International Journal of Electrical and Power Engineering 1 (5): 479-485, 2007
ISSN: 1990-7958

Backstepping Based Control of PWM DC-DC Boost Power Converters

[1]H. El Fadil and [2]F. Giri
[1]Department LA2I, EMI, Rabat, Agdal, Morocco
[2]Greyc, Ismra, 6 Bd Maréchal Juin, 14050 Caen, France

Abstract: This study focuses on the problem of controlling DC-to-DC switched power converter of Boost type. The system nonlinear feature is coped with by resorting to the backstepping control approach. Both adaptive and nonadaptive versions are designed and shown to yield quite interesting tracking and robustness performances. A comparison study shows that backstepping nonlinear controllers perform as well as passivity-based controllers. For both the choice of design parameters proves to be crucial to ensure robustness with respect to load resistance variations. From this viewpoint, adaptive backstepping controllers are more interesting as they prove to be less sensitive to design parameters.

Key words: Switched power converters, nonlinear control, lyapunov function, PWM, boost power

INTRODUCTION

There three main types of switched power converters, namely Boost, Buck and Buck-Boost. These have recently aroused an increasing deal of interest both in power electronics and in automatic control. This is due to their wide applicability domain that ranges from domestic equipments to sophisticated communication systems. They are also used in computers, industrial electronics, battery-operating portable equipments and uninterruptible power sources. From an automatic control viewpoint, a switched power converter represents an interesting case study as it is a variable-structure nonlinear system. Its rapid structure variation is coped with by using averaged models (Middlebrook and Cuk, 1976; Sira-Ramirez *et al*., 1997). Based on such models, different nonlinear control techniques have been developed. These include passivity techniques (Ramirez *et al*., 1997), feedback linearization, flatness methods (Fliess and Sira-Ramirez, 1998) and more generally, sliding mode control (El Fadil *et al*., 2006). In this study, the problem of controlling switched power converters is approached using the backstepping technique (Krstic *et al*., 1995). While, feedback linearization methods require precise models and often cancel some useful nonlinearities, backstepping designs offer a choice of design tools for accommodation of uncertain nonlinearities and can avoid wasteful cancellations. In this study, the backstepping approach is applied to a specific class of switched power converters, namely DC-DC Boost converters. In the case where the converter model is fully known the backstepping nonlinear controller is shown to achieve the control objectives i.e. output voltage tracking and robustness with respect to load resistance uncertainty. In the case of unknown model an adaptive version of the above controller is developed and shown to ensure asymptotically the control objective. Finally, a comparison study shows that a backstepping controller does as well as a passivity-based controller. For both controllers the choice of design parameters turns out to be crucial to achieve robustness with respect to load resistance variations. Inversely, the performances of adaptive backstepping controllers are less sensitive to design parameters.

BOOST CONVERTER PRESENTATION AND MODELING

Boost converter is a circuit that is constituted of power electronics components connected as shown in Fig. 1. The circuit operating mode is the so-called Pulse Width Modulation (PWM). According to this principle, time is shared in intervals of length T (also called switching period). Within any period, the T-switch is on during a period fraction, say μT, for some $0 \leq \mu \leq 1$. Then, the current in the boost inductor L increases linearly and the diode D is off at that time. During the rest of the

Corresponding Author: H. El Fadil, Department LA2I, EMI, Rabat, Agdal, Morocco

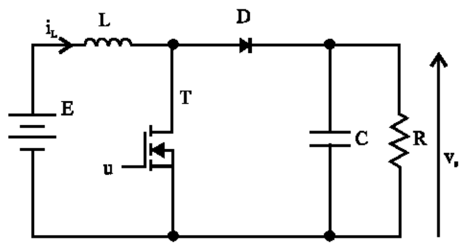

Fig. 1: Boost converter circuit

sampling period, i.e. (1-μ)T, the switch T is tuned off, consequently the energy stored in the inductor is released through the diode to the output RC circuit. It is worth noting that the value of μ varies from a switching period to another. The variation law of μ determines the value of output voltage v_s.

The averaged model of such a converter is shown to be the following (Middlebrook and Cuk, 1976; Sira-Ramirez *et al*., 1997; Middlebrook, 1989):

$$\begin{aligned} \dot{x}_1 &= -(1-\mu)\frac{x_2}{L} + \frac{E}{L} \\ \dot{x}_2 &= (1-\mu)\frac{x_1}{C} - \frac{x_2}{RC} \end{aligned} \tag{1}$$

Where, x_1 and x_2 denote the average input current (i_L) and the average output capacitor voltage (v_s), respectively. The control input for the above model is the function μ, called duty ratio function.

BACKSTEPPING NONLINEAR CONTROL OF BOOST CONVERTER

The backstepping approach is a recursive design methodology (Krstic *et al*., 1995). It involves a systematic construction of both feedback control laws and associated Lyapunov functions. The controller design is completed in a number of steps, which is never higher than the system order.

Nonlinear controller design

Direct output voltage regulation: The aim is to directly enforce x_2, the capacitor voltage output, to track a given reference voltage V_d>E. The latter is any bounded and smooth signal. Following the backstepping technique a controller is designed in two steps, since the controlled system (1) is a two-order.

Step 1: Let us introduce the output error:

$$z_1 = x_2 - V_d \tag{2}$$

Deriving $z_1$with respect to time yields and accounting for (1), implies:

$$\dot{z}_1 = \dot{x}_2 - \dot{V}_d = (1-\mu)x_1/C - x_2/(RC) - \dot{V}_d \tag{3}$$

In Eq. 3, x_1/C behaves as a virtual control input. Such an equation shows that one gets $\dot{z}_1 = c_1 z_1$(c_1>0 being a design parameter) provided that:

$$x_1/C = \left(-c_1 z_1 + x_2/(RC) + \dot{V}_d\right)/(1-\mu) \tag{4}$$

As x_1/C is just a variable and not (an effective) control input, (3) cannot be enforced at all time instants t≥0. Nevertheless, Eq. 3 shows that the desired value for the variable x_1/C is:

$$\alpha_1 = \left(-c_1 z_1 + x_2/(RC) + \dot{V}_d\right)/(1-\mu) \tag{5}$$

Indeed, if the error:

$$z_2 = \frac{x_1}{C} - \alpha_1 \tag{6}$$

vanishes (asymptotically) then the control objective is achieved i.e. z_1 vanishes in turn. The desired value α_1 is called a stabilization function.
Now, replacing x_1/C by (z_2+α_1) in (3) gives:

$$\dot{z}_1 = -c_1 z_1 + (1-\mu) z_2 \tag{7}$$

Step 2: Let us investigate the behavior of error variable z_2. In view of (6), (5) and (1), time-derivation of z_2 turns out to be:

$$\begin{aligned} \dot{z}_2 = &-(1-\mu)x_2/(LC) + E/(LC) - \dot{\mu}\alpha_1/(1-\mu) \\ &-\left\{c_1^2 z_1 - (1-\mu)c_1 z_2 + (1-\mu)x_1/(RC^2)\right. \\ &\left.-x_2/(RC)^2 + \ddot{V}_d\right\}/(1-\mu) \end{aligned} \tag{8}$$

In the new coordinates (z_1, z_2), the controlled system (1) is expressed by the couple of Eq. 7 and 8. We now need to select a Lyapunov function for such a system. As the objective is to drive its states (z_1, z_2) to zero, it is natural to choose the following function:

$$V = 0.5z_1^2 + 0.5z_2^2 \tag{9}$$

Its time-derivative along the (z_1, z_2)-trajectory, using (5), is given by:

$$\dot{V} = -c_1 z_1^2 - c_2 z_2^2 + z_2\left[c_2 z_2 + (1-\mu) z_1 + \dot{z}_2\right] \quad (10)$$

Where $c_2>0$ is a design parameter and $\dot{z}_2$ is to be replaced by the right side of (8). Equation 10 shows that the equilibrium $(z_1, z_2) = (0,0)$ is globally asymptotically stable if the term between brackets in (10) is set to zero. So doing, one gets the following control law:

$$\begin{aligned}\dot{\mu} = \Big\{ & \left[(1-\mu)^2 - c_1^2\right] z_1 + (1-\mu)(c_1 + c_2) z_2 \\ & -(1-\mu)x_1/(RC^2) + \left(1/(RC)^2 - (1-\mu)^2/(LC)\right)x_2 \\ & +(1-\mu)E/(LC) + \ddot{V}_d \Big\}/\alpha_1 \end{aligned} \quad (11)$$

Unfortunately, controller (11) is inappropriate due to its lack of stability. Indeed, the corresponding zero-dynamics (obtained by setting $z_1 \to 0$ and $z_2 \to 0$) turn out to be:

$$\dot{\mu} = R(1-\mu)^2\left[E - (1-\mu)V_d\right]/(LV_d) \quad (12)$$

The system (12) has two equilibriums at: $\mu = \mu_1 = 1$ and $\mu = \mu_2 = 1\text{-}E/V_d$. Only the second is meaningful since physically $0<\mu<1$. Furthermore, in the neighborhood of such an equilibrium ($\mu \cong \mu_2$) Eq. 12 becomes:

$$\dot{\mu} = R(1-\mu)^2(\mu - \mu_2)/L \quad (13)$$

Which clearly shows that the equilibrium $\mu = \mu_2$ is unstable. Direct output voltage regulation then turns out to be unable to deal with the control problem. Such a failure is due to the nonminimum phase nature of Boost converters.

Indirect output voltage regulation: As direct output voltage regulation has failed, the control problem will be chandelled resorting to the indirect approach. This consists in forcing output capacitor voltage regulation indirectly through the regulation of the input current. The new control objective is to enforce current x_1 to an appropriate reference I_d. The latter is chosen in such a way that if $x_1\text{-}I_d$ vanishes then so that $x_2\text{-}V_d$. From (1), it follows that the equilibrium I_d is given by:

$$I_d = V_d^2/(RE) \quad (14)$$

To this end, a controller is designed following exactly the same procedure as in the study. The obtained controller generates the duty ratio according to the following control law:

$$\begin{aligned}\dot{\mu} = \Big\{ & \left[c_1^2 - (1-\mu)^2\right] z_1 + (1-\mu)(c_1 + c_2) z_2 \\ & -(1-\mu)^2 x_1/(LC) - (1-\mu)x_2/(RLC) + \ddot{I}_d \Big\}/\alpha_1 \end{aligned} \quad (15)$$

Where,

$$z_1 = x_1 - I_d \quad (16)$$

$$z_2 = x_2/L - \alpha_1 \quad (17)$$

$$\alpha_1 = \left(c_1 z_1 + E/L - \dot{I}_d\right)/(1-\mu) \quad (18)$$

The performance of such a controller is described in the following proposition.

Proposition 1: Consider the control system consisting of the average PWM Boost model (1) in closed-loop with the controller (15), where the desired output voltage reference V_d is sufficiently smooth and satisfies $V_d>E$. Then, the equilibrium $(x_1,x_2,\mu) = (I_d, V_d, U_d)$ is locally asymptotically stable where $U_d = 1\text{-}E/V_d$

Remark: The zero-dynamics associated to the control law (15) are determined by letting $z_1 = z_2 = 0$ (i.e. $x1 = I_d$, $x_2 = E/(1\text{-}\mu)$). So doing one gets:

$$\dot{\mu} = (1-\mu)\left[(1-\mu)^2 RI_d - E\right]/(RCE) \quad (19)$$

the equilibriums of (19) are $\mu_1 = 1 - \sqrt{E/(RI_d)}$, $\mu_2 = 1 + \sqrt{E/(RI_d)}$ and $\mu_3 = 1$. The only meaningful equilibrium is μ_1 since $0<\mu<1$. In the neighborhood of μ_1 (19) becomes:

$$\dot{\mu} = 2(\mu_1 - \mu)/(RC) \quad (20)$$

This clearly shows that μ_1 is asymptotically stable and, consequently, the indirect control success.

Practical evaluation of the backstepping controller performances: The backstepping controller designed in the study has been applied to the Boost converter according to the experimental setting of Fig. 2.

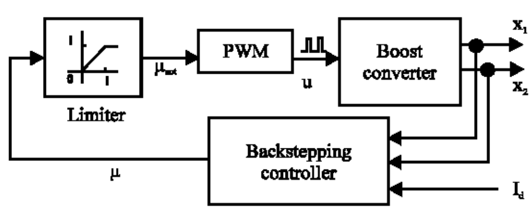

Fig. 2: Experimental bench for boost converter control

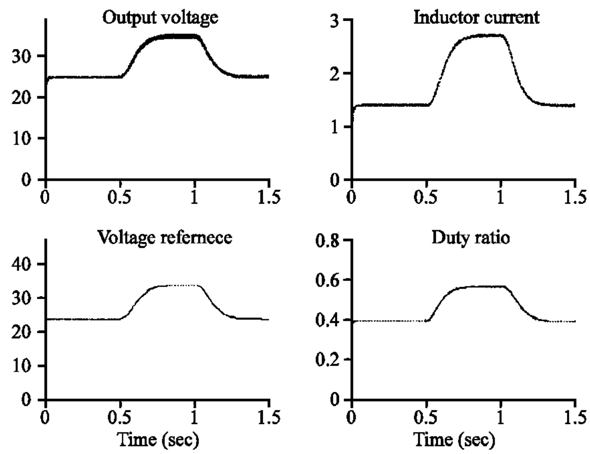

Fig. 3: Tracking Performances of the Backstepping Controller in presence of a time-varying output reference switching between 25 and 35 V

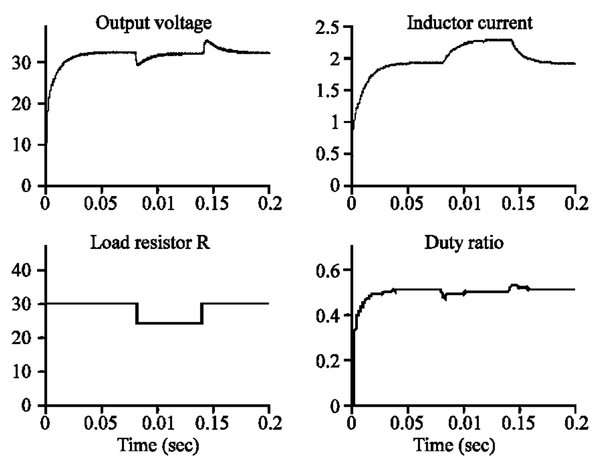

Fig. 4: Backstepping controller behavior in presence of a load resistance variation

The Boost circuit is designed with the following parameters values:
R = 30 Ω, L = 20 mH, C = 68 μF, E = 15 V.
The switching frequency is set: F = 20 KHz.

The backstepping controller parameters are: c_1 = 100 and c_2=1000.

Figure 3 shows the controller tracking behavior; the output voltage reference is a filtered square signal that switches between 25 and 35 volts. Such a behavior is quite satisfactory. Figure 4 illustrates the controller robustness with respect to a load resistance uncertainty; more precisely, the load nominal value (equal to 30 Ω) continues to be used in the control law (15), while the true load is time-varying as it switches between 25 and 30 Ω.

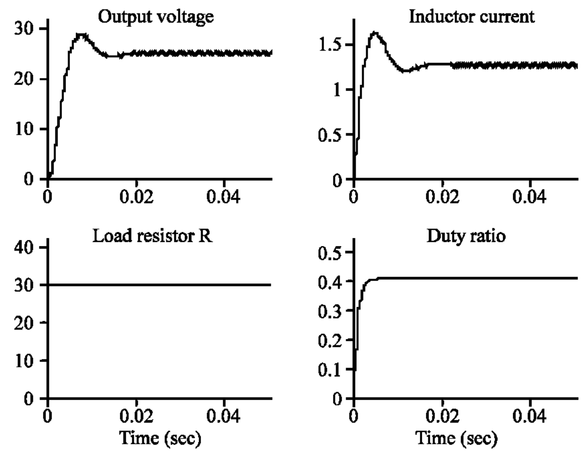

Fig. 5: Passivity-based controller: closed-loop response to a step reference voltage V_d = 25V

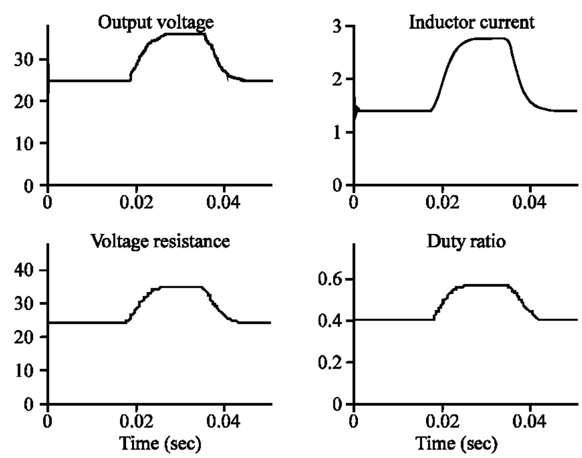

Fig. 6: Tracking behavior of the passivity-based controller

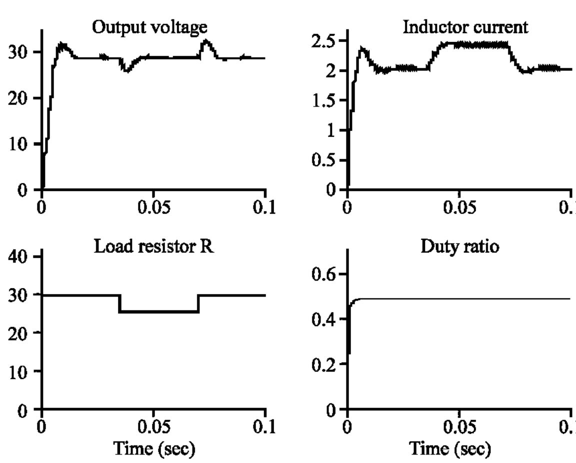

Fig. 7: Robustness of passivity-based controller with respect to load variations (between 25 and 30Ω)

Backstepping controller versus passivity-based controller: Using the passivity approach (Ramirez *et al*., 1997) a nonlinear controller has been designed in (El Fadil, 2002) for the Boost converter, to achieve indirect output voltage regulation. With the notations of (Ramirez *et al*., 1997) the involved design parameter R_1 has been set to 10^{-3}, this turned out to be the best choice. The controller thus obtained has been applied to the Boost converter of subsection 3.B. The resulting performances are summarized by Fig. 5-7. In the light of Fig. 3, 5 and 6 it is seen that, when the converter load is constant and perfectly known, the output reference tracking capabilities of the two controllers (backstepping and passivity) are globally comparable. The signal chattering that can be seen on v_s and i_L is due to the switching which is a physical feature of converters. Figure 4 and 7 show that the two controllers are equally robust to load uncertainty and variation.

BACKSTEPPING ADAPTIVE CONTROL OF BOOST CONVERTER

Controllers of the study, guarantee perform well only when the converter model is perfectly known. This particularly means that the load resistance is constant and time-invariant. When this is not the case, the controllers may still provide an acceptable behavior provided their design parameters are appropriately chosen. In practical situations, finding such an appropriate is not a simple task; it may take a too long time and necessitate many simulations. Therefore, adaptive versions of the above controllers turn out to be interesting alternatives.

Backstepping adaptive controller design: The new control design will be performed within the indirect output voltage control; for the reasons explained in subsection 3.A this means that the control objective is to enforce the Boost converter current x_1 to track its desired value $I_d = V_d^2/(RE)$. The difference with respect to study 3 lies in the fact that load resistance R is not known. To cope with such a model uncertainty the new controller will be given a learning capacity. More specifically, the controller to be designed should involve an on-line estimation of the unknown parameter

$$\theta = 1/R \tag{21}$$

The obtained estimate is denoted $\hat{\theta}$. With these notations, one gets:

$$I_d = \left(V_d^2/E\right)\theta \quad \hat{I}_d = \left(V_d^2/E\right)\hat{\theta} \tag{22}$$

where $\hat{I}_d$ denotes the estimate of I_d. Just as for the nonadaptive case, the adaptive design procedure includes two steps.

Step 1: Following closely step 1 of the design in section 3.A.2, one successively defines:

- The current error:

$$z_1 = x_1 - \hat{I}_d \tag{23}$$

- The stabilizing function:

$$\alpha_1 = \left(c_1 z_1 + E/L\right)/\left(1-\mu\right) \tag{24}$$

- The error:

$$z_2 = x_2/L - \alpha_1 \tag{25}$$

In (24) c_1 is a design parameter.

Step 2: Let

$$\theta = \hat{\theta} + \tilde{\theta} \tag{26}$$

The objective is to develop a control law that generates μ and an parameter adaptation law for $\hat{\theta}$, such that the resulting state vector $\left(z_1, z_2, \tilde{\theta}\right)$ vanishes asymptotically. To this end, it is natural to consider the following Lyapunov function candidate:

$$V = 0.5\left(z_1^2 + z_2^2 + \tilde{\theta}^2/\gamma\right) \tag{27}$$

Where, $\gamma > 0$ is any real constant, called parameter adaptation gain. Using (23) and (25), time derivation of V yields (after some algebraic manipulations):

$$\begin{aligned}\dot{V} = &-c_1 z_1^2 - c_2 z_2^2 - z_1 \dot{\hat{I}}_d \\ &+ z_2\Big[c_2 z_2 - (1-\mu) z_1 + (1-\mu) x_1/(LC) - x_2\hat{\theta}/(LC) \\ &- \dot{\mu}\alpha_1/(1-\mu) + \left\{c_1^2 z_1 + (1-\mu) c_1 z_2 + c_1 \dot{\hat{I}}_d\right\}/(1-\mu)\Big] \\ &+ \tilde{\theta}\left[-\dot{\hat{\theta}}/\gamma - x_2 z_2/(LC)\right]\end{aligned} \tag{28}$$

Where, c_2 is a design parameter. The last term in $\tilde{\theta}$, on the right side of (28), can be canceled using the following adaptation law

$$\dot{\hat{\theta}} = -\gamma x_2 z_2/(LC) \tag{29}$$

Time derivative of $\hat{I}_d$, can be obtained using (22) and (29):

$$\dot{\hat{I}}_d = -K_1 z_2 x_2 \tag{30}$$

Where,

$$K_1 = V_d^2\gamma/(ELC) \tag{31}$$

Combining (28), (29) and 30), one obtains:

$$\begin{aligned}\dot{V} = {} & -c_1 z_1^2 - c_2 z_2^2 \\ & + z_2\Big[K_1 z_1 x_2 + c_2 z_2 - (1-\mu) z_1 \\ & + (1-\mu)x_1/(LC) - x_2\hat{\theta}/(LC) \\ & - \dot{\mu}\alpha_1/(1-\mu) + \Big\{c_1^2 z_1 + (1-\mu)c_1 z_2 + c_1\dot{\hat{I}}_d\Big\}/(1-\mu)\Big]\end{aligned} \tag{32}$$

This suggests choosing μ so that the bracketed term, on the right side of (32), is equal to zero. Doing so, one gets the following control law (after some algebraic manipulations):

$$\begin{aligned}\dot{\mu} = {} & \Big\{\Big(c_1^2 - (1-\mu)^2\Big) z_1 + (1-\mu)(c_1 + c_2) z_2 + (1-\mu) K_1 z_1 x_2 \\ & + (1-\mu)^2 x_1/(LC) - (1-\mu) x_2\hat{\theta}/(LC) + c_1\dot{\hat{I}}_d\Big\}/\alpha_1\end{aligned} \tag{33}$$

The following proposition summarizes the main results of the study.

Proposition 2: Consider the control system including the average PWM Boost model (1), where R is the only unknown parameter, in closed-loop with the adaptive controller (33)-(29). If the reference output voltage V_d is smooth enough and satisfy $V_d > E$, then the closed-loop system equilibrium $(x_1, x_2, \mu) = (\hat{I}_d, V_d, U_d)$ is locally asymptotically stable where $U_d = 1 - E/V_d$.

Practical evaluation of the backstepping controller performances: The components of the controlled Boost converter have the same values as in study 3. The adaptive controller design parameters have the following values: $c_1 = 110$; $c_2 = 1000$; $\gamma = 10^{-7}$. The corresponding performances are illustrated by Fig. 8. This shows that, despite the load resistance uncertainty, the controller behavior is quite satisfactory. It is worth noting that such a good behavior is preserved when facing different variations of the load resistance i.e. there is no need to tune the design parameters values (c_1 and c_2). Unlikely,

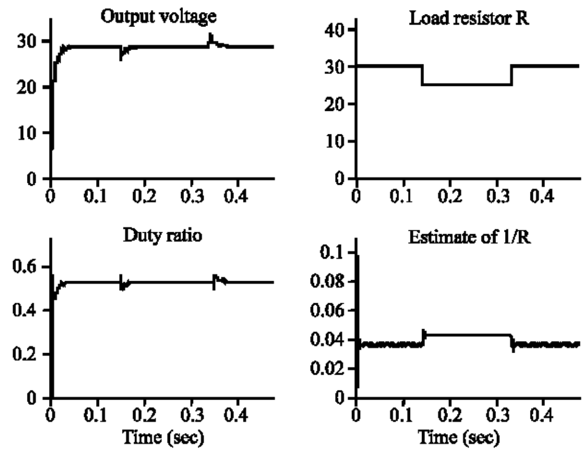

Fig. 8: Backstepping adaptive controller performances

the nonadaptive controllers (backstepping and passivity) prove to be very sensitive to the design parameters. That is, when facing a different variation of the load resistance, the performances shown in Fig. 4 and 7 deteriorate, unless c_1 and c_2 are changed accordingly. The design parameters should be tuned whenever the load resistance variation changes, which is inconvenient in practical applications.

CONCLUSION

The problem of Boost converters has been dealt with using the backstepping approach. The controller design is based on the average PWM Boost model (1). The nonminimum phase feature of the latter makes it impossible to perform a direct output voltage control. Therefore, an indirect voltage control has been resorted-to. Accordingly, the control objective is to enforce the current i_L to track the reference I_d, which in turn implies the convergence of output voltage v_s to its desired value V_d. In the case of perfectly known converter model, the control objective can be ensured using a backstepping nonlinear controller (15). This proved to be quite comparable to passivity-based controllers. In the case of unknown load resistance, an adaptive version of the backstepping controller (33)-(29) has been developed to achieve the control objective. The latter proved to be less sensitive to its design parameters, particularly c_1 and c_2, than the nonadaptive controllers.

REFERENCES

Alvarez-Ramírez, J., G. Espinosa-Pérez and D. Noriega-Pineda, 2001. Current-mode control of dc-dc power converters: A backstepping approach. Proc. IEEE. Int. Conf. Control Application, pp: 190-195.

El Fadil, H., 2002. Nonlinear Control of DC-to-DC Power Converters: Passivity and Backstepping Approaches, DESA-Rep., Department of Electrical Engineering, LA2I-Lab., Mohammadia School of Engineering, Rabat, Morocco.

El Fadil., H., F. Giri and H. Ouadi, 2006. Adaptive Sliding Mode Control of PWM Boost DC-DC Converters. In Proc. IEEE. Int. Conf, Control Appli. (CCA), Munich, Germany, pp: 3151-3156.

Fliess, M. and H. Sira-Ramirez, 1998. Regulation of non-minimum phase outputs: A flatness based approach, Springer-Verlag, New York.

Krstic M., I. Kanellakopoulos and P. V. Kokotovic, 1995. Nonlinear and adaptive control design. John Willy and Sons, New York.

Middlebrook, R.D. and S. Cuk, 1976. A general unified approach to modeling switching-converter power stages, In Proceedings of IEEE-PESC.

Middlebrook, R.D., 1989. Modeling current-programmed buck and boost regulators. IEEE Transaction Power Electron.

Sira-Ramirez, H., M. García-Esteban and A.S.I . Zinober, 1996. Dynamical adaptive pulse-width modulation control of dc to dc power converters: A backstepping approach. Int. J. Control, pp: 205-222.

Sira-Ramirez, H., R. Perez-Moreno, R. Ortega and M. Garcia-Esteban, 1997. Passivity-based controllers for the stabilisation of DC-to-DC power converters, Automatica.