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Sensorless Control of Induction Machine with Stator Resistance Estimation

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Abstract: Speed sensorless systems are intensively studied during recent years, this is mainly due to their economical benefit and fragility of mechanical sensors and also the difficulty of installing this type of sensor in many applications. These systems suffer from instability problems and sensitivity to parameter mismatch at low speed operation. In this study an analysis of adaptive observer stability with stator resistance estimation is given.

Key words: Induction motor drive, sensorless control, adaptive observer, stator resistance estimation

INTRODUCTION

In recent years the use of induction machine without shaft encoder has known a great development. Modern control techniques avoid the use of sensors because they are unreliable and have an increased cost[1]. The sensorless systems require estimation of internal state variables of the machine such as speed and rotor flux from input variables like stator voltage and stator current. The estimation methods of speed and rotor flux for sensorless vector control drives are based on adaptive observer theory[2-4]. Machine parameters change during motor operation due to temperature rise. In this study, the adaptive observer stability and stator resistance estimation are analysed.

Development of induction motor model: The induction motor is modelled from the equations below presented in the synchronous rotating reference frame

$$\begin{cases} \frac{d}{dt}\underline{\psi}_r = -(\frac{1}{T_r} + j\omega_{sl})\underline{\psi}_r + \frac{L_m}{T_r}\underline{i}_s \\ \frac{d}{dt}\underline{i}_s = \frac{L_m}{b}(\frac{1}{T_r} - j\omega)\underline{\psi}_r - (a + j\omega_s)\underline{i}_s + \frac{1}{\sigma L_s}\underline{u}_s \end{cases} \quad (1)$$

Where,

$\underline{u}_s = [u_{sd}, u_{sq}]^T$	:	Stator voltage vector
$\underline{i}_s = [i_{sd}, i_{sq}]^T$	:	Stator current vector
$\underline{\psi}_r = [\psi_{rd}, \psi_{rq}]^T$	:	Rotor flux vector
L_s, L_r, L_m	:	Stator, rotor and mutual inductance respectively.
R_s, R_r	:	Stator and rotor resistance
$\sigma = 1 - \frac{L_m^2}{L_s L_r}$	:	Leakage coefficient

ω_s, ω, ω_{sl} are the stator angular frequency, the motor angular velocity and the slip frequency, respectively.

$$b = \sigma L_s L_r \quad a = \frac{L_r^2 R_s + L_m^2 R_r}{\sigma L_s L_r^2} \quad \text{and} \quad \omega_{sl} = \omega_s - \omega$$

Adaptive observer design: The conventional full order observer is defined by:

$$\begin{cases} \frac{d}{dt}\hat{\underline{\psi}}_r = -(\frac{1}{T_r} + j\hat{\omega}_{sl})\hat{\underline{\psi}}_r + \frac{L_m}{T_r}\hat{\underline{i}}_s + G_1(\underline{i}_s - \hat{\underline{i}}_s) \\ \frac{d}{dt}\hat{\underline{i}}_s = \frac{L_m}{b}(\frac{1}{T_r} - j\hat{\omega})\hat{\underline{\psi}}_r - (\hat{a} + j\hat{\omega}_s)\hat{\underline{i}}_s + \frac{1}{\sigma L_s}\underline{u}_s \\ +G_2(\underline{i}_s - \hat{\underline{i}}_s) \end{cases} \quad (2)$$

With: $\hat{a} = \frac{1}{\sigma L_s}(\hat{R}_s + \frac{L_m^2}{L_r^2}R_r)$, $G = [G_1 G_2]^T$

In order to derive the adaptive laws for rotor speed and stator resistance Lyapunov's theorem is utilized[4]. Defining the state vector $\underline{x} = [\underline{\psi}_r \ \underline{i}_s]^T$ and its estimate $\hat{\underline{x}} = [\hat{\underline{\psi}}_r \ \hat{\underline{i}}_s]^T$.

From (1) and (2) the estimation error of the rotor flux and stator current is described by the following equation:

$$\frac{d\underline{e}}{dt} = (A + GC)\underline{e} - \Delta A\hat{\underline{x}} \quad (3)$$

Where

$$\underline{e} = \underline{x} - \hat{\underline{x}}$$

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In the observer (2) $\hat{\omega}$ and $\check{R}_s$ are speed and stator resistance estimations, respectively. They can be expressed by:

$$\hat{\omega} = \omega_s + \Delta\omega, \hat{R}_s = R_s + \Delta R_s$$

Rotor speed adaptation law: In Eq. (3) for speed estimation we have:

$$\Delta A = \begin{bmatrix} 0_{2\times2} & -\frac{L_m}{b}\Delta\omega J \\ 0_{2\times2} & \Delta\omega J \end{bmatrix} \text{ with } J = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

We define the following Lyapunov function

$$V = e^T e + \frac{\Delta\omega^2}{\lambda} \quad (4)$$

With V, energy of signal e and λ a positive scalar.

Stability of observer is obtained for:

$$\frac{dV}{dt} \langle 0 \quad (4)$$

This leads to

$$\frac{dV}{dt} = 2\underline{e}^T(A+GC)\underline{e} - 2\frac{\Delta\omega L_m}{b_s}(e_{id}\hat{\psi}_{rq} - e_{iq}\hat{\psi}_{rd}) + 2\frac{\Delta\omega}{\lambda}\frac{d\hat{\omega}}{dt} \quad (5)$$

The first term of (5) is negative semi definite[2], the speed adaptation law is find by equalizing the second term to the third term.

$$\frac{d\hat{\omega}}{dt} = \frac{\lambda L_m}{b}(e_{id}\hat{\psi}_{rq} + e_{iq}\hat{\psi}_{rd}) \quad (6)$$

Stator resistance adaptation law: In this case

$$\Delta A = \begin{bmatrix} -\frac{\Delta R_s}{\sigma L_s} I & 0_{2\times2} \\ 0_{2\times2} & 0_{2\times2} \end{bmatrix} \text{ with } I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

The Lyapunov function is defined by

$$V = e^T e + \frac{\Delta R_s^2}{\lambda} \quad (7)$$

The time derivative of V becomes

$$\frac{dV}{dt} = 2\underline{e}^T(A+GC)\underline{e} + 2\frac{\Delta R_s}{\sigma L_s}(e_{id}\hat{i}_{sd} + e_{iq}\hat{i}_{sq}) + 2\frac{\Delta R_s}{\lambda}\frac{d\hat{R}_s}{dt} \quad (8)$$

The stator adaptation law is

$$\frac{d\hat{R}_s}{dt} = -\frac{\lambda}{\sigma L_s}(e_{id}\hat{i}_{sd} + e_{iq}\hat{i}_{sq}) \quad (9)$$

Defining $e_\omega = \omega - \hat{\omega}$ and $e_{Rs} = R_s - \check{R}_s$ $\hat{\alpha}$ can be defined by $\hat{a} = a - \frac{1}{\sigma L_s} e_{R_s}$.

System describing the observer is then:

$$\begin{cases} \frac{d}{dt}\hat{\underline{\psi}}_r = -(\frac{1}{T_r} + j\omega_{sl})\hat{\underline{\psi}}_r + \frac{L_m}{T_r}\hat{\underline{i}}_s - je_\omega\hat{\underline{\psi}}_r + G_1(\underline{i}_s - \hat{\underline{i}}_s) \\ \frac{d}{dt}\hat{\underline{i}}_s = \frac{L_m}{b}(\frac{1}{T_r} - j\omega)\hat{\underline{\psi}}_r - (a + j\omega_s)\hat{\underline{i}}_s + \frac{1}{\sigma L_s}\underline{u}_s + \\ +j\frac{L_m}{b}e_\omega\hat{\underline{\psi}}_r + \frac{1}{\sigma L_s}e_{R_s}\hat{\underline{i}}_s + G_2(\underline{i}_s - \hat{\underline{i}}_s) \\ \frac{d}{dt}\hat{\omega} = -K_i\Im\{\underline{e}_i\hat{\underline{\psi}}_r^*\} \\ \frac{d}{dt}\hat{R}_s = -K_i'\Re\{\underline{e}_i\hat{\underline{i}}_s^*\} \end{cases} \quad (10)$$

using the following hypothesis

$$\frac{d}{dt}\omega = 0 \quad (11a)$$

$$\frac{d}{dt}R_s = 0 \quad (11b)$$

$$\hat{\underline{\psi}}_r \to \underline{\psi}_r \quad (11c)$$

According to (11a, b,c), the motor model (1) may be written as:

$$\begin{cases} \frac{d}{dt}\underline{\psi}_r = -(\frac{1}{T_r} + j\omega_{sl})\underline{\psi}_r + \frac{L_m}{T_r}\underline{i}_s \\ \frac{d}{dt}\underline{i}_s = \frac{L_m}{b}(\frac{1}{T_r} - j\omega)\underline{\psi}_r - (a + j\omega_s)\underline{i}_s \\ +\frac{1}{\sigma L_s}\underline{u}_s \\ \frac{d}{dt}\omega = 0 \\ \frac{d}{dt}R_s = 0 \end{cases} \quad (12)$$

Stability analysis: The study of open loop observer stability (G = 0) by linearizing the motor model (12) and the conventional full order observer (10) around an equilibrium operating point is carried out.

The new state vectors are defined as follows:

$$\underline{x} = \underline{x}_0 + \delta\underline{x} \text{ and } \hat{\underline{x}} = \hat{\underline{x}}_0 + \delta\hat{\underline{x}} \text{ with :}$$

$$\underline{x}_0 = \begin{bmatrix} \underline{\psi}_{ro} & \underline{i}_{so} & \omega_o & R_{so} \end{bmatrix}^T, \hat{\underline{x}}_o = \begin{bmatrix} \hat{\underline{\psi}}_{ro} & \hat{\underline{i}}_{so} & \hat{\omega}_o & \hat{R}_{so} \end{bmatrix}^T$$

$$\delta\underline{x} = \begin{bmatrix} \delta\underline{\psi}_r & \delta\underline{i}_s & \delta\omega & \delta R_s \end{bmatrix}^T, \delta\hat{\underline{x}} = \begin{bmatrix} \delta\hat{\underline{\psi}}_r & \delta\hat{\underline{i}}_s & \delta\hat{\omega} & \delta\hat{R}_s \end{bmatrix}^T$$

The reference frame is synchronized with the estimated rotor flux

$$(\hat{\psi}_{rqo} = 0),$$

then its two components are

$$\hat{\psi}_{rd} = \hat{\psi}_o + \delta\hat{\psi}_{rd} \text{ and } \hat{\psi}_{rq} = \delta\hat{\psi}_{rq}.$$

In these two systems, the stator pulsations are regarded as identical: $\omega_s = \hat{\omega}_s$ [5].

Defining, $\delta e = \begin{bmatrix} \delta e_\psi & \delta e_i & \delta e_\omega \, \delta e_{Rs} \end{bmatrix}^T$

the system describing the estimation error is

$$\begin{cases} \frac{d}{dt}\delta e_\psi = -(\frac{1}{T_r} + j\omega_{sl})\delta e_\psi + \frac{L_m}{T_r}\delta e_i + j e_\omega \delta\hat{\underline{\psi}}_r \\ -j\delta e_w \hat{\psi}_o - j\delta\omega_{slo} e_{\psi o} \\ \frac{d}{dt}\delta e_i = \frac{L_m}{b}(\frac{1}{T_r} - j\omega)\delta e_\psi - (a + j\omega_s)\delta e_i - j\frac{L_m}{b} e_\omega \delta\hat{\underline{\psi}}_r \\ -\frac{1}{\sigma L_s}\delta e_{R_s}\hat{\underline{i}}_{so} - j\frac{L_m}{b}\hat{\psi}_o \delta e_\omega - j\frac{L_m}{b} e_{\psi o}\delta\omega - \frac{1}{\sigma L_s} e_{Rs}\delta i_s \\ -j\delta\omega_s e_{is} - \frac{\delta R_s}{\sigma L_s} e\hat{i}_{so} \\ \frac{d}{dt}\delta e_\omega = K_i(-e_{ido}\delta\hat{\psi}_{rq} + e_{iqo}\delta\hat{\psi}_{rd} + \hat{\psi}_o \delta e_{iq}) \\ \frac{d}{dt}\delta e_{R_s} = K_i'(\delta e_{id}\hat{I}_{sdo} + \delta e_{iq}\hat{I}_{sqo} + e_{id}\delta\hat{I}_{sd} + e_{iq}\delta\hat{I}_{sq}) \end{cases} \quad (13)$$

Splitting each state in d and q components, we define the state vector

$$\delta e = \begin{bmatrix} \delta e_{\psi d} & \delta e_{\psi q} & \delta e_{id} & \delta e_{iq} & \delta e_\omega & \delta e_{Rs} \end{bmatrix}$$

The corresponding state matrix is

$$\hat{A} = \begin{bmatrix} -\frac{1}{T_r} & \omega_{slo} & \frac{L_m}{T_r} & 0 & 0 & 0 \\ -\omega_{slo} & \frac{-1}{T_r} & 0 & \frac{L_m}{T_r} & \psi_o & 0 \\ \frac{L_m}{bT_r} & \frac{L_m}{b}\omega_o & -a & \omega_{so} & 0 & -\frac{\hat{I}_{sdo}}{\sigma L_s} \\ -\frac{L_m}{b}\omega_o & \frac{L_m}{bT_r} & -\omega_{so} & -a & -\frac{L_m}{b}\psi_o & -\frac{\hat{I}_{sqo}}{\sigma L_s} \\ 0 & 0 & 0 & K_i\psi_o & 0 & 0 \\ 0 & 0 & K_i'\hat{I}_{sdo} & K_i'\hat{I}_{sqo} & 0 & 0 \end{bmatrix} \quad (14)$$

Stator resistance estimation: Firstly, the estimation of stator resistance with speed sensor is considered. The state matrix becomes:

$$\hat{A} = \begin{bmatrix} -\frac{1}{T_r} & \omega_{slo} & \frac{L_m}{T_r} & 0 & 0 \\ -\omega_{slo} & -\frac{1}{T_r} & 0 & -\frac{L_m}{T_r} & 0 \\ \frac{L_m}{bT_r} & \frac{L_m}{b}\omega_o & -a & \omega_{s0} & -\frac{\hat{I}_{sdo}}{\sigma L_s} \\ -\frac{L_m}{b}\omega_o & \frac{L_m}{bT_r} & \omega_{so} & -a & -\frac{\hat{I}_{sqo}}{\sigma L_s} \\ 0 & 0 & K_i'\hat{I}_{sdo} & K_i'\hat{I}_{sqo} & 0 \end{bmatrix} \quad (15)$$

Using the following property

$$\det(\hat{A}) = \prod_{i=1}^{5}\lambda_i \quad (16)$$

Where λ_i are the Eigen values of matrix $\hat{A}$. The system stability implies that the five Eigen values must have a negative real part. Consequently, a condition of stability for system (15) is

$$\det(\hat{A}) \langle 0 \quad (17)$$

The stability limit is given by $(\hat{A}) = 0$.

Using Mapple/Matlab and without any simplification, we find:

$$\det(\hat{A}) = -\frac{k_i'\hat{I}_s^2}{\sigma L_s}\left[\frac{R_s}{T_r^2\sigma L_s} + \omega_{slo}^2\frac{R_s}{\sigma L_s} + \omega_{slo}\omega_s\frac{L_m^2 R_r}{\sigma L_s L_r^2}\right] \quad (18)$$

The condition $(\hat{A}) = 0$ leads to:

$$\omega_s = \frac{-R_s R_r (1 + \omega_{slo}^2 T_r^2)}{\omega_{slo} L_m^2} \tag{19}$$

Same result as in[2].

These stability conditions may be expressed in the torque/pulsation plane.

Under RFOC conditions and steady state ($\hat{\psi}_{rqo} = \psi_{rqo} = 0$), we obtain

$$i_{sqo} = \frac{L_r}{p L_m \hat{\psi}_o} T_{Lo} \tag{20}$$

From system (1) in the same conditions, we find

$$\omega_{slo} = \frac{L_m}{T_r \hat{\psi}_o} i_{sqo} \tag{21}$$

Finally using $\omega_{so} = \omega_{slo} + \omega_o$ Eq. 19 becomes:

$$T_{Lo}^2 (\frac{R_s L_r^2}{L_m^2} + R_r) + p\psi_r^2 \omega_o T_{Lo} + \frac{R_s}{L_m^2} p^2 \psi_r^4) = 0 \tag{22}$$

Stability regions on the torque/pulsation plane are obtained by the resolution of Eq. 22 Fig.1.

Condition of stability (17) can be used only in regenerating mode. In order to validate this, we trace for each E.V, the locus, in the torque/speed plane, where, condition $\Re(\lambda_{i=1}^5) > 0$ is verified.

The system stability implies that the five Eigen values must have a negative real part. Figure 2 shows regions where the different poles have a positive real part, respectively in regenerating and monitoring modes.

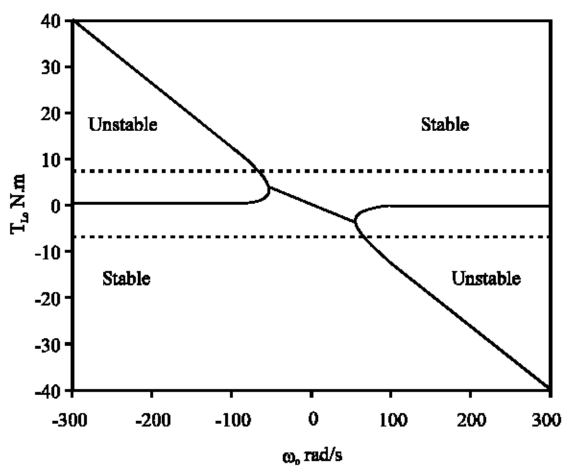

Fig. 1: Instability region in the pulsation/torque plane

Fig. 2: Stator resistance estimation: instability regions (positive real parts)

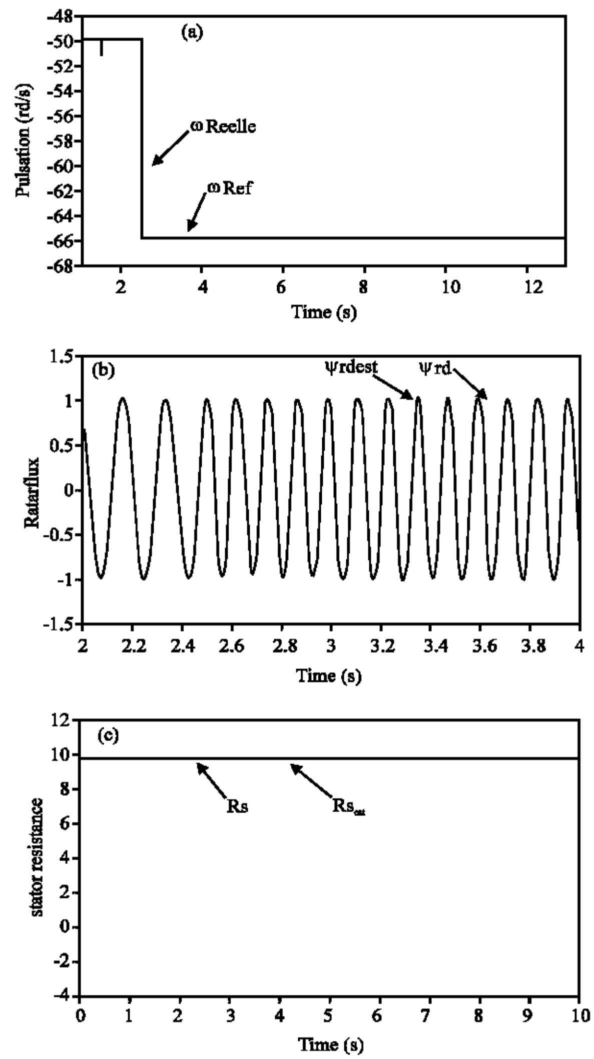

Fig. 3: Tests in regenerating mode

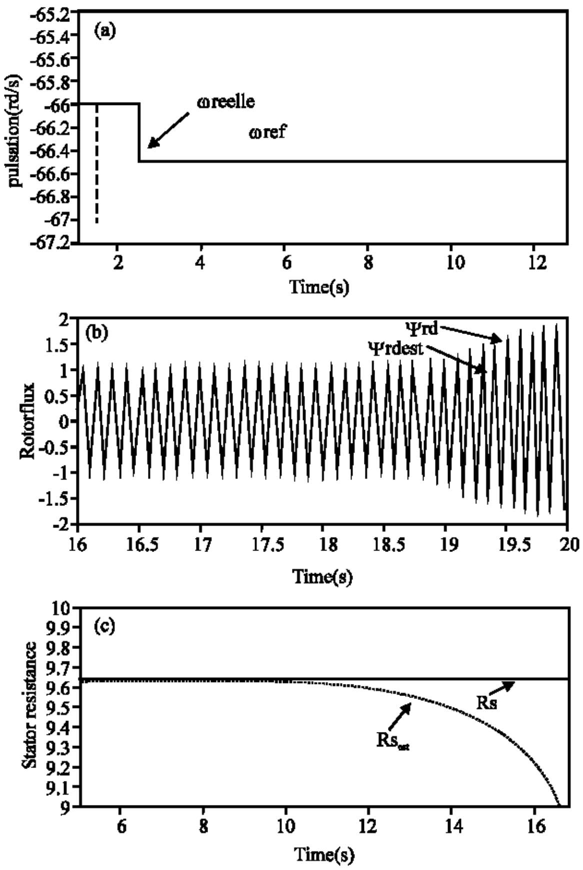

Fig. 4: Tests in regenerating mode

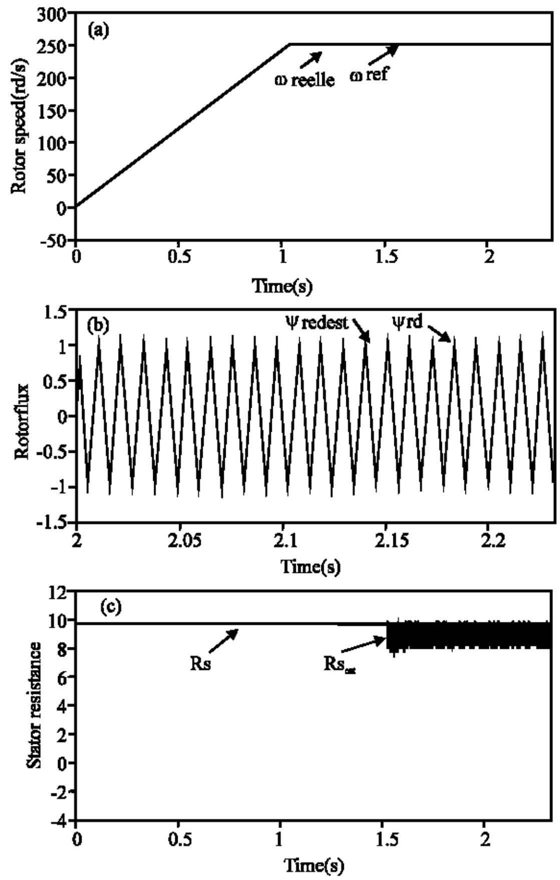

Fig. 5: Tests in monitoring mode (unstable region)

Simulation by matlab: Figure 3 and 4 illustrate simulation results for step change of pulsation reference under nominal torque.

RESULTS AND DISCUSSION

The load torque is kept at a nominal value T_{Lo} = 7 N.m. Figure 3 shows the transition of pulsation reference from stable to stable region. It can be noticed that the system is stable (speed changes from ω_0 = -50 rad/s to ω_0 = -66 rad/s, nominal torque T_{Lo} = 7 N.m, K_i = 300, K_p = 30). However, in Fig. 4 the pulsation reference have transited from stable to unstable region and the instability is occurred (speed changes from ω_0 = -50 rad/s to ω_0 = -66 rad/s , nominal torque T_{Lo} = 7 N.m, K_i = 300, K_p = 30).

Divergences on flux and resistance estimations are illustrated respectively in Fig. 4b and 4c.

Figure 5, shows simulation results for the monitoring mode (unstable region T_{Lo} = 37,2 N.m, K_i = 300, K_p = 30, Ω_0 = 250 rad/s).

In this case, it is noticed from Fig. 2 that instability regions are localized in the area where the torque and speed are higher.

CONCLUSION

In this study it has been shown that stability of observers employed in sensorless control of induction motor is not ensured. The instability appears in both regenerating and monitoring mode. The drives dynamic performance and the estimators tracking capability are strongly affected.

As perspective to continue this work it is important to study the simultaneous speed and resistance estimation.

To minimize the instability regions, the analysis of the three main approaches is required:

- Action on speed and stator resistance
- Action on feedback gain
- Simultaneous action on both speed and stator resistance and feedback gain.

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