

Adaptive Backstepping Voltage Controller Design for an PWM AC-DC Converter

¹A. Allag, ²M. Y. Hammoudi, ¹S.M. Mimoune and ²M. Y. Ayad ¹Department of Electrical Engineering, MSE Laboratory, University of Biskra, Algeria ²University of Technology Belfort-Montbéliard Belfort, Cedex, 90010 France

Abstract: This study presents a new control strategy for a three phase PWM converter, which consists of applying an adaptive backstepping control. The input-output feedback linearisation approach is based on the exact cancellation of the nonlinearity, for this reason, this technique is not efficient, because system parameters can vary. The nonlinear adaptive backstepping control can compensate the nonlinearities in the nominal system and the uncertainties. Simulation results are obtained using Matlab/Simulink. These results show how the adaptive back-stepping law updates the system parameters and provide an efficient control design both for tracking and regulation.

Key words: PWM AC-DC converter, feedback linearization adaptive backstepping control and harmonic current compensation

INTRODUCTION

Three phase ac-dc converters are widely used in industrial applications. The static power converters are nonlinear in nature and consequently they generate harmonics into the supply. As a result, the power factors of the converters are usually poor and vary with the load. Most electrical systems are designed on the basis of balanced three-phase supply at the fundamental frequency.

Many research results focusing on the control point of view have been reported $^{[1,2]}$. $In^{[1]}$ a feedback linearization technique is applied to control the voltage of three phase PWM converter. $In^{[2]}$, the elimination of the harmonics is made by the introduction of a shunt active power filter; this last was controlled by a feedback linearization technique.

Different nonlinear technique was used in power electric such as Static Compensator^[3], Uninterruptible power supply^[4], DC/DC series resonant converter^[5], Three-Phase Three-Level Neutral Point Clamped Rectifier^[2,6].

On the other hand, the adaptive backstepping control techniques was used in different other application such us permanent magnet synchronous motor control^[7], induction motor control^[8], Linear DC Brushless Motor control^[9].

A linearization technique using input-output feedback have been used to design the nonlinear controller by changing the original nonlinear dynamics into linear one^[10,11], but this technique do not take into

account the uncertainties of system parameters. Adaptive backstepping control is a newly developed technique for the control of uncertain nonlinear systems^[12-14].

This study presents the implementation of the adaptive back-stepping control law to the three phase PWM converter. First, the nonlinear model of the system is introduced the exact nonlinear Multiple-Input Multiple-Output state space model was obtained in (d,q,0) reference frame using the power balance between the input and output sides. State feedback linearisation control for a three phase PWM converter is disussed in this study.

This study persent the uncertainty in the system is the resistance of the load. The uncertain nonlinear model of PWM converter is partially linearised through an input-output feedback linearisation method when the parameter uncertainty is considered. To compensate the uncertainty, a nonlinear adaptive control technique is adopted to derive the control algorithm and the uncertainty adaptation laws can also be derived systematically based on the backstepping control technique.

Mathematical model of PWM converters: Figure 1 presents the topology of the converter under study. The dynamic model of a PWM AC-DC Converter can be described in the well known (d-q) frame through the Park transformation as follows^[9], see appendix:

Simulation results were obtained; these resultselearly show how the adaptive backstepping law updates the system parameters of PWM conveter and gave a good performance.

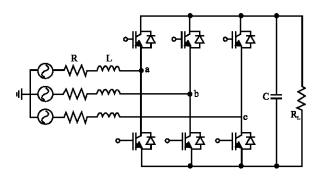


Fig. 1: Three-phase PWM converter topology

$$\begin{cases} \frac{di_{d}}{dt} = -\frac{R}{L}i_{d} + wi_{q} + \frac{1}{L}(e_{d} - v_{d}) \\ \frac{di_{q}}{dt} = -\frac{R}{L}i_{q} - wi_{q} + \frac{1}{L}(e_{q} - v_{q}) \\ \frac{dv_{dc}}{dt} = \frac{2}{3Cv_{dc}}(e_{d}i_{d} + e_{q}i_{q}) - \frac{v_{dc}}{CR_{L}} \end{cases}$$
(1)

Where i_d and i_q are the (d-q) axis currents, ed and eq are the (d-q) axis source voltage, vdc is the DC output voltage, vd and vq are the (d-q) axis converter input voltage. R and L mean the line resistance and inductance, respectively.

Input-output feedback linearization; The input-output feedback linearisation control for a PWM Converters is introduced in^[15].

We can write the system (1) as follows:

$$\dot{x} = f(x) + g_1(x) u_d + g_2(x) u_q$$
 (2)

The system parameters may deviate from the nominal values.

Let
$$\beta = 1/R_L$$
, then $\beta = \beta_n + \Delta \beta$

Where: β_n represent the nominal value of parameter β , while $\Delta\beta$ represent the error of the nominal value.

Taking into account these uncertainties, the system (2) can be changed as:

$$\dot{x} = \overline{f}(x) + \Delta f(x) + g_1 u_d + g_2 u_q$$
 (3)

Where $\bar{f}(x)$ and $\Delta f(x)$ are the nominal and uncertain matrices of f(x) respectively.

$$\overline{f}(x) = \begin{bmatrix} -\frac{R}{L}i_d + wi_q \\ -\frac{R}{L}i_q - wi_d \\ \frac{2}{3Cvdc} (e_di_d + e_qi_q) - \frac{\beta_n vdc}{C} \end{bmatrix}$$

$$\Delta f(x) = \begin{bmatrix} 0 \\ 0 \\ -\Delta\beta vdc \\ C \end{bmatrix}$$

The control objective is to make DC output voltage track the desired reference output voltage command. We should select the dc output voltage as one of the output variable. Therefore, we choose the outputs variable of the PWM converter drive system as:

$$\begin{cases} y_1 = h_1(x) = vdc \\ y_2 = L_f h_1(x) \\ y_3 = i_d \end{cases}$$

$$(4)$$

Then the dynamic model of PWM rectifier in the new coordinate is given by:

$$\begin{bmatrix} \dot{y}_{1} \\ \dot{y}_{2} \\ \dot{y}_{3} \end{bmatrix} = \begin{bmatrix} L_{\overline{f}}h_{1} + L_{\Delta f}h_{1} + L_{g1}h_{1}u_{d} + L_{g2}h_{1}u_{q} \\ L_{\overline{f}}h_{1} + L_{\Delta f}L_{\overline{f}}h_{1} + L_{g1}L_{\overline{f}}h_{1}u_{d} + L_{g2}L_{\overline{f}}h_{1}u_{q} \\ L_{\overline{f}}h_{2} + L_{\Delta f}h_{2} + L_{g1}h_{2}u_{d} + L_{g2}h_{2}u_{q} \end{bmatrix} (5)$$

Where

$$\begin{split} L_{\overline{f}} \, h_1(x) &= \frac{2}{3 \, C \, v dc} \, (e_d i_d + e_q i_q) - \frac{\beta_n \, v dc}{C} \\ L_{\Delta f} h_1(x) &= -\frac{\Delta \beta \, v dc}{C} \\ L_{g1} h_1(x) &= L_{g2} h_1(x) = 0 \; . \\ L^2_{\overline{f}} \, h_1(x) &= \frac{2 (e_d \, f_1(x) + e_q \, f_2(x))}{3 \, C \, v dc} \\ &\qquad - \left\{ \frac{2 (e_d \, i_d + e_q \, i_q)}{3 \, C \, v dc^2} + \frac{\beta_n}{C} \right\} f_3 \\ L_{\Delta f} L_{\overline{f}} \, h_1(x) &= \left\{ \frac{2 (e_d \, i_d + e_q \, i_q)}{3 \, C^2 \, v dc} + \frac{\beta_n \, v dc}{C^2} \right\} \Delta \beta \\ L_{g1} \, L_{\overline{f}} \, h_1(x) &= \frac{2 e_d}{3 \, L \, C \, v dc} \\ L_{g2} \, L_{\overline{f}} \, h_1(x) &= \frac{2 e_q}{3 \, L \, C \, v dc} \end{split}$$

$$\begin{split} & L_{\overline{f}} h_2(x) = -\frac{R}{L} i_d + w i_q \\ & L_{g1} h_2(x) = & \frac{1}{L}, L_{g2} h_2(x) = 0 \\ & L_{\Delta f} h_2(x) = 0 \end{split}$$

In order to decouple the two control inputs, we construct the new control inputs as follows:

$$\begin{bmatrix} \overline{u}_{d} \\ \overline{u}_{q} \end{bmatrix} = \begin{bmatrix} L_{g1}L_{\overline{f}} h_{1}u_{d} + L_{g2}L_{\overline{f}} h_{1}u_{q} \\ L_{g1}h_{2}u_{d} + L_{g2}h_{2}u_{q} \end{bmatrix}$$
(6)

In the new coordinate, the system can be written as follows:

$$\begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \\ \dot{y}_3 \end{bmatrix} = \begin{bmatrix} y_2 \\ L^2_{\overline{f}} h_1 \\ L_{\overline{f}} h_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \overline{u}_{d} \\ \overline{u}_{q} \end{bmatrix} + \begin{bmatrix} L_{\Delta f} h_1 \\ L_{\Delta f} L_{\overline{f}} h_1 \\ L_{\Delta f} h_2 \end{bmatrix}$$
(7)

Where:

$$L_{\Delta f} h_1(x) = \Delta \beta \left\{ -\frac{vdc}{C} \right\} = \theta \phi_1(x)$$
 (8)

$$L_{\Delta f} L_{\overline{f}} h_1(x) = \Delta \beta \left\{ \frac{2(e_d i_d + e_q i_q)}{3C^2 \text{ vdc}} + \frac{\beta_n \text{ vdc}}{C^2} \right\}$$

$$= \theta \varphi_2(x)$$
(9)

$$L_{Af}h_{2}(x) = 0 \tag{10}$$

The structural property of the new system (7) contains two decoupled subsystems. The first subsystem is:

$$\begin{cases} \dot{y}_1 = y_2 \\ \dot{y}_2 = L^2_{\overline{f}} h_1 + \overline{u}_d \end{cases}$$
 (11)

The second subsystem is:

$$\dot{y}_3 = L_{\overline{f}} h_2 + \overline{u}_{\alpha} \tag{12}$$

Where u_d and u_q are the input control of the subsystem (1) and (2), respectively.

This structure allows us to conveniently use adaptive backstepping design technique to obtain the desired controller. The uncertainties of the system now are reflected by unknown parameter θ .

Thus we obtain the compact form of the error-tracking model as follows:

$$\dot{\mathbf{y}} = \overline{\mathbf{A}}(\mathbf{x}) + \Delta \mathbf{A}(\mathbf{x}) + \mathbf{B}(\mathbf{x}) \,\overline{\mathbf{U}} \tag{13}$$

Where:

In order to obtain good transient performance, a linear reference model is defined as.

$$\dot{y}_{m} = k_{m} y_{m} + B_{m} u_{ref} \tag{14}$$

$$\begin{bmatrix} \dot{y}_{m1} \\ \dot{y}_{m2} \\ \dot{y}_{m3} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -k_{m1} & k_{m2} & 0 \\ 0 & 0 & k_{m1} \end{bmatrix} \begin{bmatrix} y_{m1} \\ y_{m2} \\ y_{m3} \end{bmatrix} \\ + \begin{bmatrix} 0 & 0 \\ k_{m1} & 0 \\ 0 & k_{m3} \end{bmatrix} \begin{bmatrix} vdc^* \\ id^* \end{bmatrix}$$

Using the reference model, the performance of the system can easily be evaluated, as the tracking problem could be changed to a regulation problem.

Define the error variables as.

$$e = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} y_1 - y_{m1} \\ y_2 - y_{m2} \\ y_3 - y_{m3} \end{bmatrix}$$
 (15)

And use the following transformation:

$$\tilde{U} = \begin{bmatrix} \tilde{u}_d \\ \tilde{u}_q \end{bmatrix} = \begin{bmatrix} \overline{u}_d + k_{m1} y_{m1} + k_{m2} y_{m2} - k_{m1} v dc^* \\ \overline{u}_q + k_{m3} y_{m3} - k_{m3} i_d^* \end{bmatrix}$$
(16)

Then the differential equations of the errors can be derived as follows:

$$\dot{\mathbf{e}} = \overline{\mathbf{A}}(\mathbf{x}) + \Delta \mathbf{A}(\mathbf{x}) + \mathbf{B}(\mathbf{x}) \tilde{\mathbf{U}} \tag{17}$$

The uncertain parameter error is defined as: $\tilde{\theta} = \theta - \hat{\theta}$

Where $\hat{\theta}$ is the estimation of θ , $\tilde{\theta}$ is the estimation error.

For the first Eq. 17, e_2 is taken as the new control input according to backstepping control technique. It can be easily obtained that, if uncertainty θ is known, the

first Eq. 17 is obviously stable by a Lyapunov function V = 1/2e² and a virtual controller á' as:

$$\alpha'(x) = -k_1 e_1 - \theta \phi_1 \tag{18}$$

However, θ is actually unknown and e2 is not the real control. Hence, an estimate $\hat{\theta}$ is used to replace θ in (18). Define the new virtual control a for e2 as:

$$\alpha(x) = -k_1 e_1 - \hat{\theta} \varphi_1 \tag{19}$$

Step 1: Define new variables as

$$z_1 = e_1, \quad z_2 = e_2 - \alpha(x), \quad z_3 = e_3$$
 (20)

The virtual control a for z2 stabilizes the first equation as follows:

$$\alpha = -k_1 z_1 - \hat{\theta} \varphi_1 \tag{21}$$

The derivatives of the new variables are written as:

$$\dot{z}_1 = z_2 + \alpha + \theta \phi_1 = -k_1 z_1 + z_2 + \tilde{\theta} \phi_1$$
 (22)

$$\dot{z}_2 = e_2 + \dot{\alpha} = L_{\bar{f}}^2 h_1(x) + \theta \phi_2 + \tilde{u}_d - k_1 \dot{z}_1 + \dot{\hat{\theta}} \phi_1$$
 (23)

Step 2: The first control Lyapunov function $V_1(z_1,z_2,\tilde{\theta}\,)$ is written as:

$$V_{1}(z_{1}, z_{2}, \tilde{\theta}) = \frac{1}{2}z_{1}^{2} + \frac{1}{2}z_{2}^{2} + \frac{1}{2\gamma}\tilde{\theta}^{2}$$
 (24)

Where \tilde{a} is the adaptation gains. The derivative $V_1(z_1, z_2, \tilde{\theta})$ of is:

$$\begin{split} \dot{V}_{1}(z_{1},z_{2},\tilde{\theta}) &= z_{1}\dot{z}_{1} + z_{1}\dot{z}_{1} + \frac{1}{\gamma}\tilde{\theta}\dot{\tilde{\theta}} \\ &= -k_{1}z_{1}^{2} + \tilde{\theta} \left\{ z_{2}\phi_{2} + z_{1}\phi_{1} + k_{1}z_{2}\phi_{1} + \frac{1}{\gamma}\dot{\tilde{\theta}} \right\} \\ &+ z_{2} \left\{ z_{1} + L_{\overline{f}}^{2}h_{1} + \hat{\theta}\phi_{2} + \overline{u}_{d} \right. \end{split} \tag{25}$$

Step 3: We can write the third equations of (14) as:

Define the Lyapunov function $V_2(z_3)$ for the third

$$\dot{V}_{1}(z_{3}) = z_{3}\dot{z}_{3} = z_{3} \{L_{\overline{t}}h_{2}(x) + \tilde{u}_{d}\}$$
 (26)

Step 4: To design the final adaptive backstepping nonlinear control for the system, we define the augmented

Lyapunov function $V(z_1, z_2, z_3, \tilde{\theta})$ as:

$$V(z_1, z_2, z_3, \tilde{\theta}) = V_1(z_1, z_2, \tilde{\theta}) + V_2(z_3)$$
 (27)

The derivative of $V(z_1, z_2, z_3, \tilde{\theta})$ is computed as:

$$\dot{V}(z_{1}, z_{2}, z_{3}, \tilde{\theta}) = -k_{1}z_{1}^{2} + \tilde{\theta} \begin{cases} z_{2}\phi_{2} + z_{1}\phi_{1} \\ +k_{1}z_{2}\phi_{1} + \frac{1}{\gamma}\tilde{\theta} \end{cases} \\
+ z_{2} \begin{cases} z_{1} + L_{\overline{f}}^{2}h_{1} + \hat{\theta}\phi_{2} + \overline{u}_{d} \\ +k_{1}(z_{2} - k_{1}z_{1}) + \hat{\theta}\phi_{1} \end{cases}$$

$$+ z_{3}(L_{\overline{f}}h_{2} + \tilde{u}_{q})$$
(28)

To make $V(z_1, z_2, z_3, \tilde{\theta}) \le 0$, the simplest way is to make the items in the square brackets in the second, third and fourth items equal to zero to cancel the uncertainties and make the fifth term equal to $_{-k_2}z_2^2$, the sixth term equal to $_{-k_3}z_3^2$. And V - 0 if and only if, z =0. Then the following results can be obtained. Control outputs:

$$\tilde{u}_{d} = e_{1} - k_{2}e_{2} - L_{\overline{f}}^{2}h_{1} - \theta\phi_{2} + k_{1}(k_{1}z_{1} - z_{2}) - \dot{\hat{\theta}}\phi_{1}$$
 (29)

$$\tilde{u}_{q} = -k_{3}e_{3} - L_{\overline{f}}h_{2}(x)$$
 (30)

Parameter adaptation laws:

$$\hat{\hat{\theta}} = -\hat{\tilde{\theta}} = \gamma (z_1 \phi_1 + k_1 z_2 \phi_1 + z_2 \phi_2)$$
 (31)

The final control inputs u_d and u_q can be easily derived through (15) and (18).

THE SIMULATION RESULTS

We implemented the controller in Matlab/Simulink to verify the stability and asymptotic tracking performance. The overall block diagram for proposed control scheme is shown in Fig. 1.

Table 1: Control scheme	
Supply's voltage and frequency	220 v(rms), 50 Hz
Line's inductor and resistance	$0.1 \text{mH}, 2 \text{m}\Omega$
DC link resistance	20Ω
Output capacitors	370 μΡ
PMW carrier frequency	1KHz

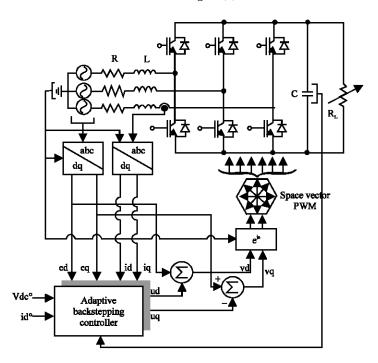


Fig. 2: Adaptive backstepping control block diagram of PWM converter

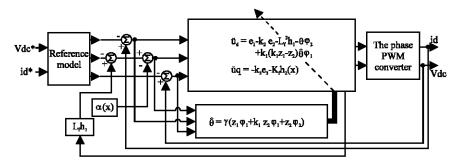


Fig. 3: Block diagram of adaptive backstepping controller

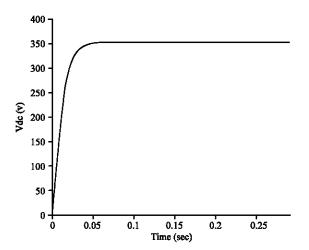


Fig. 4: Output DC link voltage response

The block diagram of the nonlinear controller and adaptation laws is shown in Fig. 2 and 3, respectively. Table 1 shows the parameter values used in the ensuing simulations.

The output DC link voltage of the PWM converter, the supply voltage and the input line current responses of the ideal input-output feedback linearization, are presented in Fig. 4 and 5, respectively.

If there is no uncertainty the actual output DC link voltage response can track the DC voltage reference resulting from the reference model perfectly. The ideal input-output feedback linearisation is based on the exact cancellation of the nonlinearity, for this reason, the ideal nonlinear control cannot deal with the system uncertainties.

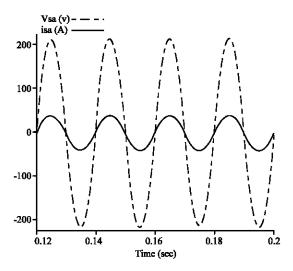


Fig. 5: Supply current and supply voltage without uncertain

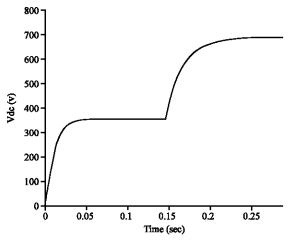


Fig. 6: Output DC link voltage response without adaptation law

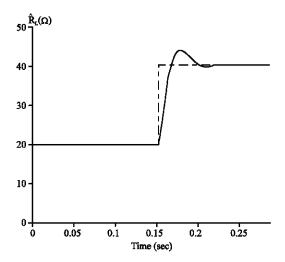


Fig. 7: Estimation of the parameter $\hat{R}_{l}(t)$

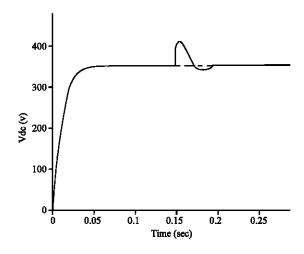


Fig. 8: Output DC link voltage response with adaptive law

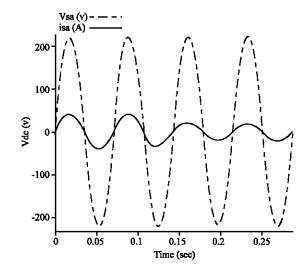


Fig. 9: Supply current and supply voltage with adaptive law

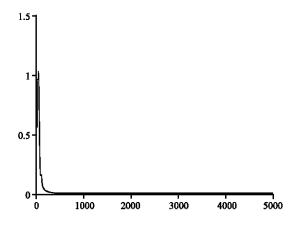


Fig. 10: Harmonic spectra of line current

Figure 6 presents the output DC link voltage of the system without an adaptation law. Figure 7-9 give the simulation results of the proposed backstepping controller with adaptation laws.

In order to test the robustness of the controller to the change of the system parameters, the resistance of the load is changed to $R_{\text{L}} = 2R_{\text{Lnom}}$ at t=0.15s, an output dc voltage drop is observed but this voltage drop is successfully rejected due to the effectiveness of the adaptation laws. The simulation results are performed with the following

CONCLUSION

We have implemented and simulated the adaptive backstepping control for an uncertain PWM converter, which provides an efficient control design for both tracking and regulation. Global asymptotic stability of the block system is guaranteed. Simulation results obtained were in good performance as it is expected. The strategy control was very robust to uncertain parameters and gave a very high power factor and small ripple in the current line supply.

APPENDIX

The voltage equation of a per-phase of PWM converter can be written as:

$$es = Ri + L\frac{di}{dt} + va$$
 (1)

The d-q voltage equations in synchronous reference frame are

$$\begin{cases} e_d = Ri_d + L\frac{di_d}{dt} - Lwi_q + v_d \\ e_q = Ri_q + L\frac{di_q}{dt} + Lwi_d + v_q \end{cases}$$
 (2)

The power balance relationship between the AC input and the DC output is given as

$$P = \frac{2}{3}(e_d i_d + e_q i_q) = v_{dc} i_{dc}$$
 (3)

With:

$$i_{dc} = C \frac{dv_{dc}}{dt} + \frac{v_{dc}}{R_{ch}} \tag{4}$$

Then:

$$\frac{2}{3}(e_{d}i_{d} + e_{q}i_{q}) = Cv_{dc}\frac{dv_{dc}}{dt} + \frac{v_{dc}^{2}}{R_{ch}}$$
 (5)

The nonlinear model of three phase PWM converter

$$\begin{cases} \frac{di_{d}}{dt} = -\frac{R}{L}i_{d} + wi_{q} + \frac{1}{L}(e_{d} - v_{d}) \\ \frac{di_{q}}{dt} = -\frac{R}{L}i_{q} - wi_{q} + \frac{1}{L}(e_{q} - v_{q}) \\ \frac{dv_{dc}}{dt} = \frac{2}{3Cv_{dc}}(e_{d}i_{d} + e_{q}i_{q}) - \frac{v_{dc}}{CR_{ch}} \end{cases}$$
(6)

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