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Using Reliability Indices-Markov Model in Electric Power Distribution Reliability Assessment

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Abstract: A Reliability Indices-Markov model for up-coming year reliability prediction suitable for Electric Power Distribution Reliability Assessment is presented. This model was developed using the IEEE Std 1366^{TM} 2003 and Markov chain. The model was calibrated with 2003 power outage data and validated using 2004 data collected from the monopolistic operator of the Nigerian power system, National Electric Power Authority (now unbundled and called Power Holding Company of Nigeria, (PHCN) Plc) on the four feeders: Uselu, Ugbowo, Eguadaiken and FGGC Road feeders radiating from the 2×15 MVA transformers, 33/11kV Ugbowo injection substation. A maximum error in prediction of -0.12% was obtained in the Eguadaiken feeder while a minimum error of -5.5% is obtained in Ugbowo feeder. Due to these low error margins, the model can be used confidently.

Key words: Markov, reliability, distribution, electricity, outages

INTRODUCTION

Electricity system is a critical infrastructure designed to supply electrical energy and its continued and reliable functioning is essential to any nation's economy and citizen's way of life. The electricity system impacts on other networked infrastructure such as water, communications and transportation etc., since they depend on it for their operation.

Electricity reliability means that electric power service should be delivered to the consumer with a high degree of assurance. Historically, most important attention regarding reliability analysis has been put into power generation rather than into distribution system reliability (Billinton and Allan, 1996). Distribution systems make the greatest contribution to the unavailability of Electric Power Supply to customers (Billinton and Allan, 1996) and the liberalization of the power sector would make distribution system reliability of more interest now than ever before. Various studies have shown that about 75, 80 and 85% of electric power interruptions are due to distribution system problems (Billinton and Allan, 1996; National Energy Board Canada, 2004; North West Power Planning Council, 2000; Vicentini *et al*., 2004). Thus, there is need for the development of fast predictive model that will enable electric power distribution companies to predict up-coming year reliability in order to decide on where the next financial investment should be.

Despite the fact that about 80% of the power interruptions occur in the distribution systems, the reliability analysis of these systems had not received much attention until few years ago (Vicentini *et al*., 2004). Though reliability is a broad area, it has two main sub-divisions: System adequacy and system security (Goel and Billinton, 1993; Billinton *et al*., 1987; VPX, 1998).

There are two main techniques to assess electric power distribution reliability: Simulation and analytical techniques. In the simulation technique such as Monte Carlo simulation (NEPLAN Reliability analysis, 2000), it is highly time consuming and expensive because it has to simulate a huge number of failures. Also, since the simulation of probabilistic events generates variable results, in effect simulating the variability of real life, it is usually necessary to perform a number of runs in order to obtain estimates of means and variance of the output parameters of interest, such as availability, number of repairs arising and repair facility utilization (O`Connor, 2002). Analytical technique is sub-divided into network and Markov modeling. Since distribution networks are largely radial, the methods are simple to understand and implement but lack prediction ability. Markov modeling is a well define approach with fast computer run time when all the states are defined.

The states can be defined by the various network techniques but they are complex. Thus, there are various

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simple reliability indices developed by Institute of Electrical and Electronics Engineers for electric power utilities to quantitatively measure the level of performance of electric power supply. In this study, these reliability indices are used to define the states and transition states of the Markov model for Electric Power distribution reliability assessment.

MATERIALS AND METHODS

Historical power outage data between 2003 and 2004 from the monopolistic operator of the Nigerian power system, National Electric Power Authority (now unbundled and called Power Holding Company of Nigeria, (PHCN) Plc) on the four feeders: Uselu, Ugbowo, Eguadaiken and FGGC Road feeders radiating from the 2×15 MVA transformers, 33/11kV Ugbowo injection substation located in Benin City, Edo State of Nigeria were collected and analysed using Microsoft Access database (PHCN, 2003, 2004). The average customers connected to each of the feeders are shown in Table 1. This study was carried out between 2005 and 2006.

The reliability assessment was based on the IEEE Std 1366^{TM} 2003 reliability indices and Markov modeling. These methods are discussed as follows.

IEEE Std 1366^{TM} 2003: The Institute of Electrical and Electronics Engineers (IEEE) worried by the inconsistency in reporting distribution services reliability indices drew up IEEE Std 1366^{TM} 1998.The IEEE Std 1366^{TM} 1998 is a guide for Electric Power Distribution Reliability Indices. It was reversed in 2003 and called IEEE Std 1366^{TM} 2003.

The purpose of this standard is to provide (IEEE Std 1366^{TM}, 2003):

- Uniformity and consistency in reporting practices in the electric power distribution industry.
- Personnel with tools necessary for the comparison of various distribution companies.

One of the indices called Average System Availability Index (ASAI) defined in the IEEE Std 1366^{TM} 2003 was used in this study. It represents the fraction of time that a customer has received power during the reporting period, usually of one year. Mathematically, this is given as:

$$\text{ASAI} = \frac{8760C - \sum_{i=1}^{m} \sum_{j=1}^{k} C_{ij} T_{ij}}{8760C} \qquad (1)$$

Table 1: No. of customers per feeder

	Year	
Feeders	2003	2004
FGGC road	1826	2227
Ugbowo	4994	5436
Eguadaiken	2808	3107
Uselu	6490	6767
Total	16118	17537

Where:

C = Total number of customers connected.

T_{ij} = Cumulative interruption duration (minutes) for customers load affected by jth restoration step associated with ith interruption,

$$T_{ij} = \sum_{k=1}^{j} T_{ij}$$

C_{ij} = Number of customers restored during jth restoration step

Markov modeling: A Markov process is a stochastic system for which the occurrence of a future state depends only on the immediately preceding state. Because of this reason, the Markovian process is characterized by a lack of memory. Therefore, a discrete parameter stochastic process, {X (t); t = 0, 1, 2...}, or a continuous parameter stochastic process, {X (t); t≥0}, is a Markov process if it has the following Markovian property (Gonen, 1986):

$$P\left\{X(t_n) \geq x_n \middle| X(t_1)=x_1, X(t_2)=x_2;\ldots,X(t_{n-1})=x_{n-1}\right\} = P\left\{X(t_n) \leq x_n \middle| X(t_{n-1})=x_{n-1}\right\} \qquad (2)$$

for any set of n time points, $t_1 < t_2 < \ldots < t_n$ in the index set of the process and any real numbers X_1, $X_{2,\ldots,}X_{n.}$ The probability of

$$Px_{n-1},x_n = P\left\{X(t_x)=x_n \middle| X(t_{n-1})=x_{n-1}\right\} \qquad (3)$$

is called the transition probability and represents the conditional probability of the system being in $X_{n'}$ at t_n, given it was X_{n-1} at t_{n-1}. It is also called the one-step transition probability due to the fact that it represents the system between t_{n-1} and t_n. Of course, one can define a k-step transition probability as:

$$P_{x_n,x_{n+k}} = P\left\{X(t_{n+k})=x_{n+k} \middle| X(t_n)=x_n\right\} \qquad (4a)$$

or as

$$P_{x_{n-k},x_n} = P\left\{X(t_n)=x_n \middle| X(t_{n-k})=x_{n-k}\right\} \qquad (4b)$$

A Markov chain is defined by a sequence of discrete-valued random variables, $\{X(t_n)\}$, where t_n is discrete-valued or continuous. Therefore, one can also define the Markov chain as the Markov process with a discrete state space. Define:

$$P_{ij} = P\{X(t_n) = j | X(t_{n-1}) = i\} \quad (5)$$

as the one-step transition probability of going from state i at t_{n-1} to state j at t_n and assume that these probabilities do not change over time. The term used to describe this assumption is stationarity. If the transition probability depends only on the time difference, then the Markov chain is defined to be stationary in time. Therefore, a Markov chain is completely defined by its transition probabilities, of going from state i to state j, given in a matrix form:

$$P = \begin{array}{cc} & \text{To state } j \\ \text{From state } i & \left[\begin{array}{c|cccccc} & 0 & 1 & 2 & 3 & \text{——} & n \\ 0 & P_{00} & P_{01} & P_{02} & P_{03} & \text{——} & P_{0n} \\ 1 & P_{10} & P_{11} & P_{12} & P_{13} & \text{——} & P_{0n} \\ 2 & P_{20} & P_{21} & P_{22} & P_{23} & \text{——} & P_{2n} \\ 3 & P_{30} & P_{31} & P_{32} & P_{33} & \text{——} & P_{3n} \\ \hline n & P_{n0} & P_{n1} & P_{n2} & P_{n3} & \text{——} & P_{nn} \end{array}\right] \end{array} \quad (6)$$

The number of states for n-components system is given as 2^n possible combinations of operating and failed states (Ebeling, 2005). Basically, the various components in the substation are in series. Thus, the substation can only be in two states: Operating and failure states. Therefore, the associated transition matrix can be expressed as

$$P = \begin{bmatrix} P_{11} \, P_{10} \\ P_{01} \, P_{00} \end{bmatrix} \quad (7)$$

Where:

P_{11} = Probability of being in state 1 at time t, given that it was in state 1 at time zero.
P_{00} = Probability of being in state 0 at time t, given that it was in state 0 at time zero.
P_{10} = Probability of being in state 0 at time t, given that it was in state 1 at time zero.
P_{01} = Probability of being in state 1 at time t, given that it was in state 0 at time zero.

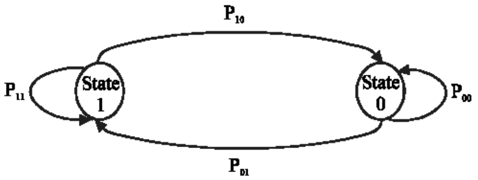

Fig. 1: State transition diagram for a two-state system

The associated state transition diagram is shown in Fig. 1.

Here, states 1 and 0 represent the system being up (operating) and down (fail), respectively.

Applying Chapman-Kolmogorov equations we have

$$p^{(n)} = p^{(0)} \, p^{(n)} \quad (8)$$

Where:

$p^{(n)}$ = State probabilities at time, t_n
$p^{(0)}$ = Initial state probabilities at time, t_0
$p^{(n)}$ = Transition matrix after n-step transition.

In more basic terms, Eq. 8 can be given in statement form as:

$$\begin{aligned} &\text{State probabilities at time, } t_n = \\ &(\text{Initial position matrix at time } t_0) \times \\ &(\text{Transitionalmatrix after n} - \text{step transition})^n \end{aligned} \quad (9)$$

RESULTS AND DISCUSSION

The expression of Eq. 1 was used to evaluate the ASAI of each feeder and the results are presented in Table 2.

The Average System Availability Index (ASAI) was combined with Markov model to develop a predictive model for the four feeders. The ASAI evaluated from the 2003 data gives the element P_{11}, the transition from up (1) state to down (0) state, P_{10} is given as 1 - $P_{11.}$ As a result of bad data, an error factor is used to correct the unavailability index in order to obtain P_{00}. Transition from up (0) state to down (1) state, i.e., P_{01} is given as 1 - P_{01}.The state space diagram was then constructed and consequently the transitional matrix.

The probability of power availability in 2004 was then predicted using the leading matrix (0, 1) and the transition matrix as the trailing matrix. The value of n is 2 because we are predicting 2004. Percentage error in prediction was calculated based on the system availability index evaluated from 2004 power outage data. These analyses are presented as follows.

Table 2: The 2003 and 2004 ASAI per feeder

	Year	
Feeders	2003	2004
FGGC road	0.78984	0.78879
Ugbowo	0.73215	0.69483
Eguadaiken	0.75915	0.75938
Uselu	0.79300	0.78797

Table 3: The prediction error of the model

Feeders	Data based 2004 ASAI	Predicted 2004 ASAI	Prediction error (%)
FGGC road	0.7888	0.7956	-0.35
Ugbowo	0.6948	0.7330	-5.50
Eguadaiken	0.7594	0.7603	-0.12
Uselu	0.7880	0.7936	-0.70

FGGC Road feeder:

$P_{11} = 0.7898$, $P_{01} = 1-0.7898 = 0.2102$
Total data = 849
Bad data = 15
Error factor = 15/849 = 0.018
$P_{00} = 0.2102-0.2102(\text{error factor})$
$= 0.2102-0.2102(0.018) = 0.2064$
$P_{01} = 1- P_{00}=1-0.2064 = 0.7986$

Using Eq. 7 and the FGGC Road feeder state space diagram in Fig. 2 , the transition matrix is given as:

$$P = \begin{bmatrix} 0.78980.2102 \\ 0.79860.2064 \end{bmatrix}$$

Recall Eq. 8 and substituting we have,

$$\begin{array}{ccc} \text{State} & 1 & 0 \\ & [0 & 1] \end{array} \text{x} \begin{bmatrix} 0.7898 & 0.2102 \\ 0.7986 & 0.2064 \end{bmatrix}^2 = \begin{array}{ccc} \text{State} & 1 & 0 \\ & 0.7956 & 0.2105 \end{array}$$

A program written in M-file environment of MATLAB® was used to evaluate this matrix multiplication. Thus, the predicted 2004 ASAI is 0.7956.

$$\text{Prediction error} = \left(\frac{\text{ASAI}_{2004} - \text{ASAI}_{\text{Predicted}}}{\text{ASAI}_{2004}} \right) \times 100$$
$$= \left(\frac{0.7888 - 0.7956}{0.7888} \right) \times 100 = -0.35\%$$

In the same way, other feeders were analysed and the results are presented in Table 3.

The Reliability indices-Markov model predicted up-coming year reliability for all the feeders using 2003 data. The prediction was validated using 2004 data. A maximum error in prediction of -0.12% was obtained in the Eguadaiken feeder while a minimum error of -5.5% is obtained in Ugbowo feeder. Due to these low error margins, the model can be used confidently in electric power distribution reliability assessment.

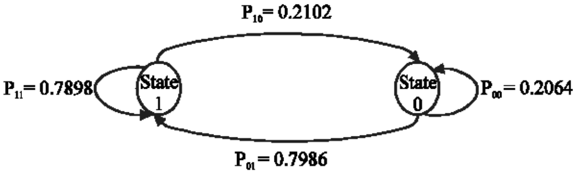

Fig. 2: FGGC Road feeder state space diagram

CONCLUSION

Reliability assessment studies are crucial for distribution systems. A Reliability indices-Markov model has been developed for up-coming year reliability prediction. The model was calibrated with 2003 power outage data and validated using 2004 data collected from the monopolistic operator of the Nigerian power system, National Electric Power Authority (now unbundled and called Power Holding Company of Nigeria, (PHCN) Plc) on the four feeders: Uselu, Ugbowo, Eguadaiken and FGGC Road feeders radiating from the 2×15 MVA transformers, 33/11kV Ugbowo injection substation. A maximum error in prediction of -0.12% was obtained in the Eguadaiken feeder while a minimum error of -5.5% is obtained in Ugbowo feeder. Due to these low error margins, the model can be used confidently.

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