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Numerical Comparison Between Electromagnetic Forces Calculation Methods

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Abstract: To calculate magnetic global force on bodies several approaches can be used with the Finite Element Method (FEM). They are based on Maxwell's stress tensor, or on the Lorentz formula or on the virtual work principle. The accuracy of the results for a given mesh is an important criterion for choosing between them. Description and comparison of these methods applied to the calculation of magnetic global forces on current carrying conductor is done.

Key words: Numerical comparison, electromagnetic, calculation methods, FEM, conductor

INTRODUCTION

Both for vibrations and electromechanical conversion, the determination of forces is of greatest important. The choice of the most appropriate method is essential for an efficient modelling of the system. High accuracy or the saving of computing time is determinant. Sevral methods allow the calculation of the magnetic global forces on electromagnetic devices (Coulomb, 1983; Rezig, 2002). They rely on different principle (Lorentz formula, Maxwell stress tensor, virtual work principle) and give, theoretically, the same result for the global force calculation. As force is calculated from finite element solution (approximate solution), the results depend on the accuracy of different methods. Thus, for the same finite element mesh, results can be sharply differing. The aim of this study is to apply these methods in the computation of forces on current carrying conductors. The accuracy and computing time for different finite element meshes are discussed. Precision is not only criterion to be taken into account, for some application, the ease of implementation may be more important.

MATERIALS AND METHODS

Lorentz formula: Consider a current carrying coil with constant permeability, the force density within coil is (Belahcen, 1999):

$$f = J \wedge B = \frac{1}{\mu_0}(\text{rot } B) \wedge B \qquad (1)$$

Where the Ampere low ($J = \text{rot}(H)$) and $B = \mu_0 H$ have been used.

The expression ($f = J \wedge B$) gives the density of force acting on the current carrying conductors. By integrating over the conductor volume, we get the total force.

Maxwell stress tensor method: For the calculation of forces by the Maxwell stress tensor, one can use either a surface integration or volume integration (Coulomb, 1983; Rezig, 2002). Each way presents some advantages with respect to the other.

Surface integration method: The force on the body can be calculated by integrations of the Maxwell stress tensor on a surface surrounding the body (Rezig, 2002) (in 2 dimensions modelling). The terms of the Maxwell stress tensor is defined as:

$$\tau_{ij} = \frac{1}{\mu_0} B_i B_j - \delta_{ij} \frac{1}{2\mu_0} B^2.n \qquad (2)$$

is the Koneke delta. The global force is given by:

$$F = \int_V \text{div}\, \tau \, dV \qquad (3)$$

After the use of Green's theorem the global force is evaluated by Rezig (2002), Medeiros (1998) and Belahcen (1999):

$$F = \int_s [\frac{1}{\mu_0}(B.n)B - \frac{1}{\mu_0} B^2.n]\, dS \qquad (4)$$

Where: S is an arbitrary surface surrounding the object. Theoretically the choice of this surface should not

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influence the results of the force calculation. Thought, as the force is obtained with an approximate finite element solution, the results depend on the choice of this surface. To overcome the definition of the surface integration, a volume integration method is proposed by Medeiros (1998) and Belahcen (1999). The advantage of this method is that the same FEM mesh is used to calculate the force and electromagnetic computation.

Volume integration method: With this method, the force on a direction i on a node k is given by:

$$f_k = -\int_{\Omega} [B_k]^T \{\tau\}\, d\Omega \qquad (5)$$

Ω: The volume of the elements that contains the node k.

$$[B_k]^T = \begin{bmatrix} \frac{\partial N_k}{\partial x} & 0 & 0 & \frac{\partial N_k}{\partial y} & 0 & \frac{\partial N_k}{\partial z} \\ 0 & \frac{\partial N_k}{\partial y} & 0 & \frac{\partial N_k}{\partial x} & \frac{\partial N_k}{\partial z} & 0 \\ 0 & 0 & \frac{\partial N_k}{\partial xz} & 0 & \frac{\partial N_k}{\partial y} & \frac{\partial N_k}{\partial x} \end{bmatrix} \{t\} = \begin{Bmatrix} t_{xx} \\ t_{yy} \\ t_{zz} \\ t_{xy} \\ t_{yz} \\ t_{zx} \end{Bmatrix}$$

τ_{ij}: Terms of the tensor of Maxwell given by the Eq. 2.
N_k: The nodal shape functions.

The global force is given by the summation of the forces on all nodes of the object. This method is very adapted to FEM code and does not require the definition of any surface of integration.

Virtual work methods: The method virtual work has consisted of taking the derivative of the magnetic energy with respect to the displacement with a constant vector potential. The magnetic energy is defined as Coulomb (1983):

$$W = \int_V \int_0^B H\, dB\, dV_e \qquad (6)$$

And the force is given by

$$F = -\frac{\partial W}{\partial s} = -\frac{\partial}{\partial s}\left(\int_0^B H\, dB\, dV_e \right) \qquad (7)$$

After approximation with finite elements method this equation can be written in the reference element as:

$$F = \int_{V_e} \left[\frac{1}{\mu_e} B \frac{\partial B}{\partial s} \det(J) + \int_0^H H dB dV \frac{\partial \det(J)}{\partial s} \right] dV_e \qquad (8)$$

This gait to apply the virtual work principle is very difficult to implement for this reason, we presented in this research an author gait to apply this principle, witch based on the use of a new formulation of magnetic energy given by:

$$W = \int_0^A J' \, dA \qquad (9)$$

The finite element system is written as: M.A = J
The nodal force is then:

$$F = -\frac{\partial W}{\partial s} = -\int_0^A A^t \frac{\partial M(A,s)}{\partial s} dA \qquad (10)$$

For an element of the mesh the elementary matrix $[M^e]$ is given by:

$$[M_{ij}^{\,e}] = \frac{\nu}{4\Delta}\left[b_i b_j + c_i c_j \right]$$

$$\text{Where}: b_i = y_i - y_k \text{ and } c_i = x_k - x_j$$

The differentiation of matrix M is done analytically and all elements that are not function of A are taken outside the integral. Only one integration is made for the part that is A dependent.

RESULTS AND DISCUSSION

The methods described above are implemented and compared hereafter to allow the most appropriate choice. Theoretically, all methods give the same global force, hence any method provided that an adequate mesh is chosen, will converge to the analytical solution.

One of models (Fig. 1) used in this study consists of current carrying conductors with a square cross section (Medeiros, 1998).

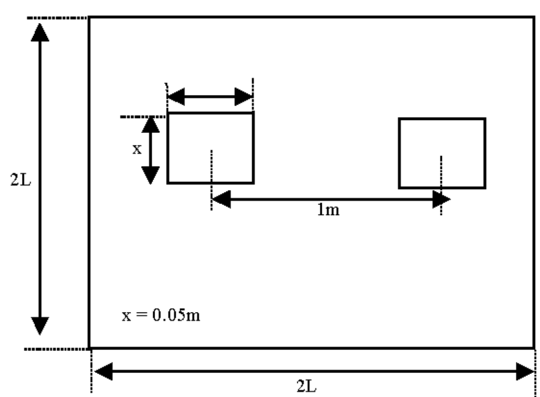

Fig. 1: Geometry of the model

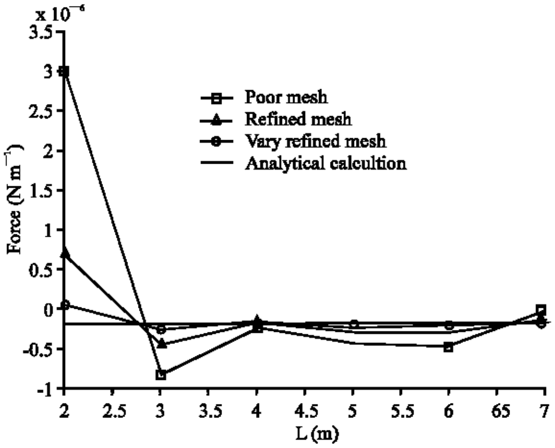

Fig. 2: Computed force with Lorentz method versus distance L to remote boundary

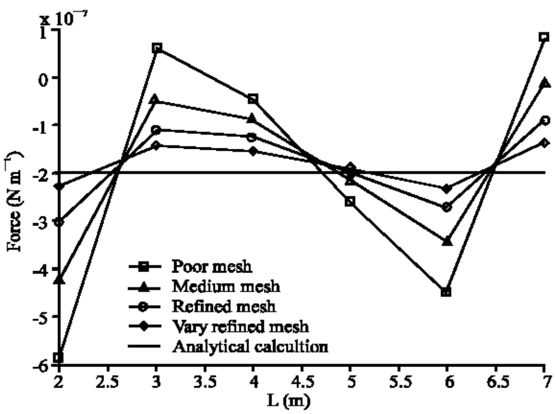

Fig. 3: Computed force with surface Maxwell stress method versus distance L to remote boundary

The global force calculated by the methods described above can change with the mesh discretization and the boundary of the mesh so that several meshes were prepared to investigates these problems.

To investigate the effect of mesh discretization the mesh was adaptively refined. The result is shown in the following figures, where it can be seen that the computed force by the different methods converges to its analytical value as the number of mesh refinement is increased.

To investigate the effect of boundary, the computed force is plotted in the (Fig. 2-5) against the distance L from the conductor to its remote boundary. The remote boundary of L = 7 m used in earlier calculations was found to be adequate.

In Maxwell stress method and when the force is obtained with an approximate finite element solution, the results depend on the choice of the path of integration. For this reason the computed force by Maxwell stress method was plotted for two cases considered.

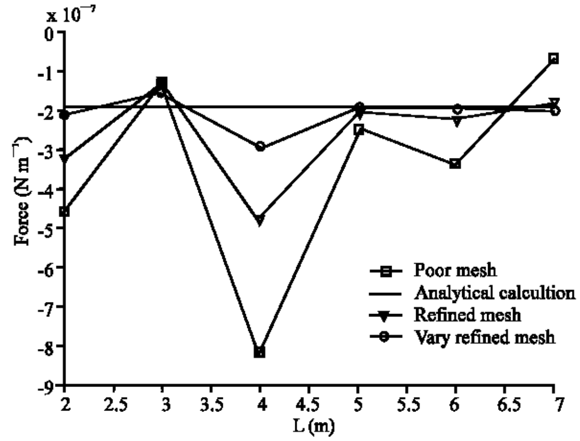

Fig. 4: Computed force with volume Maxwell stress method versus distance L to remote boundary

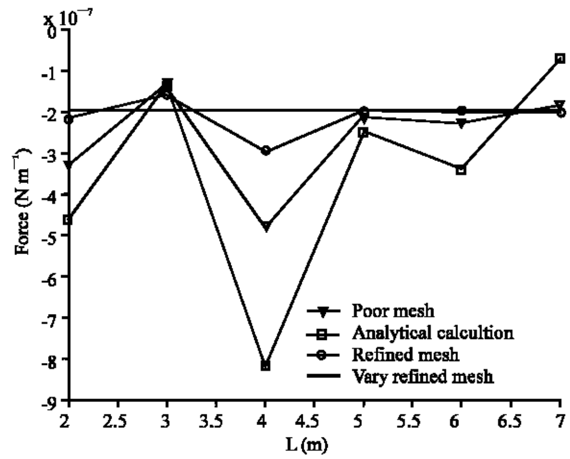

Fig. 5: Computed force with virtual work method versus distance L to remote boundary

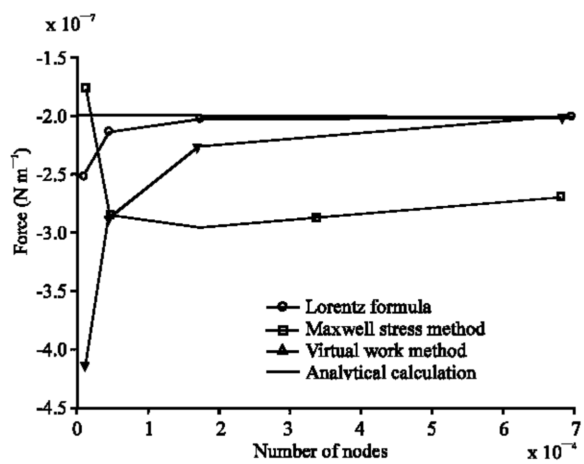

Fig. 6: Computed force versus number of nodes in mesh

In the first the terms of Maxwell tensor is integrated on an arbitrary path surrounding the conductor Fig. 6. In

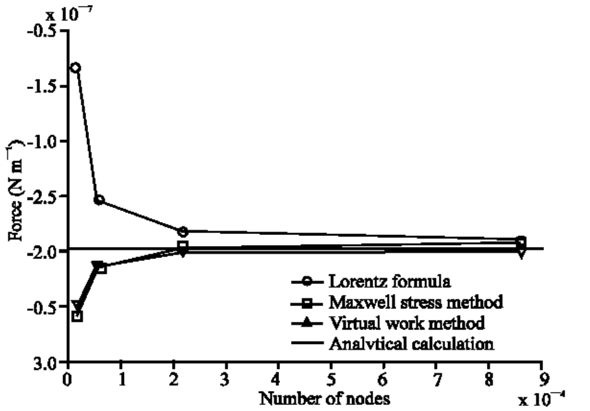

Fig. 7: Computed force versus number of nodes in mesh

the second the terms of Maxwell tensor is integrated on using centroid path surrounding the conductor Fig. 7.

It is obvious that the force computed using centroid path is better than the case of the arbitrary path.

CONCLUSION

Different methods for the calculation of the global force have been presented, analysed and compared in relation to the ease of implementation, computation time and accuracy.

It has shown that the most precise methods are those based on volume integration (Lorentz formula, Maxwell stress tensor with volume integration, virtual work methods). The method based on Maxwell's stress tensor with an arbitrary surface integration is the fastest ones, but when the surface integration is changed the result obtained by Maxwell stress method with centroid path were mush better than for an arbitrary path.

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