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Heuristic Search Technique for Stochastic Multi-Objective Generation Scheduling Based on Exact B-Coefficients

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Abstract: This study presents the application of the decision making methodology based on fuzzy set theory to determine the optimal generation schedule of multi-objective problem with due consideration of uncertainties in system input data and system load. The stochastic models are converted into their deterministic equivalent by taking their expected values. To determine trade-off relationship between conflicting objectives in the non-inferior domain, the weighting method is exploited. A new heuristic search technique based on binary successive approximation method is devised to search weights assigned to the objectives and incremental cost to obtain the non-inferior solution. Binary coded strings are used to represent weights assigned to the objectives as well as the incremental cost and the continuous values are obtained to represent a point in the search space through mapping. Once the trade-off has been obtained, fuzzy set theory helps The Decision Maker (DM) to choose the optimal operating point over the trade-off curve and adjust the generation schedule in the most preferred economic manner. This method has shown improvement because the weights are searched for more significant digits in fixed number of iterations. The validity of the proposed method has been examined on a sample system and the results are compared with the similar existing methods.

Key words: Stochastic multi-objective optimization, economic load dispatch, fuzzy set, evolutionary method, successive approximation method

INTRODUCTION

The environmental/economic load dispatch problem involves allocation of generations to different thermal units to minimize the cost of generation, while satisfying the equality and inequality constraints of the power systems and keeping pollution within limits. In a large number of real-life problems, a decision-maker is faced with multiple goals. The levels of attainment of these goals are to be expressed in the form of qualitative performance criteria, some of which can be selected as optimization objectives. Normally, the decision making input system data were assumed to be well behaved and deterministic. But in practical situations the input system data cannot be predicted and estimated with hundred percent certainties. It is bound to vary depending upon the uncertainties due to load changes, load forecasting errors, ageing of equipment, measurement errors etc. So the single datum used in the generation scheduling procedure can be incorrect in real life circumstances. Due to these variations, the optimum solution found out using deterministic data cannot result into practically optimum solution. It is worthwhile to assume the system data as variable and uncertain for more realistic approach (El-Hawary and Mbamalu, 1991; Dhillon *et al*., 1993; Chang and Fu, 1998; Bijwe *et al*., 2005).

With the increasing public awareness of the environmental protection and the passage of clean air Act Amendments of 1990 have forced the utilities to modify their design or operational strategies to reduce pollution of the thermal power plants (Talaqe *et al*., 1994; Huang and Huang, 2003). An excellent review by Chowdhary and Rahman (1990) updates the developments in the area. The cost and emission functions are conflicting functions in that minimizing pollution maximizes cost and vice versa. So, multiple criteria must be considered simultaneously to attain meaningful, practical, optimal schedule of operation. Nanda *et al*. (1982) proposed a goal programming technique to solve the optimal load dispatch problem for

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thermal generating units running with natural gas and fuel oil. Karmanshahi *et al.* (1990) presented a decision making methodology to determine the optimal generation dispatch and environmental marginal cost for power system operation with multiple conflicting objectives. Economic Emission Load Dispatch (EELD) problem has been solved through an interactive fuzzy satisfying method using fuzzy logic based, surrogate worth trade-off method (Dhillon and Kothari, 2000) evolutionary search for weightage pattern assigned to objectives (Brar *et al.*, 2002) and evaluating the best weights for objectives (Lakhwinder *et al.*, 2006).

In this study the authors have formulated multi-objective generation scheduling problem as a stochastic multi-objective problem with explicit recognition of uncertainties in the system production cost coefficients, emission coefficients and system load, which are treated as random variables. The study considers deviations proportional to the expectations of the square of the unsatisfied load as another objective. The objectives are clubbed in a single objective with the help of the weighting method.

In this study, a new search technique based on the successive approximation method is proposed to search the optimal weight pattern in the non-inferior domain. Further for a known weight combination, the generation schedule is also obtained by successive approximation method in which incremental cost, •$_p$ is represented by binary coded string. Fuzzy methodology has been exploited for solving a decision making problem involving multiplicity of objectives and selection criterion for best compromised solution. The objectives are quantified by eliciting the corresponding membership function. The shape of fuzzy membership function may be decided by the DM and generally depends upon the type of the problem. The best compromised solution is one, which provides maximum satisfaction level from the participating objectives/goals during the search of weights. Stochastic multi-objective optimization problem has also been solved by using exact B-coefficients.

STOCHASTIC MULTI-OBJECTIVE OPTIMIZATION PROBLEM FORMULATION

Five important non-commensurable objectives in electrical thermal power system are undertaken. These are economy, environmental impacts because of No_x, SO_2 and CO_2 emissions. The multi-objective optimization problem is defined as:

$$\text{Minimize cost } J_1 = \sum_{i=1}^{N}(a_{i1}P_i^2 + b_{i1}P_i + c_{i1})\text{Rs h}^{-} \quad (1)$$

$$\text{Minimize NO}_x \text{ emission } J_2 = \sum_{i=1}^{N}(a_{i2}P_i^2 + b_{i2}P_i + c_{i2})\text{kg h}^{-1} \quad (2)$$

$$\text{Minimize CO}_2\ J_3 = \sum_{i=1}^{N}(a_{i3}P_i^2 + b_{i3}P_i + c_{i3})\text{emission kg h}^{-1} \quad (3)$$

$$\text{Minimize SO}_2 \text{ emission } J_4 = \sum_{i=1}^{N}(a_{i4}P_i^2 + b_{i4}P_i + c_{i4})\text{kg h}^{-1} \quad (4)$$

Subject to power balance equation

$$\sum_{i=1}^{N} P_i = P_D + P_L \quad (5)$$

Power limits

$$P_i^L \le P_i \le P_i^U \quad i = 1,2,...,N \quad (6)$$

Where

a_{i1}, b_{i1} and c_{i1} are cost coefficients of ith unit
a_{i2}, b_{i2} and c_{i2} are No_x emission coefficients of ith unit
a_{i3}, b_{i3} and c_{i3} are SO_2emission coefficients of ith unit
a_{i4}, b_{i4} and c_{i4} are emission coefficients of ith unit
P_i is real power generation of ith unit.
P_D is the power demand.
P^l_i are the lower and upper limits of real power, respectively.
N is the number of generators
$P_L$is the transmission loss and is expressed through the simplified well known loss formula expression as a quadratic function:

$$P_L = \sum_{i=1}^{N}\sum_{j=1}^{N} P_i B_{ij} P_j + \sum_{i=1}^{N} B_{io} P_i + B_{oo} P_o \quad (7)$$

With

$$B_{ij} = \frac{R_{ij}}{|V_i||V_j|}\frac{\cos(\theta_i - \theta_j)}{\cos\varphi_i \cos\varphi_j};\ i = 1,2,\ldots,N_b,\ j = 1,2,\ldots,N_b(1h)$$

$\theta_i = \delta_i - \varphi_i$; i=1,2,..,N_b and

$$\varphi_i = \tan^{-1}\frac{Q_i}{PI_i}$$

•$_i$ is voltage angle at ith bus. N_b is number of buses.
$PI_i = P_i - P_{di}$; i =1,2,…,N_b

Pi_i is the injected power at i^{th} bus. P_{di} is the load demand at i^{th} bus

$$B_{oo} = \sum_{i=1}^{N_b}\sum_{j=1}^{N_b} P_{di}B_{ij}P_{dj} \text{ and } B_{io} = -\sum_{j=1}^{N_b}(B_{ij}+B_{ji})P_{dj}$$

The stochastic model of multi-objective problem is formulated by considering cost coefficients, emission coefficients and load demand as random variables. Then the generator output automatically becomes random. Random variables are considered as normally distributed and statistically dependent to each other. By taking expectations, the stochastic model can be converted into its deterministic equivalent. The expected value of a function can be obtained by expanding the function, employing Taylor's series, about the mean. Deterministic equivalent of stochastic multi-objective optimization problem is stated as:

Minimize expected cost

$$\bar{J}_1 = \sum_{i=1}^{N} \bar{a}_{i1}\bar{P}_i^2 + \bar{b}_{i1}\bar{P}_i + \bar{c}_{i1} + \bar{a}_{i1}\,\mathrm{var}(P_i) + 2\bar{P}_i\,\mathrm{cov}(a_{i1},P_i) + \mathrm{cov}(b_{i1},P_i) \qquad (8)$$

Minimize expected NO_X emission

$$\bar{J}_2 = \sum_{i=1}^{N} \bar{a}_{i2}\bar{P}_i^2 + \bar{b}_{i2}\bar{P}_i + \bar{c}_{i2} + \bar{a}_{i2}\,\mathrm{var}(P_i) + 2\bar{P}_i\,\mathrm{cov}(a_{i2},P_i) + \mathrm{cov}(b_{i2},P_i) \qquad (9)$$

Minimize expected CO_2 emission

$$\bar{J}_3 = \sum_{i=1}^{N} \bar{a}_{i3}\bar{P}_i^2 + \bar{b}_{i3}\bar{P}_i + \bar{c}_{i3} + \bar{a}_{i3}\,\mathrm{var}(P_i) + 2\bar{P}_i\,\mathrm{cov}(a_{i3},P_i) + \mathrm{cov}(b_{i3},P_i) \qquad (10)$$

Minimize expected SO_2 emission

$$\bar{J}_4 = \sum_{i=1}^{N} \bar{a}_{i4}\bar{P}_i^2 + \bar{b}_{i4}\bar{P}_i + \bar{c}_{i4} + \bar{a}_{i4}\,\mathrm{var}(P_i) + 2\bar{P}_i\,\mathrm{cov}(a_{i4},P_i) + \mathrm{cov}(b_{i4},P_i) \qquad (11)$$

Minimize expected variance of power

$$\bar{J}_5 = \left(\sum_{i=1}^{N} \mathrm{var}(P_i) + \sum_{i=1}^{N-1}\sum_{j=i+1}^{N} 2\,\mathrm{cov}(P_i,P_j) \right) \qquad (12)$$

$$\text{Subject to } \sum_{i=1}^{N} \bar{P}_i = \bar{P}_D + \bar{P}_L \qquad (13)$$

$$\bar{P}_i^L \le \bar{P}_i \le \bar{P}_i^U, \quad i = 1,2,\ldots,N \qquad (14)$$

Where $\bar{P}_i$ is the expected real power generation of i^{th} generator,

$\bar{a}_{i1}$, $\bar{b}_{i1}$ and $\bar{c}_{i1}$ are the expected cost coefficients of i^{th} unit.

$\bar{a}_{i2}$, $\bar{b}_{i2}$ and $\bar{c}_{i2}$ are the expected NO_X emission coefficients of i^{th} unit

$\bar{a}_{i3}$, $\bar{b}_{i3}$ and $\bar{c}_{i3}$ are the expected SO_2 emission coefficients of i^{th} unit

$\bar{a}_{i4}$, $\bar{b}_{i4}$ and $\bar{c}_{i4}$ are the expected CO_2 emission coefficients of i^{th} unit

$\bar{P}_D$ is the expected power demand.

$\bar{P}_i^L$ and $\bar{P}_i^U$ are the expected lower and upper limits of real power, respectively.

N is the number of generators

$\bar{P}_L$ is the expected transmission loss and is given as:

In this study, variance and covariance are replaced by the coefficients of variation and correlation, respectively. In general variance and covariance are defined as:

$$\mathrm{var}(x) = C^2(x)\bar{x}^2 \qquad (15)$$

$$\mathrm{cov}(x,y) = R(x,y)C(x)C(y)\bar{x}\,\bar{y} \qquad (16)$$

Where C(x) and C(y) are the coefficients of $\bar{x}$ and $\bar{y}$ variation and are the expected values of variables x and y, respectively. R (x, y) is correlation coefficient and varies from-1 to 1.The zero value of coefficient of variation implies no randomness, in other words, the complete certainty about the value of random variables. Using (15) and (16), the multi-objective optimization problem defined by (8-14) can be rewritten as:

$$\text{Minimize}\left[\bar{J}_1, \bar{J}_2, \bar{J}_3, \bar{J}_4, \bar{J}_5\right]^T \qquad (17)$$

$$\text{Subject to } \sum_{i=1}^{N} \bar{P}_i = \bar{P}_D + \bar{P}_L \qquad (18)$$

$$\bar{P}_i^L \le \bar{P}_i \le \bar{P}_i^U, \quad i = 1,2,\ldots,N \qquad (19)$$

$$\text{Where } \bar{J}_j = \sum_{i=1}^{N} (\bar{A}_{ij}\bar{P}_i^2 + \bar{B}_{ij}\bar{P}_i + \bar{C}_{ij}) \quad j = 1,2,3,4$$

$$\text{With } \bar{A}_{ij} = [1 + C^2(P_i) + 2R(a_{ij},P_i)C(a_{ij})C(P_I)]\bar{a}_{ij}$$

$$\bar{B}_{ij} = [1 + R(b_{ij},P_i)C(b_{ij})C(P_i)]\bar{b}_{ij} \text{ and } \bar{C}_{ij} = \bar{c}_{ij}$$

$$\overline{J}_5 = \sum_{i=1}^{N}\sum_{1}^{N}\overline{P}_i T_{ij}\overline{P}_j \text{ with} \tag{20}$$
$$T_{ii} = C^2(P_i)\; T_{ij} = R(P_i,P_j)C(P_i)C(P_j);\; i \neq j$$

$$\overline{P}_L = \sum_{i=1}^{NG}\sum_{j=1}^{NG}\overline{P}_i U_{ij}\overline{P}_j + \sum_{i=1}^{N}\overline{B}_{io}\overline{P}_i + \overline{B}_{OO}\overline{P}_O \tag{21}$$

With

$$U_{ii} = \left(1.0 + C(P_i)^2 + 2.0R(P_i,B_{ii})C(P_i)C(B_{ii})\right)\overline{B}_{ii}$$
$$U_{ij} = \left(1 + R(P_i,P_j)C(P_i)C(P_j) + 2R(P_i,B_{ij})C(P_i)C(B_{ij})\right)\overline{B}_{ij}\;\; ; i \neq j$$

SOLUTION APPROACH

To generate the non-inferior solutions of the multi-objective problem, the weighting method is used. In this method, the multi-objective optimization problem is converted into a scalar optimization problem as:

$$\text{Minimize} \quad \sum_{n=1}^{5} w_n \overline{J}_n \tag{22}$$

$$\text{Subject to} \quad \sum_{i=1}^{N}\overline{P}_i = \overline{P}_D + \overline{P}_L \tag{23}$$

$$\overline{P}_i^L \le \overline{P}_i \le \overline{P}_i^U, \quad i = 1,2,\ldots,N \tag{24}$$

$$\sum_{n=1}^{5} w_n = 1,\; w_n \ge 0 \tag{25}$$

To solve the scalar optimization problem, the Lagrangian function is defined as:

$$L(\overline{P}_i,\lambda_P) = \sum_{n=1}^{5} w_n \overline{J}_n + \sum_{i=1}^{N} \lambda_P(\overline{P}_D + \overline{P}_L - \sum_{i=1}^{N}\overline{P}_i) \tag{26}$$

Where λ_p is the Lagrangian multiplier.

The necessary conditions to minimize the unconstrained Lagrangian function are:

$$\frac{\partial L}{\partial \overline{P}_i} = \sum_{n=1}^{5} w_n \frac{\partial \overline{J}_n}{\partial \overline{P}_i} + \lambda_p\left(\frac{\partial \overline{P}_L}{\partial \overline{P}_i} - 1\right) = 0.\; i = 1,2,\ldots,N \tag{27}$$

$$\frac{\partial L}{\partial \lambda_p} = \overline{P}_D + \overline{P}_L - \sum_{i=1}^{N}\overline{P}_i = 0 \tag{28}$$

The above equations can be rewritten as:

$$\sum_{j=1}^{N} X_{ij}\overline{P}_j = Y_i \qquad i = 1,2,\ldots,N \tag{29}$$

Where

$$X_{ii} = \sum_{k=1}^{4} 2\, w_k \overline{A}_{ik} + 2\,(w_5 T_{ii} + \lambda_p U_{ii})$$
$$X_{ij} = \sum_{k=1}^{N} 2\,(w_5 T_{iJ} + \lambda_p U_{iJ}) \qquad ; i \neq j$$
$$Y_i = \lambda_p\,(1 - \overline{B}_{io}) - \sum_{k=1}^{4} w_k \overline{B}_k$$

As λ_p is known during the search, is $\overline{P}_i$ obtained by solving above simultaneous equations using Guass Elimination method. The search of λ_p is terminated when (28) is satisfied.

DECISION MAKING

Considering the imprecise nature of the DM's judgment, it is natural to assume that the DM may have fuzzy or imprecise goals for each objective function. The fuzzy sets are defined by equations called membership functions. These functions represent the degree of membership in certain fuzzy sets using values from 0 to 1. The membership value 0 indicates incompatibility with the sets, while 1 means full compatibility. By taking account of the minimum and maximum values of each objective function together with the rate of increase of membership satisfaction, the DM must determine the membership function $\mu(J_i)$ in a subjective manner. It is assumed that $\mu(J_i)$ is a strictly monotonic linear decreasing and continuous function and is defined as:

$$\mu(J_i) = \begin{cases} 1 & ; J_i \le J_i^{min} \\ \dfrac{J_i^{max} - J_i}{J_i^{max} - J_i^{min}} & ; J_i^{min} \le J_i \le J_i^{max} \\ 0 & ; J_i \ge J_i^{max} \end{cases} \tag{30}$$

Where J_i^{min} and J_i^{max} are the minimum and maximum values of ith objective function in which the solution is expected. The value of membership function suggests how far (in the scale from 0 to 1) a non-inferior solution has satisfied the $\overline{J}_i$ objective. The decision regarding the best solution is made by the selection of minimax of membership function as defined below (Tapia and Murtagh, 1991):

$$\mu^D = \text{Max}\begin{bmatrix} \text{Min}\{\mu(J_j)^k;\; j = 1,2,\ldots,5\}; \\ k = 1,2,\ldots,2^L + 1 \end{bmatrix} \tag{31}$$

The function μ_D in (31) can be treated as a membership function for non-dominated solutions. The solution which attains highest membership μ^K_D in the

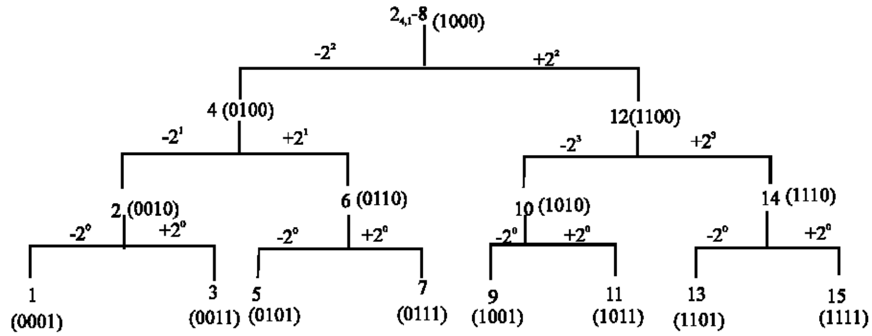

Fig. 1: Operation diagram of the successive approximation method

fuzzy set so obtained can be chosen as best solution or the one having highest cardinal priority ranking.

ALGORITHM FOR HEURISTIC SEARCH OF INCREMENTAL COST

The operation diagram of the successive approximation method is shown in Fig. 1. It has been shown in the diagram that all the possibilities from 1-15 have been included when four binary bits are used to represent either the incremental cost or the weights. In the proposed method the number of binary bits to represent the incremental cost and weights has been selected as thirty and 6, respectively to get accurate results. The successive approximation strategy to search the incremental cost, • is elaborated here.
The stepwise procedure is outlined below:

1. Read NB, number of binary digits to represent, •.
2. Set binary digit counter, i = 1
3. N = $^{NB-1}$
4. Increment i; i = i+1
5. If (i• NB) then go to 10
6. Determine N_1 and N_2 as
   6.1 $N_1 = N + 2^{NB-i}$
   6.2 $N_2 = N - 2^{NB-i}$
7. Determine $•_1$ and $•_2$ as

$$\lambda_1 = \lambda^{min} + \frac{N_1}{2^{NB}-1}\left(\lambda^{max} - \lambda^{min}\right)$$

Determine P_i^1; i = 1,2, …, N from (29) using Gauss Elimination method

7.1.2 Determine $\Delta P_D^1 = \left| P_D + P_L - \sum_{i=1}^{N} P_i^1 \right|$

7.2 $\lambda_2 = \lambda^{min} + \frac{N_2}{2^{NB}-1}\left(\lambda^{max} - \lambda^{min}\right)$

7.2.1 Determine P_i^2; i = 1,2, … , N from (29) using Gauss Elimination method

7.2.2 Determine $\Delta P_D^2 = \left| P_D + P_L - \sum_{i=1}^{N} P_i^2 \right|$

8. If $(\Delta P_D^1 \langle \Delta P_D^2)$ Then set $N = N_1$ and $\Delta P_D = \Delta P_D^1$
   Else set $N = N_2$ and $\Delta P_D = \Delta P_D^2$
9. If $(\Delta P_D \le \varepsilon)$ then continue else go to 4
10. Stop.

Algorithm for heuristic search of weights: Heuristic evolutionary method is proposed to search the optimal weight combination. In this method (2^L+1) weight combinations are simulated at 2^L corner points of an L-dimensional hypercube centered on initial point W^c_i. (2^L+1) non-inferior solutions are generated and membership functions are obtained using (30). The best or preferred non-inferior solution is obtained using (31). To continue the iterative process, another hypercube is formed around the preferred point. Successive approximation strategy to search the weights is elaborated here. The weights are generated as given below:

$$\alpha_i^j = \alpha_i^c + \gamma_i^j \quad ;i = 1,2,...,L \quad ;J = 1,2,...,2^L \qquad (32)$$

Where $\gamma_i^j = \pm 2^{NW-k}$

With NW is the number of binary bits used to represent weights.
$•_i^j$ is the scalar weights, $•_i^c$ is the initial value of weights.

Table 1: Generation of weight combinations at hypercube corners (Three objectives)

Hypercube corners	Possible combinations of three binary bits $b_2\ b_1\ b_0$	Distance of hypercube corners from centre point $\bullet^c_1 \bullet^c_2 \bullet^c_3$	Possible generated weights at the hypercube corners
1	0 0 0	-• -• -•	$\bullet^c_1$-• $\bullet^c_2$-• $\bullet^c_3$-•
2	0 0 1	-• -• +•	$\bullet^c_1$-• $\bullet^c_2$-• $\bullet^c_3$+•
3	0 1 0	-• +• -•	$\bullet^c_1$-• $\bullet^c_2$+• $\bullet^c_3$-•
4	0 1 1	-• +• +•	$\bullet^c_1$-• $\bullet^c_2$+• $\bullet^c_3$+•
5	1 0 0	+• -• -•	$\bullet^c_1$+• $\bullet^c_2$-• $\bullet^c_3$-•
6	1 0 1	+• -• +•	$\bullet^c_1$+• $\bullet^c_2$+• $\bullet^c_3$+•
7	1 1 0	+• +• -•	$\bullet^c_1$+• $\bullet^c_2$+• $\bullet^c_3$-•
8	1 1 1	+• +• +•	$\bullet^c_1$+• $\bullet^c_2$+• $\bullet^c_3$+•

$\bullet^j_i$ Weights are mapped in the range of 0-100.

$$\beta_i^j = \beta_i^{min} + \frac{\alpha_i^j}{2^{NW-1}}\left(\beta_i^{max} - \beta_i^{min}\right); \quad i = 1,2,...,L \ ; \ j = 1,2,...,2^{L-1} \tag{33}$$

The normalized weights, w^j_i are obtained as:

$$w_i^j = \frac{\beta_i^j}{\sum_{i=1}^{L}\beta_i^j} \ ; \ j = 1,2,...,2^{L-1} \tag{34}$$

Where $\bullet_i^{min}$ and $\bullet_i^{max}$ are the minimum and maximum value of the weights $\bullet_i^J$, respectively (0-100). • is the distance of the corners of the hypercube from the point around which hypercube is generated. A matrix has been generated from possible combinations of binary bits. '0' bit is replaced by -• and '1' bit is replaced by +• . As an illustration the generation of weight combinations for three objectives has been shown in Table 1. For three objectives 2^3 (eight) different possible weight combinations can be obtained. In general the different possible weight combinations are 2^L. To implement the heuristic evolutionary search, the stepwise algorithm is outlined as below:

1. Input the data.
2. Find the minimum and maximum values of objectives; J_i^{min} and J_i^{max} i=1,2,…,L.
3. Set the initial centre $\bullet^c_i = 2^{NW-1}$ where NW is the number of bits to represent weights w_i^j.
4. Set the initial value of membership function $\mu^p = 0$.
5. Initialize iteration counter, r = 0.
6. Increment iteration counter, r = r + 1.
7. Generate weight combinations at the edges of hypercube as given by (32).
8. Initialize iteration counter, k = 0.
9. Increment iteration counter, k = k + 1.
10. Generate the non-inferior solutions for kth weight combination by implementing the algorithm .
11. Find membership function of the objectives $\mu(J_i)^K$; I = 1,2,..., L freom (30).
12. Find the intersection of the membership function, $\mu_k^{min} = Min\{\mu(J_i)^K; I = 1,2,...,L\}$
13. If (k• 2^L +1), then go to step 9.
14. Find maximum satisfied membership function, $\mu^D = Max\ \{\mu_K^{Min}; K = 1,2,..., 2^L + 1\}$
15. Choose weight combination having maximum satisfied membership function μ^D as a centre of hypercube.
16. If (r• NW) then go to step 18, else continue.
17. If (μ^D• μ^p) then $\mu^P = \mu^D$ and $\bullet^c_i = \bullet^{co}_i$; I = 1,2,..., L, else go to step 6.
18. Stop.

TEST SYSTEM AND RESULTS

The proposed algorithm has been implemented on a six-generator system. The fuel cost, NO_X emission, CO_2 emission and SO_2 emission coefficients are taken from Dhillon and Kothari (2000) along with expected transmission loss coefficients. The generation schedule has been obtained for power demand of 1800 MW. The validity of proposed method is also illustrated on 11-bus, 17-lines power system, comprising of three generators (Lakhwinder *et al.*, 2006).

Stochastic multi-objective generation scheduling: In this case the 5 objectives are considered. These are economy, environmental impacts because of NO_X, SO_2 and CO_2 emissions and variance of power which have weights w_1, w_2, w_3, w_4 and w_5, respectively. The cost and emission coefficients are treated as random variables. The minimum and maximum values of the objectives are calculated by giving full weightage to one objective at a time and no weightage to the other objectives.

Six different cases are considered to realize the effect of variance and covariance of the random variables to each other (pair wise).

Case I: All the variables are independent to each other

$$C(a_{ij}) = C(b_{ij}) = C(P_i) = 0.05 \text{and}$$
$$R(P_i, P_j) = R(a_{ij}, P_i,) = R(b_{ij}, P_i) = 0.0$$

Table 2: Best solution for the best weight combination under different cases

Case No.	$\overline{J}_1$ (Rs h•^{1})	$\overline{J}_2$ (kg h•^{1})	$\overline{J}_3$ (kg h•^{1})	$\overline{J}_4$ (kg h•^{1})	$\overline{J}_5$ (MW^2)	P_L(MW)	• P_D	μ_D
I	18760.19	2205.26	11246.16	59551.80	1669.09	136.645	0.0000610	0.5657
II	18789.09	2217.51	11263.44	59957.04	7823.2	135.103	0.0001526	0.5507
III	18723.73	2198.89	11224.35	59263.18	4497.98	136.598	0.0000153	0.5605
IV	18790.43	2327.17	11265.48	59354.60	6547.99	139.607	0.0000916	0.9928
V	18753.38	2209.75	11242.05	59685.08	7823.12	135.095	0.0000153	0.5506
VI	18803.69	2215.74	11272.21	59905.49	5520.86	136.301	0.0000916	0.5634

Table 3: Generation schedule for different cases

Case No.	P_1(MW)	P_2(MW)	P_3(MW)	P_4(MW)	P_5(MW)	P_6(MW)
I	238.312	288.005	465.572	385.745	366.275	219.736
II	241.703	284.094	473.501	359.932	355.823	220.050
III	238.670	288.662	465.114	358.657	365.685	219.809
IV	249.049	314.759	425.480	373.750	346.768	229.483
V	241.719	284.104	473.489	359.909	355.706	220.168
VI	239.642	286.546	467.026	358.719	364.141	220.227

Case II: All the variables are correlated to each other such that;

$$C(a_{ij}) = C(b_{ij}) = 0.1\,,\ C(P_i) = 0.05 \text{and}$$
$$R(P_i, P_j) = R(a_{ij}, P_i,) = R(b_{ij}, P_i) = 0.5$$

Case III: All the variables are correlated to each other such that;

$$C(a_{ij}) = C(b_{ij}) = 0.1\,,\ C(P_i) = 0.05 \text{and}$$
$$R(P_i, P_j) = R(a_{ij}, P_i,) = R(b_{ij}, P_i) = 0.8$$

Case IV: All the variables are independent to each other.

$$C(a_{ij}) = C(b_{ij}) = C(P_i) = 0.1 \text{and}$$
$$R(P_i, P_j) = R(a_{ij}, P_i,) = R(b_{ij}, P_i) = 0.0$$

Case V: The cost and emission variables are independent but power generations are dependent to each other.

$$C(a_{ij}) = C(b_{ij}) = 0.1\,,\ C(P_i) = 0.05,$$
$$R(a_{ij}, P_i) = R(b_{ij}, P_i) = 0.0\,,\ R(P_i, P_j) = 0.8$$

Case VI: All the variables are correlated to each other.

$$C(a_{ij}) = C(b_{ij}) = 0.1\,,\ C(P_i) = 0.05$$
$$R(P_i, P_j) = R(a_{ij}, P_i,) = R(b_{ij}, P_i) = 0.5$$

The best solutions are obtained for the above mentioned cases in which diverse values of coefficients of variation and correlation coefficients are considered. The results corresponding to the different cases have been tabulated in Table 2 and 3. From the results it can be seen that there is significant variation in the objectives and generation schedules under different cases when diverse values of coefficients of variation and correlation coefficients are considered.

Table 4: B-coefficients

$B_{11} = 4.802226\times10^{-02}$	$B_{12} = 7.185836\times10^{-03}$	$B_{13} = 1.283460\times10^{-02}$
$B_{21} =-7.185854\times10^{-03}$	$B_{22} = 3.379933\times10^{-02}$	$B_{23} = 3.336367\times10^{-03}$
$B_{31} =-1.283459\times10^{-02}$	$B_{32} = 3.336425\times10^{-03}$	$B_{33} = 5.342060\times10^{-02}$
$BO_1 =-7.217036\times10^{-03}$	$BO_2 = 6.733713\times10^{-03}$	$BO_3=1.555294\times10^{-02}$
	$BOO = 3.465590\times10^{-02}$	

From the results, it is evident that as the coefficient of variation of power, *C(P$_i$)* is increased there is considerable increase in the values of expected cost and variance of power. Also, with the variation of Correlation Coefficient (CC) there is considerable variation in expected cost and variance of power. The expected cost increases as CC is changed from negative to positive value whereas the variance of power increases almost linearly as CC is varied from negative to positive value. Due to these variations, there is a need to determine the optimum solution by taking into account the statistical variation of system parameters.

Multi-objective generation scheduling based on exact B-coefficients: The validity of proposed method is illustrated on 11-bus, 17-lines, comprising of three generators (Lakhwinder *et al*., 2006). Four objectives, economy, No_x, SO_2 and CO_2 emissions are considered which have weights, respectively and weight w_5 is set to zero. The random variables are considered independent and uncorrelated to each other. The best obtained solution is given in below. B-coefficients obtained corresponding to best solution is given in Table 4. Comparison with other methods adopted in Brar *et al*. (2002) and Lakhwinder *et al*. (2006) is given in Table 5. Table 6 and 7 give the load flow solution and lines flows, respectively.

Table 5: Comparison of optimal values of weights and objectives

Method	Parameters	Cost($ h^{-1})	NO_x emission(Kg h^{-1})	So_xemission(Kg h^{-1})	CO_x emission(Kg h^{-1})
Proposed method	Weights	0.30435	0.21739	0.23913	0.23913
	Objectives	4559.694	680.241	2735.675	6.64726
Tapia and Murtagh	Weights	0.4700	0.1770	0.1770	0.1760
(1991)	Objectives	4587.9210	666.9317	2752.396	8.039393
	Weights	0.1639	0.1745	0.1645	0.4971
	Objectives	4593.1370	655.0237	2755.4950	8.604041

Table 6 Load flow solution at the best schedule

Bus	Voltage		Injected power	
No	Magnitude V_i (p.u.)	Angle $\bullet_i$ (rad)	Real PI_i (p.u.)	Reactive Q_i (p.u.)
1		0.1052519	1.2355270	0.0355645
2	1.070	0.1037145	1.3719050	0.3836708
3	1.088	0.1044669	0.7395561	0.1279518
4	1.062	-0.0545564	-0.2500043	-0.0500086
5	0.9655043	-0.1644830	-0.2499949	-0.0499905
6	0.8956835	-0.1197684	-0.1000159	-0.0200075
7	0.9300811	-0.0294966	-0.4000041	-0.0999989
8	0.9945043	-0.0746856	-0.9000573	-0.4500146
9	0.9601473	-0.1369145	-0.6999480	-0.3499784
10	0.8994597	0.0017453	-0.2499915	-0.0499990
11	1.0264090	0.0	-0.2605073	-0.0499990

Table 7: Real and reactive power line flows

Line No.	SB-EB	Real power	Reactive power	Line No.	SB-EB	Real power	Reactive power
1	1-9	0.5393819	0.2249603	1	1-9	-0.4924556	-0.1271570
2	1-11	0.6961447	-0.1893960	2	1-11	-0.6735976	0.2386486
3	2-3	0.0139765	0.0168716	3	2-3	-0.0136040	-0.0849776
4	2-7	0.5811101	0.1663201	4	2-7	-0.5495334	-0.1213611
5	2-10	0.7768186	0.2004786	5	2-10	-0.7494256	-0.1351938
6	3-4	0.7531602	0.2129294	6	3-4	-0.7091768	-0.1118799
7	4-6	0.2262017	0.0295339	7	4-6	-0.2204638	-0.0494129
8	4-8	0.0651641	-0.0227742	8	4-8	-0.0647067	-0.0130268
9	4-9	0.1678065	0.0551118	9	4-9	-0.1621648	-0.0885431
10	5-6	-0.1180742	-0.0639667	10	5-6	0.1204480	0.0294056
11	5-9	-0.1319206	0.0139767	11	5-9	0.1330354	-0.0265221
12	7-8	0.3081397	0.1134589	12	7-8	-0.3025704	-0.1147464
13	7-10	-0.1586106	-0.0920969	13	7-10	0.1611283	0.0690117
14	8-9	0.1842183	0.0820488	14	8-9	-0.1783631	-0.1077560
15	8-10	-0.3581163	-0.1475566	15	8-10	0.3707974	0.1559688
16	8-11	-0.3588821	-0.2567328	16	8-11	0.3790127	0.2717629
17	10-11	-0.0324915	-0.1397860	17	10-11	0.0340778	0.0895844

CONCLUSION

A new heuristic search technique based on binary successive approximation method has been developed for the solution of the multi objective optimization problem. The solution set of the formulated problem is non-inferior due to contradictions among objectives taken and has been generated through weighting method. In order to overcome the limitation of the interactive method it is proposed to search the optimal Weight pattern with the help of successive approximation method. In this method the solution is guaranteed within the fixed number of iterations. The accuracy of this method does not depend on initial guess whereas the accuracy of other methods is a function of initial guess. The weighting pattern that attains maximum satisfaction level from the membership function of the participating objectives have been designated the best achieved solution. The comparison of results reveals that the proposed search method gives the comparable results in terms of achieved satisfaction level in known number of iterations. Further the proposed method provides the facility to consider the inaccuracies and uncertainties in the multi-objective generation scheduling problem. The practical utility of the stochastic formulation is illustrated through numerical example in diverse cases. The results show that the proposed method is capable of obtaining higher quality solutions.

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