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A New Non Linear Multivariable Control of an Induction Motor

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Abstract: In this study we study the non-linear control of an Induction Motor (IM). So we applied the technique of input-output linearization to the (IM), how is based on the differential geometry, where we can linearized the model of the (IM) which is strongly nonlinear, then we study the internal dynamics of linear system, we control separately flux and speed, finally we estimate the rotor flux and observe the reference torque. Our contribution is the exploitation of the space vector to obtain the outputs. The simulation is realised with an MLI inverter.

Key words: Nonlinear control, induction motor, internal dynamics, observer reference torque

INTRODUCTION

The induction machine is the workhorse of industry. It is more rugged, reliable, compact efficient and cheap in comparison to other motors used in similar applications.

The Induction Motors (IM) constitute a theoretically interesting and practically important class of nonlinear systems. The control task is further complicated by the fact that induction motors are subject to unknown (load) disturbances and change in values of parameters during its operation (Delaleau *et al.*, 2001; Chiason, 1993; Marinon, 1993).

Several techniques of control are used for (IM), The technique of Control Oriented Flux (FOC) which permits the decoupling between input and output variables, so (IM) is assimilate to continuous current motor, this method has a difficulty is how exactly oriented the flux. However feedback linearization amounts to cancelling the nonlinearities in a nonlinear system so that the Closed-Loop (CL) dynamics is in a linear form (Jaques *et al.*, 2000; Isiodori, 1989).

A goal of nonlinear control is to can controlled separately flux and the speed, the motor model is strongly nonlinear then it's composed to the autonomous and mono-variables too under systems, so every under system presented an independence loop of control for each variables is given.

Also for high speeds, field weakening is necessary in order to avoid the saturation of the stator voltages. Since field weakening depends on the speed, the dynamic of the flux may interfere with the dynamic of the speed. This coupling of the flux and speed dynamics can be eliminated by considering an input-output scheme (Tarbouchi and Le, 1996). and in order to have the relation between the inputs and outputs we use the space vectors which is our contribution in this study.

MODEL OF THE INDUCTION MOTOR

The state equations in the stationary reference frame of an induction motor can be written as (Maaziz *et al.*, 2002; Benyahia, 2001):

$$\dot{X} = F(X) + GU \tag{1}$$

$$Y = H(X) \tag{2}$$

With

$$X = \begin{bmatrix} i_{s\alpha} & i_{s\beta} & \varphi_{r\alpha} & \varphi_{r\beta} & \Omega \end{bmatrix}^T \quad U = \begin{bmatrix} u_{s\alpha} & u_{s\beta} \end{bmatrix}$$

$$F(X) = \begin{bmatrix} -\gamma i_{s\alpha} + \frac{K}{Tr}\varphi_{r\alpha} + p\Omega K f_{r\beta} \\ -\gamma i_{s\beta} - p\Omega K \varphi_{r\alpha} + \frac{K}{Tr}\varphi_{r\beta} \\ \frac{M}{T_r} i_{s\alpha} - \frac{1}{Tr}\varphi_{r\alpha} - p\Omega_{r\beta} \\ \frac{M}{T_r} i_{s\beta} + p\varphi_{r\alpha} - \frac{1}{Tr}\varphi_{r\beta} \\ p\frac{M}{JL_r}\left(\varphi_{r\alpha} i_{s\beta} - \varphi_{r\beta} i_{s\alpha} - \frac{1}{J}(C_r + f\Omega\right) \end{bmatrix}$$

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$$G = \begin{bmatrix} \frac{1}{\sigma L_s} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{\sigma Ls} & 0 & 0 & 0 \end{bmatrix}$$

And

$$K = \frac{M}{\sigma LsLr}, \sigma = 1 - \frac{M^2}{L_s L_r} \gamma = \frac{R_s}{\sigma L_s} + \frac{R_r M^2}{\sigma L_s L_r^2}$$

The variables which are controlled are the flux and the electromagnetic torque.

$$Y(X) = \begin{bmatrix} h_1(X) \\ h_2(X) \end{bmatrix} = \begin{bmatrix} \varphi_r^2 \\ T_{em} \end{bmatrix} \quad (2)$$

FEEDBACK LINEARIZATION OF IM

Relative degree of the flux:

$$h_1(X) = (\varphi_{r\alpha}{}^2 + \varphi_{r\beta}{}^2) = \phi^2_r \quad (3)$$

$$L_f h_1(X) = \frac{2}{T_r} \begin{bmatrix} M(\varphi_{r\alpha} i_{s\beta} + \varphi_{r\beta} + i_{s\beta}) - \\ (\varphi^2_{r\alpha} + f^{\,2}_{r\beta}) \end{bmatrix} \quad (4)$$

$$L_{g1} L_f h_1 = 2R_r K \varphi_{r\alpha} \quad (5)$$

$$\begin{aligned} L^2{}_f h_1(X) = & \left(\frac{4}{T_r{}^2} + \frac{2K}{T_r{}^2} M \right) (\varphi_{r\alpha}{}^2 + \varphi^2{}_{r\beta}) - \\ & \left(\frac{6M}{T_r{}^2} + \frac{2\gamma M}{T_r^2} \right) (\varphi_{r\alpha} i_{s\alpha} + \varphi_{r\beta} i_{s\beta}) \\ & + \frac{2Mp\Omega}{T_r} (\varphi_{r\alpha} i_{s\beta} - \varphi_{r\beta} i_{s\alpha}) \\ & + 2\frac{M^2}{T^2{}_r} (i^2_{s\alpha} + i^2_{s\beta}) \end{aligned} \quad (6)$$

$$L_{g2} L_f h_1 = 2R_r K \varphi_{r\beta} \quad (7)$$

The degree f $h_1(x)$ is $r_1 = 2$.

Relative degree of torque:

$$h_2(X) = T_{em} \quad (8)$$

$$T_{em} = J\frac{d^2\Omega}{dt} + f\frac{d\Omega}{dt} + \dot{T}_l \quad (9)$$

$$L_f h_2(X) = -\frac{pM}{L_r} \begin{bmatrix} \left(\frac{1}{T_r} + \gamma \right) \left(\varphi_{r\alpha} i_{s\beta} - \varphi_{r\beta} i_{s\alpha} \right) + \\ p\Omega \left(\varphi_{r\alpha} i_{s\alpha} - \varphi_{r\beta} i_{s\beta} \right) \\ + p\Omega K \left(f^{\,2}_{r\alpha} - f^{\,2}_{r\beta} \right) \end{bmatrix} \quad (10)$$

$$L_{g1} L_f h_2 = -pK\varphi_{r\beta} \quad (11)$$

$$L_{g2} Lfh_2 = pK\varphi_{r\alpha} \quad (12)$$

The degree of h2 (x) is $r_2 = 1$.

Global relative degree: The global relative degree is lower than the order n of the system ($r = r_1 + r_2 = 4 < n = 5$).
The system is said partly linearized

Decoupling matrix: The matrix defines a relation between the input (U) and the output (Y(X) is given by the expression (13):

$$\begin{bmatrix} \ddot{h}_1(X) \\ \ddot{h}_2(X) \end{bmatrix} = \begin{bmatrix} \frac{d^2\varphi_r^2}{dt} \\ \frac{dT_{em}}{dt} \end{bmatrix} = A(X) + D(X) \begin{bmatrix} u_{s\alpha} \\ u_{s\beta} \end{bmatrix} \quad (13)$$

Where

$$A(X) = \begin{bmatrix} L_f^2 h_1 & L_f h_2 \end{bmatrix}$$

The decoupling matrix is:

$$D(X) = \begin{bmatrix} L_{g1} L_f h_1 & L_{g2} L_f h_1 \\ L_{g1} L_f h_2 & L_{g2} L_f h_2 \end{bmatrix}$$

And

$$\det(D) = \frac{2pR_r KM}{J\sigma L_s I_r} (\varphi^2_{r\alpha} + \varphi^2_{r\beta}) \neq 0$$

The nonlinear feedback provide to the system a linear comportment input/output

$$\begin{bmatrix} u_{s\alpha} \\ u_{s\beta} \end{bmatrix} = D(X)^{-1}\left[-A(X) + \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}\right] \quad (14)$$

To get the outputs (2), we use this transformation

$$\ddot{y}_1 = \frac{1}{\sqrt{\tilde{y}_1}}\left(\frac{1}{2}\ddot{\tilde{y}}_1 - \frac{1}{4\tilde{y}_1}\dot{\tilde{y}}_1^2\right) \quad (15)$$

For sake of simplicity, it is more convenient to work with space vectors.
The equation motion is given by

$$J\frac{d\omega_r}{dt} = pI_m(\varphi_r^*) - f\omega_r - T_{em} \quad (16)$$

Where $I_m(\cdot)$: Imaginary part and $\varphi_r = \varphi e^{j\varphi}$
The rotor electrical equation is given by

$$\frac{d\varphi_r}{dt} - jp\omega_r\varphi_r + R_r i_r \quad (17)$$

The motor produced by the motor is given by

$$T_{em} = pI_m\left(\phi_r i_r^*\right) = \frac{p}{R_r}\phi^2\left(\delta - p\omega_r\right) \quad (18)$$

Set $\alpha = \delta\text{-}p\theta$ Substituting T_{em} from (18) in (16), the equation of motion becomes

$$J\frac{d\omega_r}{dt} = \frac{p}{R_r}\phi^2\dot{\alpha} - f\omega_r - T_L \quad (19)$$

Assuming that the disturbance $T_L = 0$, we have:

$$\alpha = \frac{R_r}{p}\int_0^t\left(\frac{J\dot{\omega}_r}{\phi^2} + \frac{f\omega_r}{\phi^2}\right)d\tau \quad (20)$$

Considering the output Eq. 2

$$\alpha = \frac{R_r}{p}\int_0^t\left(\frac{J\dot{y}_2}{y_1^2} + \frac{fy_2}{y_1^2}\right)d\tau \quad (21)$$

The rotor flux is given by:

$$\phi_r = y_1\exp\left(j\frac{R_r}{p}\int_0^t\left(\frac{J\dot{y}_2}{y_1^2} + \frac{fy_2}{y_1^2} + \frac{p^2}{R_r}y_2\right)d\tau\right) \quad (22)$$

Using (17) and (22), the rotor current can be expressed as

$$\begin{aligned} i_r &= -\frac{1}{R_r}\left(\frac{d\phi_r}{dt} - jp\omega_r\phi_r\right) \\ &= -\frac{1}{R_r}\left[\dot{y}_1 + j\frac{R_r}{p}\left(\frac{J\dot{y}_2}{y_1} + \frac{fy_2}{y_1}\right)\right]e^{j(\alpha+p\theta)} \end{aligned} \quad (23)$$

The rotor flux space is given by

$$\phi_r = Mi_s + L_r i_r \quad (24)$$

The stator current space vector is then given by

$$i_s = \frac{1}{M}\left[y_1 + T_r A\right]e^{j(\alpha+p\theta)} \quad (25)$$

Where:

$$A = \dot{y}_1 + j\frac{R_r}{p}\left(\frac{J\dot{y}_2}{y_1} + \frac{fy_2}{y_1}\right) = \dot{y}_2 + j\dot{\alpha}y_1 \quad (26)$$

The stator flux space vector is given by

$$\phi_s = L_S i_S + Mi_r \quad (27)$$

From (23) and (25), (27) becomes

$$\varphi_s = \left(\frac{L_S}{M}y_1 + \frac{1}{KR_r}A\right)e^{j(\alpha+p\theta)} \quad (28)$$

Finally, the stator voltage can be expressed as

$$\begin{aligned} u_s &= u_{s\alpha} + ju_{s\beta} = Ri_s + \frac{df_s}{dt} \\ &= \frac{R_s}{M}\left(y_1 + \frac{AL_r}{R_r}\right)e^{j(\alpha+p\theta)} + \\ &\quad \frac{d}{dt}\left\{\left(\frac{L_s}{M}y_1 + \frac{A}{KR_r}\right)e^{j(\alpha+p\theta)}\right\} \end{aligned} \quad (30)$$

The α, β components of the stator voltage are

$$\begin{aligned} u_{s\alpha} &= \frac{R_S}{M}\begin{pmatrix} y_1\cos\delta + \frac{L_r}{R_r}\dot{y}_1\cos\delta \\ -\frac{L_r}{R_r}y_1\dot{\alpha}\sin\delta \end{pmatrix} \\ &\quad \frac{L_S}{M}\left(\dot{y}_1\cos\delta - y_1\dot{\delta}\sin\delta\right) + \frac{1}{KR_r} \\ &\quad \begin{pmatrix} \ddot{y}_1\cos\delta - y_1\dot{\alpha}\dot{\delta}\cos\delta - y_1\ddot{a}\sin\delta \\ -\dot{y}_1\dot{\delta}\sin\delta - \dot{y}_1\dot{\alpha}\sin\delta \end{pmatrix} \end{aligned} \quad (31)$$

$$u_{s\beta} = \frac{R_S}{M}\begin{pmatrix} y_1\sin\delta + \frac{L_r}{R_r}\dot{y}_1\sin\delta \\ + \frac{L_r}{R_r}y_1\dot{\alpha}\cos\delta \end{pmatrix}$$
$$\frac{L_S}{M}\left(\dot{y}_1\sin\delta + y_1\dot{\delta}\cos\delta\right) + \frac{1}{KR_r}$$
$$+\begin{pmatrix} \ddot{y}_1\sin\delta - y_1\dot{\alpha}\dot{\delta}\sin\delta + y_1\ddot{\alpha}\cos\delta \\ +\dot{y}_1\dot{\delta}\cos\delta + \dot{y}_1\dot{\alpha}\cos\delta \end{pmatrix} \quad (32)$$

CONTROL FLUX AND TORQUE OF LINEAR SYSTEM

The internal outputs (V_1, V2) are definite

$$V_1 = \frac{d^2\varphi^2{}_r}{dt} = -K_{11}(\varphi_r{}^2 - \varphi^2{}_{ref}) - K_{12}(\frac{d}{dt}\varphi^2{}_r - \frac{d}{dt}\varphi^2_{ref}) + \frac{d^2\varphi^2_{ref}}{dt} \quad (33)$$

$$V_2 = \frac{d^2T_{em}}{dt} = -K_{22}(T_{em} - T_{ref}) \quad \frac{dT_{ref}}{dt} \quad (34)$$

The errors of the track in (CL) are:

$$\ddot{e}_1 + k_{12}\dot{e}_1 + K_{11}e_1 = 0 \quad (35)$$

$$\dot{e}_2 + K_{22}e_2 = 0 \quad (36)$$

With:

The coefficients K_{11}, K_{12}, K_{22}, are chosen to satisfy asymptotic stability and excellent tracking.

$$\begin{bmatrix} u_{s\alpha} \\ u_{s\beta} \end{bmatrix} = D(X)^{-1}\left[-A(X) + \begin{pmatrix} -K_{11}e_1 - K_{12}\frac{d}{dt}\varphi_r{}^2 \\ -K_{22}e_2 + \frac{dT_{ref}}{dt} \end{pmatrix}\right] \quad (37)$$

The plant will be transformed to another representation that is given by;

$$z = \begin{bmatrix} h_1(X) & L_f h_1(X) & h_2(X) & \arctan(\frac{\varphi_{r\beta}}{\varphi_{r\alpha}}) & \Omega \end{bmatrix}$$

FLUX ESTIMATOR AND OBSERVER REFERENCE TORQUE

Flux estimator: The flux components are not accessible for the measure, then it will be estimate, in our case we used a simple rotor flux estimator, based on the model of machine, in order to exhibit the robustness of the proposed control (Clerc, 1999).

$$i_{\tilde{d}s} = i_{\alpha s}\cos\tilde{\theta}$$
$$i_{\tilde{q}s} = -i_{\beta s}\sin\tilde{\theta} \quad (38)$$

$\tilde{d}$: Is the axis of rotor flux

$$\tilde{\varphi}_r = M|i_{mr}| \quad (39)$$

With

$$T_r\frac{d|i_{mr}|}{dt} + |i_{mr}| = i_d$$

$$\tilde{\omega} = \omega_r + \frac{i_{\tilde{q}s}}{T_r|i_{mr}|}$$

Reference torque observer: The measure of the error between the speed measured and the estimate one, is presented as an input of PI regulator so the output is given by: (Clerc and Grellet, 1991; Pioufle, 1993) (Fig. 1 and 2)

$$T_{ref} = \frac{(k_1 s + k_2)}{Js^2 + (f + k_1)s + k_2} \quad (40)$$

k_1 and k_2 and are determined by a poles position.

$$i_{\tilde{d}s} = i_{\alpha s}\cos\tilde{\theta}$$
$$i_{\tilde{q}s} = -i_{\beta s}\sin\tilde{\theta} \quad (41)$$

$\tilde{d}$: Is the axis of rotor flux

$$\tilde{\varphi}_r = M|i_{mr}| \quad (42)$$

With

$$T_r\frac{d|i_r|}{dt} + |i_{mr}| = i_d$$

$$\tilde{\omega} = \omega_r + \frac{i_{\tilde{q}s}}{T_r|i_{mr}|}$$

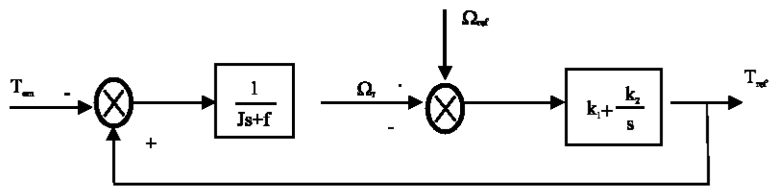

Fig. 1: Block diagram of reference torque observer

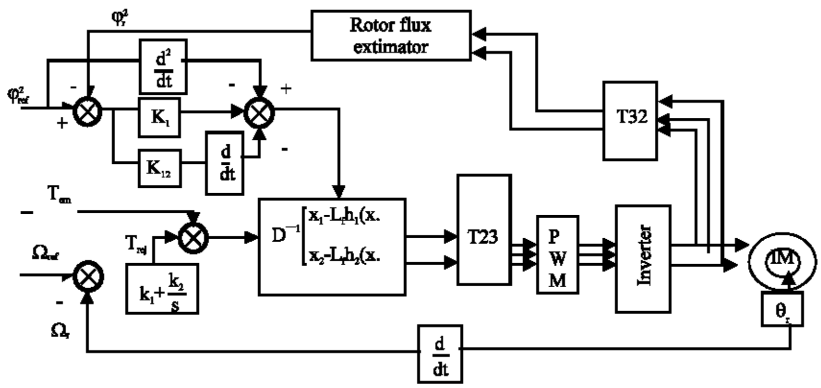

Fig. 2: Block diagram of the nonlinear control

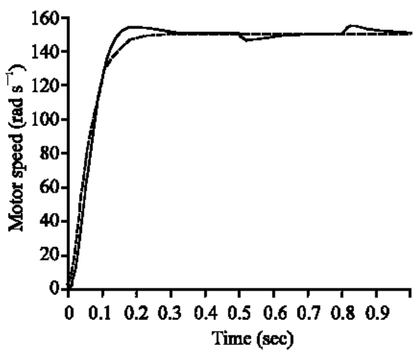

Fig. 3: Motor speed

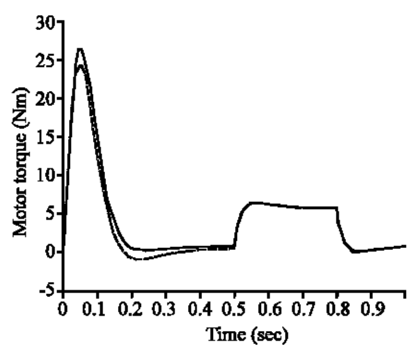

Fig. 4: Motor torque

SIMULATION

The results are shown in Fig. (3-6) that response of speed for an echelon of 150 (rad/s), has a good tracking, for the flux norm remains tight to their reference, the test is given by application of the load at (0.5s) and its effect is rejected after few seconds. The second test Fig. (7-10) represented the variation of the speed among two values -150 (rad s^{-1}) and +150 (rad s^{-1}) and the last one Fig. (11-16) is the difference of the inertia at 100% of its value, we remarked that this variation effect on the response of the speed and the flux, then the parameters machine variation represented a problem for the nonlinear control.

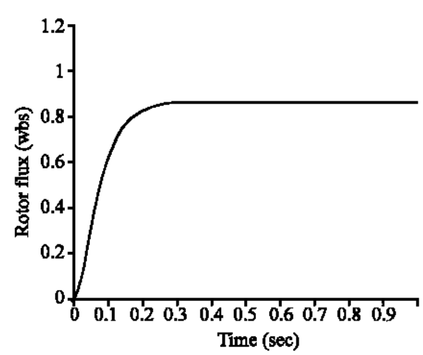

Fig. 5: Rotor flux

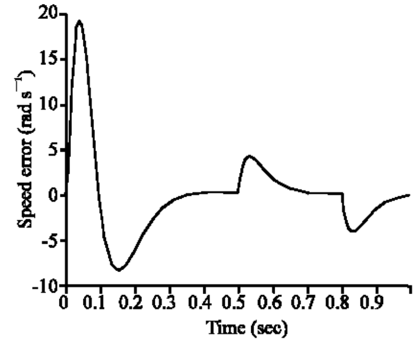

Fig. 6: Speed error

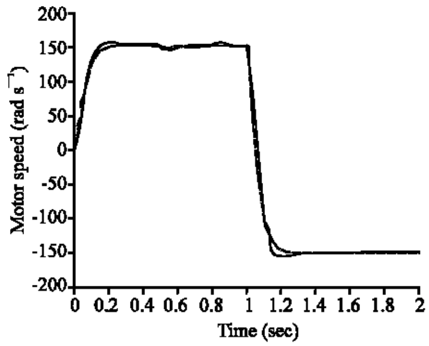

Fig. 7: Motor speed

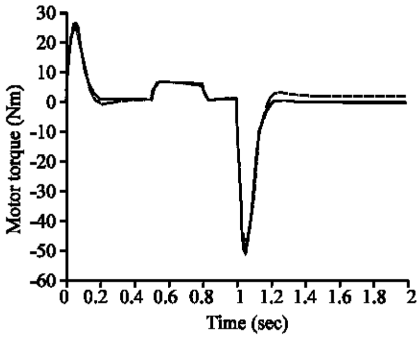

Fig. 8: Motor torque

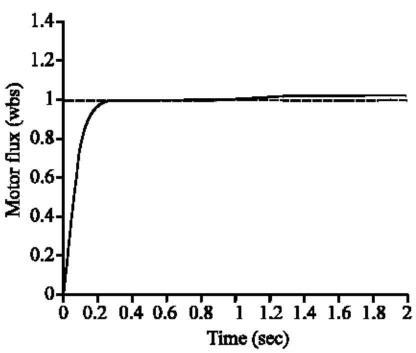

Fig. 9: Rotor flux

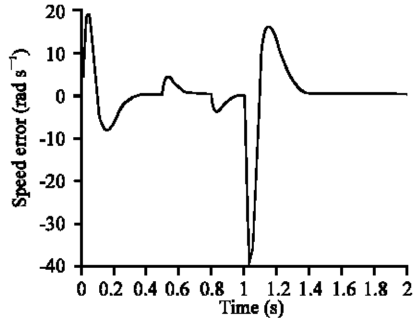

Fig. 10: Speed error

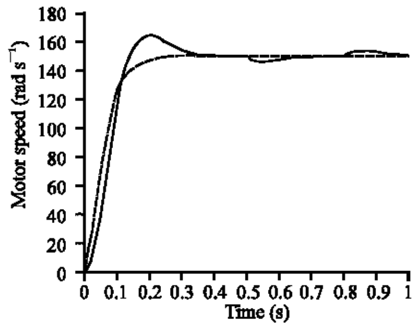

Fig. 11: Motor speed

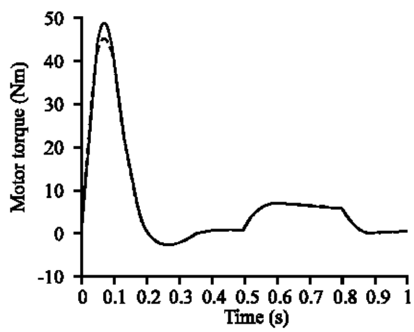

Fig. 12: Motor torque

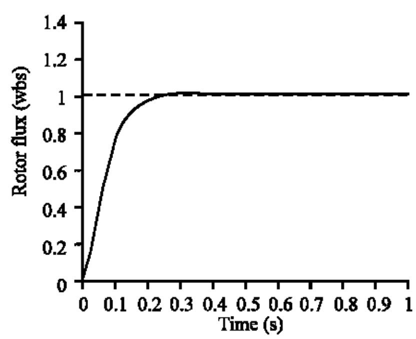

Fig. 13: Rotor flux

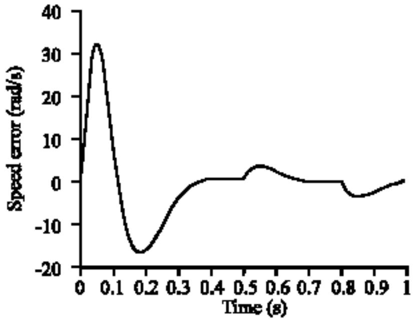

Fig. 14: Speed error

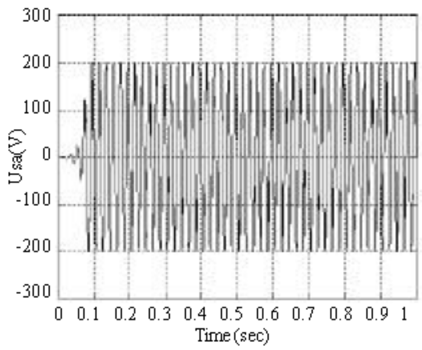

Fig. 15: Direct stator voltage

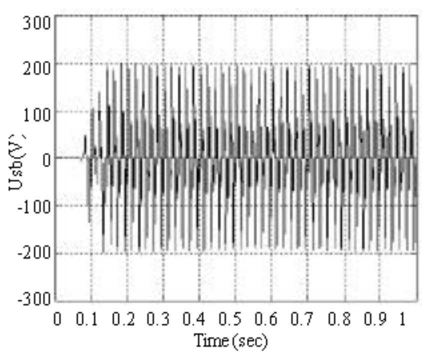

Fig. 16: Indirect stator voltage

CONCLUSION

The non-linear control gives a good tracking for the speed with basing of its static and dynamic properties. The results show that the decoupling between the parameters of IM is very excellent. This technique gives a better amelioration for the performances of system, nevertheless the problem of the parameters machine variation witch represent a difficulty for this control and the simple regulators (PI), can not resist to this variation, which necessities a robust one. In order to minimize the dependence of the system to parameter variations and external perturbations and reduce the number of variable states, a new model of the original system is proposed, this simple model will be suitable to cognitive approach such as fuzzy modelling and robust fuzzy controller could be applied to the induction motor basing in this new model. The two expression of the stator voltage can be used as the reference model for the predictive control and we will estimate the load torque by the Kalman filter.

Appendix A; List of principal symbols

R_s, R_r : Stator and rotor resistance.
L_s, L_r : Stator and rotor inductance.
M : Mutual inductance.
Ω : Motor speed.
ϕ : Rotor flux norm.
I_{mr} : Is the magnetic current.

Appendix B; Machine parameters

1.1KW, 220/380V, 50Hz, 1500 rpm.
$R_r = 3.6\Omega$, $J = 0.01$ kgm^2, $p = 2$,
$R_s = 8.0\Omega$, $f = 0.005$Nms, $M = 0.452$H
$L^r = 0.47$H, $L_s = 0.47$H, $T_{nom} = 5$ Nm
$\varphi r\alpha\beta = 1.14$Wbs.

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