain for $\partial\Delta E/\partial I_{cqj}$:

$$\frac{\partial \Delta E}{\partial I_{cqj}} = 3\,(\,2R\,TL_f\int_0^{h_j} I_s\,F_q(x)\,dx - 4RTh_j\sum_{k=1}^{j-1} I_{cqk} - 4RT\sum_{k=j+1}^{n} h_k I_{cqk} - 2\,R\,T\,h_j I_{cqj}) \tag{A2}$$

Finally we obtain for $\partial S/\partial I_{cqj} = 0$, once a certain number of arrangements operated:

$$2h_j\sum_{k=1}^{j-1} I_{cqk} + 2\sum_{k=j+1}^{n} h_k I_{cqk} + h_j I_{cqj} = \frac{k_p + k_e TL_f}{k_p + k_e T}\int_0^{h_j} I_s F_q(x)dx - \frac{k_{cf}}{2R(k_p + k_e T)} \tag{A3}$$

where : $\sum_{k=1}^{j-1} I_{cqk} = 0$

for j=1 and

$\sum_{k=j+1}^{n} I_{cqk} = 0$

for j=n.

(A3) can be written in the matrix form as it follows:

$$\begin{pmatrix} h_1 & 2h_2 & 2h_3 & . & . & . & 2h_n \\ 2h_2 & h_2 & 2h_3 & . & . & . & 2h_n \\ 2h_3 & 2h_3 & h_3 & . & . & . & 2h_n \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ 2h_n & 2h_n & 2h_n & . & . & . & h_n \end{pmatrix} \begin{pmatrix} I_{cq1} \\ I_{cq2} \\ I_{cq3} \\ . \\ . \\ . \\ I_{cqn} \end{pmatrix} = A \begin{pmatrix} \int_0^{h_1} I_s F_q(x)dx \\ \int_0^{h_2} I_s F_q(x)dx \\ \int_0^{h_3} I_s F_q(x)dx \\ . \\ . \\ . \\ \int_0^{h_n} I_s F_q(x)dx \end{pmatrix} - \begin{pmatrix} B \\ B \\ B \\ . \\ . \\ . \\ B \end{pmatrix} \tag{A4}$$

where:

$$A = \left(k_p + k_e TL_f\right) / \left(k_p + k_e T\right)$$

and

$$B = k_{cf} / 2R\left(k_p + k_e T\right)$$

Optimisation of the locations: For the different derivatives to the locations , we obtain:

$$\frac{\partial \Delta P_i}{\partial h_j} = \begin{cases} 2R\,I_{cqi}\;I_s F_q(h_i) - 2\,R\,I_{cqi}\sum_{k=1}^{i-1} I_{cqk} - R\,I_{cqi}^2 & \text{if } j = i \\ 0 & \text{if } i \rangle j \\ -2\,R\,I_{cqi}\;I_{cqj} & \text{if } i \langle j \end{cases}$$

$$\frac{\partial \Delta E_i}{\partial h_j} = \begin{cases} 2RI_{cqi}\;TL_f I_s F_q(h_i) - 2\,R\,TI_{cqi}\sum_{k=1}^{i-1} I_{cqk} - RTI_{cqci}^2 & \text{if } j = i \\ 0 & \text{if } i \rangle j \\ -2RTI_{cqi}\,I_{cqj} & \text{if } i \langle j \end{cases}$$

and then:

$$\frac{\partial \Delta P}{\partial h_j} = 3(2RI_{cqj}I_sF_q(h_j) - 4RI_{cqj}\sum_{k=1}^{j-1} I_{cqc} - RI_{cqj}^2) \quad (A5)$$

$$\frac{\partial \Delta E}{\partial h_j} = 3(2RTL_f I_{cqj}I_sF_q(h_j) - 4R\,TI_{cqj}\sum_{k=1}^{j-1} I_{cqk} - R\,TI_{cqj}^2) \quad (A6)$$

At last $\partial S/\partial h_j$ gives after having ordered it:

$$F_q(h_j) = \frac{2(k_p + k_eT)}{(k_p + k_eTL_f)I_s}\sum_{k=1}^{j-1} I_{cqk} + \frac{k_p + k_eT}{2(k_p + k_eTL_f)I_s} I_{cqj} \quad (A7)$$

REFERENCES

1. Cook, R.F., 1964. Calculating loss reduction afforded by shunt capacitor application. IEEE Trans. Power Apparatus and Syst., pp: 1227-1230.
2. Nelson E. Chang, 1994. Determination of primary-feeder losses. IEEE Trans. Power Apparatus and Sys., 87: 1991-1994.
3. Nelson E. Chang, 1969. Location shunt capacitors on primary feeder for voltage control and loss reduction. IEEE Trans. Power Apparatus and Syst. 88: 71-77.
4. Nelson E. Chang, 1972. Generalized equations on loss reduction with shunt capacitor. IEEE Winter Meeting New York, 4: 2189-2195.
5. Grainger, J.J., S.H. Lee, A.M. Byrd and K.N. Clinard, 1980. Proper Placement of Capacitors for Losses reduction on Distribution Primary Feeders. Proc. Amer. Power Conf., 42: 593-603.
6. Grainger, J.J. and S.H. Lee, 1981. Optimum size and location of shunt capacitors for reduction of losses on distribution feeders. IEEE Trans. Power Apparatus and Sys., 100: 1105-1118.
7. Grainger, J.J. and S.H. Lee, 1982. Capacity release by shunt capacitor placement on distribution feeders: A new voltage-dependent model. IEEE Trans Power Apparatus and Sys., 101: 1236-1244.
8. Duran, H., 1982. Optimum number, location and size of shunt capacitors in radial distribution feeders, A dynamic programming approach. IEEE Trans Power Apparatus and Sys., 87: 10-13.
9. Tharwat H. Fawzi and Al, 1983. New approach for the application of shunt capacitors to the primary distribution feeders. IEEE Trans. Power Apparatus and Sys., 102: 10-13.
10. Ponnavaiko, M. and K.S. Prakasa Rao, 1983. Optimal choice of fixed and switched shunt capacitors on radial distributors by the method of local variations. IEEE Trans. Power Apparatus and Sys., 102: 1607-1615.
11. Baran, M.E. and F.F. Wu, 1989. Optimal capacitor placement on radial distribution systems. IEEE Trans. Power delivery, 4: 725-734.
12. Haque, M.H., 1999. Capacitor placement in radial distribution systems for loss reduction. IEE Proc. Gener. Transm. Distrib., 146: 501-505.
13. Mekhamer, S.F., M.E. El-Hawary, S.A. Soliman, M.A. Moustafa and M.M. Mansour, 2002. Reactive power compensation of radial distribution feeders: A New approach. Trans. Dist. Conf. Eexhib, Asia Pacific IEEE/PES, 1: 285-290.
14. Mekhamer, S.F., M.E. El-Hawary, S.A. Soliman, M.A. Moustafa and M.M. Mansour, 2002. New heuristic strategies for reactive power compensation of radial distribution feeders. IEEE Trans. Power delivery, 17: 1128-1135.
15. Kenji Iba, 1994. Reactive power optimisation by genetic algorithms. IEEE Trans. Power Sys., 9: 685-692.
16. Srinivasan Sundhararajan and Anil Pahva, 1994. Optimal selection of capacitors for radial distribution systems using a genetic algorithm. IEEE Trans. Power Sys., 9: 1499-1507.
17. Mohamed, A.S. Masoum, Marjan Ladjevardi, Akbar Jafarian and Ewalds F. Fuchs, 2004. Optimal placement, replacement and sizing of capacitor banks in distorted distribution networks by genetic algorithms. IEEE Trans. Power delivery, 19: 1128-1135.
18. Benemar Alencar de Souza, Helton do Nascimento Alves, B.A. de Souza, H.N. Alves and H.A. Ferreira, 2004. Microgenetic algorithms and fuzzy logic applied to optimal placement of capacitor banks in distribution networks. IEEE Trans. Power Sys., 19: 942-947.
19. Chiang, H.D., J.C. Wang, O. Cockings and H.D. Shin, 1990. Optimal capacitor placements in distribution systems; Part1: New formulation and the overall problem. IEEE Trans. Power delivery, 5: 634-642.
20. Chiang, H.D., J.C. Wang, O. Cockings and H.D. Shin, 1990. Optimal capacitor placements in distribution systems; Part2: Solution algorithms. IEEE Trans. Power delivery, 5: 643-649.