International Journal of Electrical and Power Engineering 1 (4): 438-442, 2007

Fuzzy Sets-Based Nonlinear Modelling of Induction Machines: An Alpha-Cut Approach

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Abstract: This study is devoted to nonlinear modelling of induction machines by using the fuzzy set concepts. Induction machines are known as complex nonlinear systems in which time-varying parameters entail an additional difficulty for control and diagnostic purposes. Based on the fact that nonlinear system problems can be solved by using fuzzy set concepts, an induction machine model expressed in terms of nonlinear state space model is investigated and discussed. Fuzzy sets are used to describe uncertainty, nonlinearity and time-variance of the system parameters. This leads to nonlinear state space model with fuzzy parameters. The simulation results show that the proposed approach can be effectively applied to induction machine control and diagnostic problems. To our knowledge, this approach is new in the field of electric machine modelling, control and diagnostic.

Key words: Nonlinear modelling, induction machines, fuzzy sets, fuzzy parameters, alpha-cuts, Zadeh extension principle

INTRODUCTION

It is well known that the induction machine, due to its high reliability, relatively low cost and modest maintenance requirements, is one of the most widely used machines in industrial applications. However, induction machine is also known as a complex nonlinear system in which time-varying parameters entail additional difficulties for control and diagnostic purposes (Holtz, 2002; Bensaker *et al*., 2004). Uncertainties and a lack of information are important problems arising in nonlinear time-varying system modelling. Traditional knowledge-based modelling techniques require a deep understanding of the induction machine system, exact model and precise numeric values for parameters and measurements. Furthermore, the obtained models don't take into account the vagueness and uncertainty present in parameters or in the measurement process (Binh and Ross, 2005).

Taking into account the previous drawbacks, this paper investigates a new fuzzy-sets based approach for nonlinear modelling of induction machines.

Fuzzy sets theory is a powerful tool for modelling uncertainty and for processing vague or subjective information in mathematical models. The main advantage of fuzzy models is their ability to describe expert knowledge, experience of engineers and know how of operators, in a descriptive human like way, by using simple rules to represent relationships between linguistic variables (Fetz *et al*., 1999; Selekwa and Collins , 2005; Rouben, 2006).

Recently, fuzzy models have been used to efficiently approximate nonlinear systems in various fields, including decision making, system control and monitoring. Fuzzy modelling and control are typical examples of techniques that make use of human knowledge (Babuska and Abonyi, 2000; Abonyi *et al*., 2000). Human knowledge is qualitative in its nature and often imprecise. Fuzzy modelling is one of the so-called "intelligent" modelling methodologies, which employ techniques motivated by human intelligence to develop models for dynamic systems (Huang and Shen, 2003).

FUZZY SET BASIC CONCEPTS

Let X, be a set. A fuzzy subset of X is defined by its membership function, $\mu_A(x)$, $0M\leq\mu_A(x)\leq 1$, describing the degree of membership of the value x in A. In other words, the membership function assigns each member x of X the valuation $\mu_A(x)$, which is the degree to which the element belongs to the subset. A The membership function $\mu_A(x)$ can also be interpreted (Oberguggenberger and Pittschmann, 1999) as:

- The membership degree of the element x belongs to the set A.

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- The truth value of the statement that the element x belongs to the set A.
- The degree of possibility that the variable A takes the value x.

A fuzzy subset A of X is called normalized if:

$$\sup\{\mu_A(x) : x \in X\} = 1 \quad (1)$$

The sets $A_\alpha = \{x \in X : \mu_A(x) \ge \alpha, 0 \le \alpha \le 1\}$ are the (strict) α-level sets of A. The α-level sets A_a, called also the alpha-cut sets, are the classical (crisp) subsets of X which correspond to a closed interval for each given value of α

$$[A_\alpha] = [A_L^\alpha \ A_R^\alpha] \quad (2)$$

Where A^a_L and A^a_R represent, respectively the left and the right portion of the membership function at thelevel (Oberguggenberger and Russo, 2001).

The most used membership function to describe uncertainty in model parameters is the triangular or the triangular shaped function. A triangular (shaped) membership function is generally defined by three real numbers a, b and c. Where the base of the triangle is the interval [a c] and its vertex is at x = b. The membership function for the triangular fuzzy parameter θ will be written as θ = (a,b,c) and it can be defined mathematically by:

$$\theta(x) = \begin{cases} \dfrac{x-a}{b-a} & \text{if } 0 \le x \le b \\ \dfrac{x-c}{b-c} & \text{if } b \le x \le c \end{cases} \quad (3)$$

To be a triangular shaped membership function, it is required the corresponding graph to be continuous and monotonically increasing on the interval [a b] and monotonically decreasing on the interval [b c]. The core of a fuzzy parameter is the set of values where its membership value equals one. It corresponds to the nominal (crisp) value of the parameter.

Figure 1 shows a triangular membership function of a fuzzy parameter θ (Theta) and its alpha-cuts.

From Fig. 1, one can see that a fuzzy parameter, as a fuzzy number, can be viewed as a set of valued intervals, which is the projection of the alpha-cuts on the support of the membership function (universe of discourse).

Figure 1 shows also that higher the value of the higher the confidence in the fuzzy parameter. The risk encored in taking the confidence of level α can be defined as:

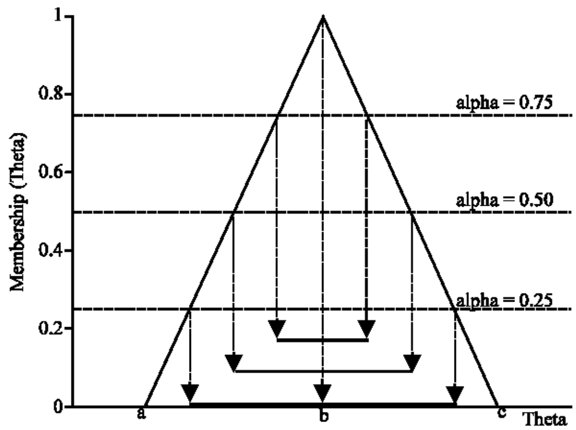

Fig.1: Triangular fuzzy parameter and its alpha-cuts

$$r = 1 - \alpha \quad (4)$$

For a given function f: X→Y, the Zadeh extension principle defines how the function f acts on a fuzzy subset of X providing the fuzzy subset f(A) of Y (Oberguggenberger and Pittschman, 1999; Oberguggenberger, 2004).

In other words, the fluctuations described by the α-level sets of $\mu_{f(A)}$ are those that arise from mapping the α-level sets of $\mu_{f(A)}$. Also the membership degree $\mu_{f(A)}(y)$ of a value y is the maximum of all degrees $\mu_{(A)}$ (x) of those giving y = f (x).

SYSTEMS FUZZY MODELLING

Modelling complex system techniques can be roughly divided in two groups. The first group is concerned with the so-called black-box model techniques in which the model parameters may not be related to the system physical parameters as neural network models. The second group is concerned with the so-called knowledge-based models which can be in their turn divided into two approaches.

The first approach concerns the analytical or quantitative knowledge-based models and the second approach concerns the qualitative knowledge-based models. In qualitative knowledge-based model approaches, the knowledge is derived in terms of facts and rules to describe the system behaviour.

With the development of the theory of qualitative reasoning it becomes possible to control and supervise dynamic systems and to diagnose their faults. Fuzzy sets theory is a powerful tool for modelling uncertainty and for processing subjective information in mathematical models. It is increasingly used as a means for modelling and evaluating the influence of imprecisely known parameters in mathematical, technical and physical models. Three main types of fuzzy model structures have been presented in the study (Edgar and Postlethwaite, 1999; Abonyi *et al.*, 1999).

Rule-based model: This model type is used in fuzzy control where the system behaviour is described by a set of fuzzy If-then rules of the following form:

- IF Condition THEN Conclusion

It can be divided in its turn into two approaches: The linguistic (Mamdani) model and the Takagi–Sugeno model. In Linguistic fuzzy model *(Mamdani)* both the condition and the conclusion are fuzzy propositions. In Takagi–Sugeno fuzzy model the condition is a fuzzy proposition and the conclusion is a crisp function expressed as a linear combination of the input variables appearing in respective conditions (Abonyi *et al.*, 1999).

Fuzzy relational model: This type of model represents an alternative to rule-based system model preserving the system qualitative characteristics. It can be interpreted by If-then statements too (Abonyi *et al.*, 2000; Edgar and Postlethwaite, 1999).

Fuzzy functional model: This model has a hybrid structure with fuzzy sets representing data in condition part and a linear function expressing the input-output state relationships in conclusion part. It can be seen as a generalization of the Takagi-Sugeno model.

This study uses a fuzzy functional model approach. This can represent faithfully nonlinear dynamical systems with time varying parameters, taking into account the imprecise knowledge of parameters and their fluctuations as in (Oberguggenberger and Pittschmann, 1999).

INDUCTION MACHINE FUZZY FUNCTIONAL MODEL

It is well known that the induction machine, due to its high reliability, relatively low cost and modest maintenance requirements, is one of the most widely used machines in industrial applications. However, induction machine is also known as a complex nonlinear system in which time-varying parameters entail additional difficulties for control and diagnostic purposes. Different models have been presented in the literature (Holtz, 2002; Bensaker *et al.*, 2004). The choice of a model structure depends on the problem at hand and on the available measurements. A complete model, generally, takes into account the rotor angular velocity as well as the rotor (stator) resistance variations. An example of such model is described, in an arbitrary rotating d-q Park reference frame, by the following nonlinear time-varying state space model (Bensaker *et al.*, 2004):

$$\frac{d}{dt}\begin{pmatrix} i_s \\ \varphi_r \end{pmatrix} = \begin{bmatrix} A_{11} & A_{21} \\ A_{21} & A_{22} \end{bmatrix}\begin{pmatrix} i_s \\ \varphi_r \end{pmatrix} + \begin{bmatrix} B_2 \\ 0_2 \end{bmatrix} u_s \qquad (5)$$

$$\frac{d\omega_r}{dt} = -\frac{f}{J}\omega_r + \frac{1}{J}\frac{p}{2}(T_e - T_l) \qquad (6)$$

$$T_e = p\frac{m}{l_r} i_s^T J_2 \varphi_r \qquad (7)$$

$$\begin{pmatrix} i_r \\ \varphi_s \end{pmatrix} = \begin{bmatrix} -L_r^{-1}M & L_r^{-1} \\ (L_s - ML_r^{-1}M) & ML_r^{-1} \end{bmatrix}\begin{pmatrix} i_s \\ \varphi_r \end{pmatrix} \qquad (8)$$

$$A_{11} = \Omega_a - (L_s - ML_r^{-1}M)^{-1}(R_s + ML_r^{-1}A_{21}),$$
$$A_{12} = (L_s - ML_r^{-1}M)^{-1}(\Omega_a ML_r^{-1} - ML_r^{-1}A_{22}),$$
$$A_{21} = R_r L_r^{-1}M,\ A_{22} = \Omega_{ar} - R_r L_r^{-1},$$
$$B_0 = (L_s - ML_r^{-1}M)^{-1}$$

The model parameters are:
With:

$$\Omega_a = \begin{bmatrix} 0 & \omega_a \\ -\omega_a & 0 \end{bmatrix},\ \Omega_{ar} = \begin{bmatrix} 0 & (\omega_a - \omega_r) \\ -(\omega_a - \omega_r) & 0 \end{bmatrix},$$
$$R_s = \begin{bmatrix} r_s & 0 \\ 0 & r_s \end{bmatrix},\ R_r = \begin{bmatrix} r_r & 0 \\ 0 & r_r \end{bmatrix},\ L_s = \begin{bmatrix} l_s & 0 \\ 0 & l_s \end{bmatrix},$$
$$L_r = \begin{bmatrix} l_r & 0 \\ 0 & l_r \end{bmatrix},\ M = \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix},\ J_2 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$
$$\text{and } I_0 = \begin{bmatrix} I_2 & 0_2 \\ 0_2 & 0_2 \end{bmatrix}$$

The indexes s and r refer to the stator and the rotor components, respectively. $\varphi = (\varphi_d\ \varphi_q)^T$ is the flux vector, $u = (u_d\ u_q)^T$ is the current vector, r is the voltage vector, is the resistance, l is the inductance, m is the mutual inductance, ω_r is the rotor angular velocity, ω_a is the arbitrary rotating reference frame angular velocity, f is the friction coefficient, J is the moment of inertia coefficient, P is the number of pair pole. T_e is the electromagnetic torque, T_l is the mechanical load torque. I_2 denotes the 2×2 identity matrix and denotes the 2×2 null matrix.

In the above nonlinear time-varying model, the parameters cannot be determined analytically with sufficient precision. They depend on properties of operating conditions, about which only incomplete and vague information are available. For these parameters, the industrial engineer can usually provide lower and upper bounds at various risk levels, using his experience and extrapolating data of former projects.

One method of treating this uncertainty is to use fuzzy sets theory. Fuzzy set theory allows considering uncertain parameters as fuzzy numbers with some membership functions. Fuzzy numbers, defined as having an interval of confidence and levels of presumption, provide an effective means by which system parameters can be represented and interpreted. In general, a range of real numbers might be used to represent a fuzzy parameter approximately, in the style of interval analysis. In this way, parameters whose values are not known precisely can be treated and the operator experience, knowledge and judgement can be incorporated into the system model (Binh and Ross, 2005; Rouben, 2006).

RESULTS AND DISCUSSION

For the simulation experiments, we make the following assumptions on the fuzzy parameter A_{ij} as in (Oberguggenberger and Pittschmann, 1999; Fetz *et al.*, 1999):

- μ_A (x) at exactly the central point of the triangular shaped membership function, which is assumed to be continuous, strictly increasing to the left and strictly decreasing to the right of the central value x.
- All the alpha-cut sets A_a, $(0 \leq \alpha \leq 1)$ as well as the support of μ_A (x), universe of discourse, are compact.

Without loss of the generality, in this experiment we consider the influence of only one fuzzy parameter.

Figure 2 shows the speed induced fuzzy parameter. The membership function of the considered speed induced fuzzy parameter is cut horizontally at a finite number of α-levels between 0 and 1 with a period of 0.25. For each α-level of the parameter, the model is run to determine the minimum and maximum possible values of the output $X(t) = F(t, A_{ij})$.

which correspond to the left and the right solution of the state Eq. 5.

If the considered fuzzy parameter fluctuates in the sets $[A_{ij}]_\alpha$ with degree of possibility α, then the state of the system X(t) at time t is confined to the level set $[X (t)]_a$. Figure 3 gives, as an example, the left and right solutions around the nominal solution of the stator current component Isd only for α = 0.25.

Figure 4 represents the global solution for the stator current component to the different alpha-cut values. Similar results are obtained for the reminding state variables of the induction machine. In fact the global solution is reconstructed by considering the envelope of the trajectories arising from parameter values in level set $[A_{ij}]_\alpha$.

It is shown that the evolution of the system 5-8 can be described by 2n equations for the endpoints of the intervals determined by the corresponding alpha cut. Oberguggenberger and Pittschmann (1999) demonstrate that the global unique solutions always exist and the solution operator F, mapping each parameter to the solution X (t) = F(t, A) exist as well. Furthermore, the solution X (t) will be a fuzzy quantity as well (Fetz *et al.*, 1999).

This way the assessment of the variations of the fuzzy parameters are faithfully processed and reflected in the fuzzy output $[X (t)]_a$. In addition, the interpretation using α-level cuts shows that this concept is in accordance with risk assessment ideas: If the initial parameters vary in certain region $[\theta]_\alpha$ at risk level α, the variations in the solution are covered precisely by the family of functions with membership degree α to the fuzzy

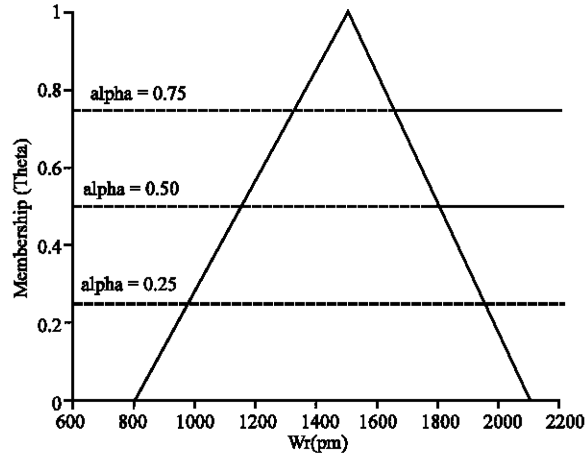

Fig. 2: Speed membership function

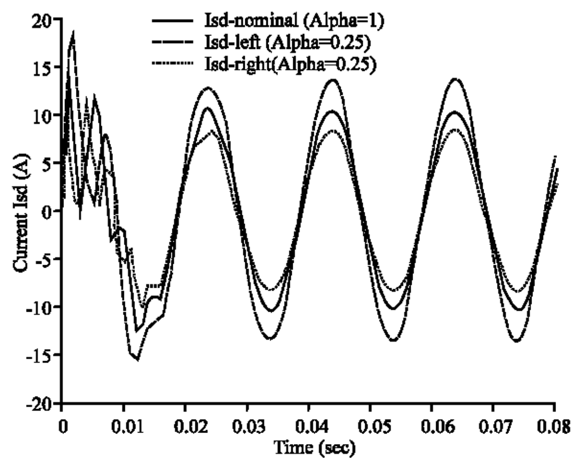

Fig. 3: Left and right solutions of the stator current

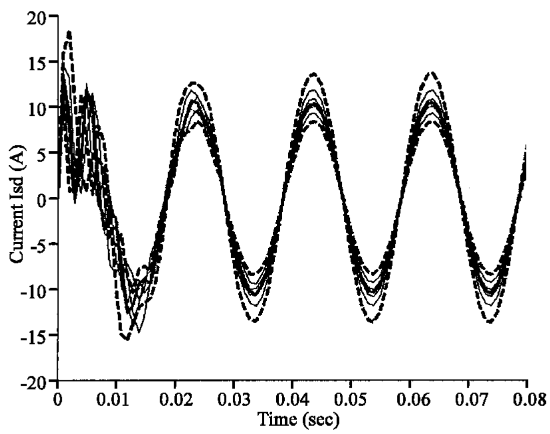

Fig. 4: Stator current global solution to induced speed fuzzy parameter

solution $[X]_\alpha = F([\theta]_\alpha$. These ideas can be significantly applied in control and diagnostic problems related to induction machines. These will be investigated in our future work.

CONCLUSION

The study has investigated the nonlinear modelling of induction machines by using fuzzy set concepts for control and diagnostic purposes. A nonlinear state space model with fuzzy parameters is used to point out the merits of the proposed approach. Fuzzy parameters are assumed to be triangular shaped membership function. The alpha cut or the alpha level approach is used for the simulation experiments. The simulation results show the effectiveness of the approach.

ACKNOWLEDGEMENT

The authors wish to thank all the reviewers for their eventual comments and suggestions.

This research was supported totally by the electromechanical systems laboratory under the project N° J0201120060047.

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