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An Adaptive Linear Quadratic Regulator Applied to Buck-Series Resonant Inverter for Induction Heating

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Abstract: An adaptive linear quadratic regulator of a cascade buck chopper-full bridge inverter is presented. A full bridge series resonant inverter based on Zero-Voltage Switching (ZVS) and an adaptive control is used as a power supply for induction heating. An adaptive control does not only keep the output current and the voltage in phase but also makes the system operating in zero-voltage-switching mode for constant and variable load.

Key words: Adaptive control, LQ regulator, resonant converter, induction heating, kalman filter

INTRODUCTION

High frequency and high power voltage source inverter using switching devices such as MOSFETs, IGBTs or SITs have been developed for induction heating applications such as melting and surface hardening. They are also capable to operate at the output frequency of over 100 kHz[1]. MOSFET devices are popularly used as power components of high frequency power supply. The gate-drain and gate-source parasitic capacitance have a severe effect on switching speed. Three non-ideal commutation phenomena occur when MOSFETs are used as power switches:

- A surge current flows through the MOSFET caused by the reverse-recovery component of the freewheeling drain-source diode during turn-on. This surge current generates switching losses, gradient of current and electromagnetic interference noise.
- During turn on, the parasitic drain-source capacitor discharges produce an additional current. This can typically only be reduced by using resonant converter techniques.
- The load resistance and inductance vary hugely during the heating processes. This variation changes the resonant frequency of the resonant circuit since the load inductance and resistance are part of a resonant tank. In order to get the maximum output energy, the first harmonic of the inverter must be in phase with the load current in turn we have to adjust the DC-DC converter in such away that the current reference follows the load current[2].

In this study, an adaptive Linear Quadratic Regulator (LQR) is proposed such that the changing resonant frequency of the equivalent load can be tracked and meanwhile to keep the ZVS condition. The LQR regulator is a useful tool in modern optimal control design, consisting of explicit matrix design equation easily solved in a digital computer. In the proposed controller, a least square estimator (RLS) identifies the plant parameters which are used to compute the LQR gains periodically. The quadratic cost function parameters are chosen in order to reduce the energy of the control signal. A Kalman filter is used to estimate the inductor current state[3].

System configuration: A series resonant circuit fed by the voltage source inverter using MOSFETs with fixed frequency and a DC-DC converter regulates the output power where the amplitude of the output voltage is controlled. The output resonant equivalent circuit were constructed by the output capacitor, output transformer and work coil. The load is modelled by the equivalent impedance witch is varied during the heating process Fig. 1.

The power switches of the inverter are turned on and off during each a constant interval T, the output capacitor voltage v_{Cr} and the inductor current i_{Lr} are chosen as the state variables, such as:

$$\dot{x} = A\,x + B\,u,\;\; y = C\,x \tag{1}$$

where

$$x = \begin{bmatrix} v_{C_r} & i_{L_r} \end{bmatrix}^T, A = \begin{bmatrix} -\dfrac{1}{R_r C_r} & \dfrac{1}{C_r} \\ -\dfrac{1}{L_r} & 0 \end{bmatrix}, \tag{2}$$

$$B = \begin{bmatrix} 0 \\ \dfrac{1}{L_r} \end{bmatrix} \text{ and } C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

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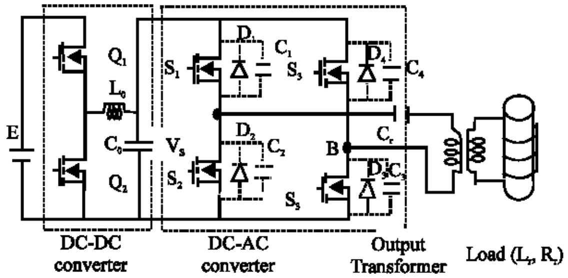

Fig. 1: Equivalent circuit of a series resonant inverter

Then a discrete time model of the plant with a Zero Order Hold (ZOH) and a simple period T_S is given by:

$$x(k+1) = A_d\ x(k) + B_d\ u(k), y(k) = C_d\ u(k) \quad (3)$$

A_d, B_d and C_d are obtained from the system transfer function:

$$A_d = \begin{bmatrix} 0 & -D' \\ 1 & -C' \end{bmatrix}; B_d = \begin{bmatrix} B' \\ A' \end{bmatrix}; C_d = C \quad (4)$$

where,

$$\begin{cases} A' = 1 - e^{-aTs}\cos(bT_S) - \dfrac{a}{b} e^{-aTs}\sin(bT_S) \\ B' = e^{-2aTs} + \dfrac{a}{b} e^{-aTs}\sin(bT_S)\ - e^{-aTs}\cos(bT_S) \\ C' = \ -2e^{-aTs}\cos(bT_S) \\ D' = e^{-2aTs} \end{cases} \quad (5)$$

and

$$a = \xi\omega_n,\ b = \omega_n\sqrt{1-\xi^2},\ \omega_n = \frac{1}{\sqrt{Lc}} \text{ and } \xi = \frac{1}{2Rc\omega_n} \quad (6)$$

Linear quadratic regulator: An adaptive controller with integrator is proposed in Fig. 2, it has the objective of tracking the discrete sinusoidal r(k) reference in each sample instant T_S and also fulfill the adaptation gains during the load changes.

The augmented state variables used in the LQR regulator are the measured capacitance voltage $v_c(k)$, the estimated inductor current $\hat{\imath}_L(k)$, the integrated tracking error v(k) and the discrete reference r(k) all through a feedback action.

Each variable has a weighting K gains tuned in function of the estimated parameters $\theta(k)$, which contains the plant parameters identified by the RLS algorithm, such that:

$$K = [K_1 \quad K_2 \quad K_3 \quad K_4 \quad K_5] \quad (7)$$

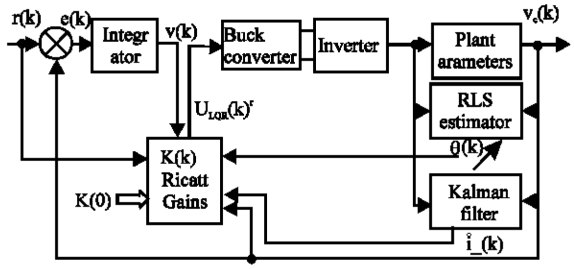

Fig. 2: Block diagram of the closed loop system

K: is the Ricatti gains matrix.
Then, the closed loop system is defined as:

$$Z(k) = \left[v_c(k)\ \ i_l(k)\ \ v(k)\ \ r(k)\ \ \dot{r}(k)\right]^T \quad (8)$$

and the LQR control is given by:

$$u_{LQR} = -K\ Z(k) \quad (9)$$

The system must be represented in the form:

$$Z(k+1) = G\ Z(k) + H\ u_{LQR}(k) \quad (10)$$

where each state variable is calculated by a difference equation. The two first variables of vector Z(k) are obtained by (3).
The signal v(k) represents the output integrator:

$$v(k+1) = e(k+1)\ +\ v(k) \quad (11)$$

where the error is given by:

$$e(k) = r(k) - y(k) \quad (12)$$

From (3), (11) and (12), it results the difference equation for the integrated error state:

$$v(k+1) = v(k) + r(k+1) - C_d\ G\ Z(k) - C_d\ H\ u_{LQR}(k) \quad (13)$$

The sinusoidal reference signals in continues time is generated as:

$$\ddot{r} + \omega^2\ r = 0 \quad (14)$$

when it starts with the initial values r(0) and $\dot{r}(0) = \omega\ V_{max}$, where V_{max} is the sine wave amplitude and ω is the operating angular frequency of the inverter. The state space variables of the reference generator are:

$$\begin{bmatrix} \dot{r}(t) \\ \ddot{r}(t) \end{bmatrix} = R \begin{bmatrix} r(t) \\ \dot{r}(t) \end{bmatrix} \text{where } R = \begin{bmatrix} 0 & 1 \\ \omega^2 & 0 \end{bmatrix} \quad (15)$$

In the discrete form, using a sample time T_S, the subsystem (15) is given by:

$$n(k+1) = R_d\, n(k) \quad (16)$$

where

$$R_d = e^{RTs} \text{ and } n(k) = \begin{bmatrix} r(k) & \dot{r}(k) \end{bmatrix}^T \quad (17)$$

Then, using the state Eq. 3, 13 and 16, the closed loop system representation becomes:

$$\begin{bmatrix} x(k+1) \\ v(k+1) \\ n(k+1) \end{bmatrix} = \begin{bmatrix} A_d & 0 & 0 \\ -C_d B_d & 1 & C_d R_d \\ 0 & 0 & R_d \end{bmatrix} \begin{bmatrix} x(k) \\ v(k) \\ n(k) \end{bmatrix} + \begin{bmatrix} B_d \\ C_d B_d \\ 0 \end{bmatrix} u_{LQR} \quad (18)$$

$$y(k) = \begin{bmatrix} C_d & 0 & 0 \end{bmatrix} \begin{bmatrix} x(k) & v(k) & n(k) \end{bmatrix}^T \quad (19)$$

The optimal gains of the control law (9) are those which minimize the following cost function:

$$J = \frac{1}{2} \sum_{K=1}^{N} \left\{ Z^T(k)\, Q Z(k) + u_{LQR}^T(k)\, R\, u_{LQR}(k) \right\} \quad (20)$$

where Q and R are chosen as definite positive matrices.

The k gains can be obtained by evaluating the Riccati equation as follows[4]:

$$S(k) = G^T\, S(k+1)\, G + Q - \left[H^T\, S^{-1}(k+1)\, G \right]^T \left[R + H^T\, S(k+1)\, H \right]^{-1} \left[H^T\, S(k+1)\, G \right] \quad (21)$$

One always wishes to bring Z(N) to zero such that:

$$\lim_{N \to \infty} Z(N) \equiv 0 \quad (22)$$

Therefore, the solution when the problem has an infinite time ($N \to \infty$), which leads the matrix of Ricatti to become a constant such that:

$$\lim_{N \to -\infty} S(N) \equiv S \quad (23)$$

We replace S(k+1)and S(k) by S in the expression (21) found previously, we find:

$$S = G^T\, S\, G + Q - \left(G^T\, S\, H\right)\left(R + H^T\, S\, H\right)^{-1}\left(H^T\, S\, G\right) \quad (24)$$

where,

$$K_\infty = \left(H^T\, S_{LQR}\, H + R\right)^{-1} H^T\, S_{LQR}\, G \quad (25)$$

Such that, K_∞ is the sub-optimal gain.

Recursive least square estimator: To estimate the plant parameters when the load conditions are variables, a RLS algorithm is employed[5]. From the discrete transfert function, the difference equation of the estimated output is written as follow:

$$y(k) = -\theta_1 y(k-1) - \theta_2 y(k-2) + \theta_3 u(k-1) + \theta_4 u(k-2) \quad (26)$$

We assume that the vector of parameters θ has been defined such that the system may be represented by:

$$y(k) = \theta^T(k)\, \psi(k-1) \quad (27)$$

where

$$\theta(k) = \begin{bmatrix} \theta_1 & \theta_2 & \theta_3 & \theta_4 \end{bmatrix} \quad (28)$$

and

$$\psi(k) = \left[-y(k-1) - y(k-2) u(k-1) u(k-2) \right] \quad (29)$$

The RLS gains are calculated using:

$$L(k) = \frac{P(k-1)\, \psi(k)}{1 + \psi(k)^T\, P(k-1)\, \psi(k)} \quad (30)$$

where we have defined the covariance matrix:

$$P(k) = P(k-1) - \frac{P(k-1)\, \psi(k)\, \psi(k)^T\, P(k-1)}{1 + \psi(k)^T\, P(k-1)\, \psi(k)} \quad (31)$$

and the plant parameters are recursively obtained by:

$$\hat{\theta}(k) = \hat{\theta}(k-1) + L(k)[\, y(k) - \psi^T\, \hat{\theta}(k-1)] \quad (32)$$

where

$$\hat{A}_d = \begin{bmatrix} 0 & -\hat{\theta}_2 \\ 1 & -\hat{\theta}_1 \end{bmatrix}, \hat{B}_d = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \hat{C}_d = \begin{bmatrix} \hat{\theta}_3 & \hat{\theta}_4 \end{bmatrix} \quad (33)$$

Then, it is possible to identify the plant parameters to a range of different loads and to substitute the matrices (33) into the system (18) to precede with the LQR gains design in real time.

The discrete kalman filter: As only the capacitor voltage is measured variable state, a Kalman filter is used to estimate the inductor current state[6]. The Kalman filter addresses the general problem in estimating the state of a discrete-time controlled process that is governed by the linear stochastic difference equation:

$$x(k+1)=A_d x(k)+B_d\ u_{LQR}(k)+w(k) \tag{34}$$

$$y(k)=C_d\ x(k)+v(k) \tag{35}$$

The variables w(k) and v(k) represent the random process and measurement noise respectively. They are assumed to be uncorrelated to each other and with normal probability distributions such that:

$$E\left[w(k)^T w(k)\right]=R_W>0 \tag{36}$$

$$E\left[v(k)^T v(k)\right]=R_V>0 \tag{37}$$

$$E\left[w(k)^T v(k)\right]=0 \tag{38}$$

In practice, the process noise covariance and measurement noise covariance matrices might change with each time step or measurement; however, here we assume that they are constant.

The specific equations for the time and measurement updates are presented below[5,6]:

- Discrete Kalman filter time updates equations

$$\bar{x}(k)=A\ \hat{x}(k\text{-}1)+B\ u(k\text{-}1) \tag{39}$$

$$M(k)=A_d\ P(k-1)\ A_d^T+B_d\ R_w\ B_d^T \tag{40}$$

- Discrete Kalman filter measurement updates equations

$$K_k(k)=M(k)\ C_d^T\left[R_V+C_d\ M(k)_d\right]^{-1} \tag{41}$$

$$P(k)=(I-K_k\ C_d)\ M_d(k) \tag{42}$$

$$\hat{x}(k)=\bar{x}(k)+L_k(k)\ \left(y(k)-C_d\ \hat{x}(k)\right) \tag{43}$$

After each time and measurement update pair, the process is repeated with the previous a posteriori estimates used to predict the new a priori estimates.

SIMULATION RESULTS

The fundamental parameters of the buck-single phase full-bridge inverter for induction heating applications and controller parameters are show Table 1.

To verify the tracking capability of the LQR control scheme in the closed loop system we have performed a simulation results. Figure 3 and 4 show, respectively the response of reference and the output current with LQR regulator for abrupt change of load. The load current frequency is well matched to the reference frequency assuring ZVS condition.

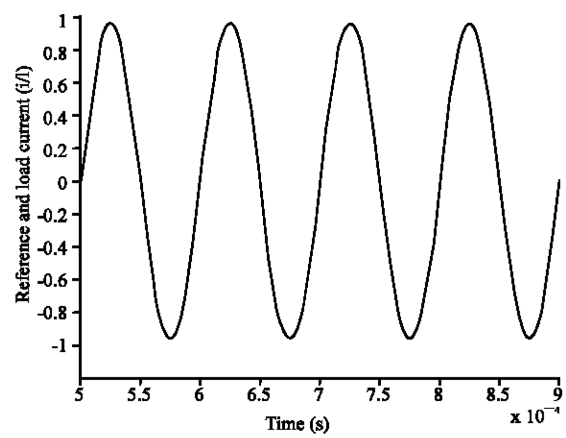

Fig. 3: Output current (solid) and reference andcurrent (dashed) to the linear load

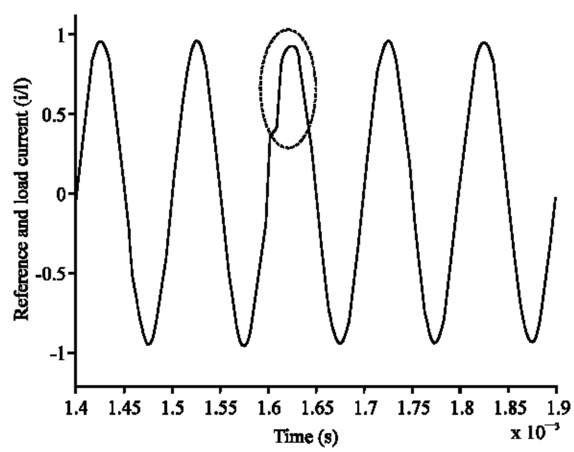

Fig. 4: Output current (solid) and reference current (dashed) to the abrupt change of load

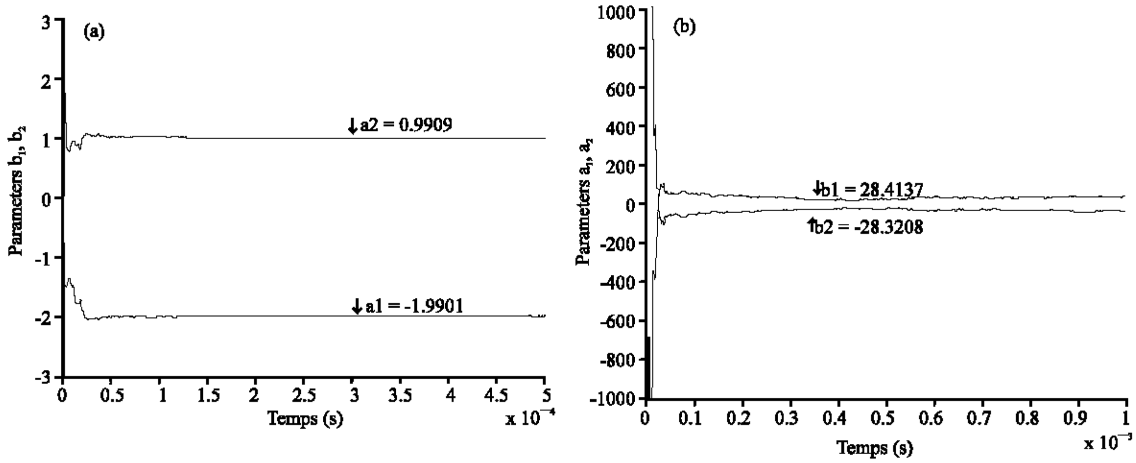

Fig. 5: Estimate parameters corresponding to the simulation

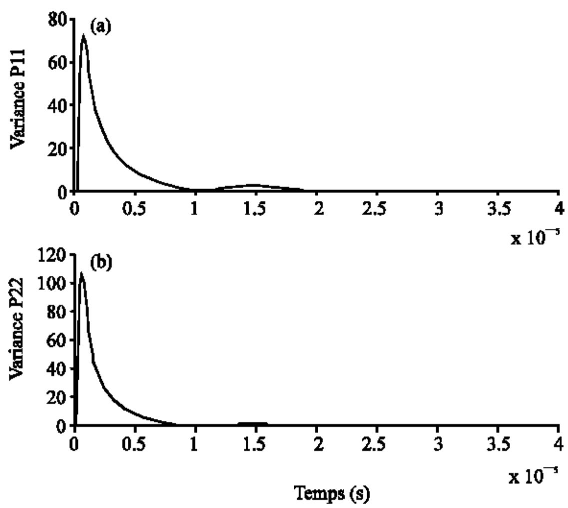

Fig. 6: Evolution of variance of the matrix elements P

Table 1: System parameters

Load inductance	Lr = 260 μH
Load capacitance	Cr = 1 μF
Load resistance	Rr = 10 Ω
DC input voltage	Vs = 300 V
Reference voltage	Vref = 300 V (peak), 10 kHz
Sample time	T_S = 0.05 μs
States weighting	Q = diag [2.10^6 , 10^5 , 5.10^3 , 1, 1]
Control weighting	R = 500
LQR gains (to the linear load)	K = [9.54 6.63 -0.014 -0.014 -0.0]

Figure 5a and b shows the applicability of the RLS algorithm which is shows the efficiency of determining the unknown parameters $\theta_1\theta_2\ \theta_3\ \theta_4$.

CONCLUSION

A Linear Quadratic Regulator was developed to the buck-full bridge series resonant inverter for induction heating application, the LQR gains are calculated by minimizing a cost function.

The RLS estimator identifies the plant parameters which are used to compute the LQR gains periodically for constant and variable loads. The discrete control adaptive law has shown good results to constant and variable loads with a moderate switching frequency.

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