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A New Approach to Model Reference Adaptive Control for Nonlinear Systems Using Virtual Linearization

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Abstract: Model reference adaptive control is applied to linear time varying systems and to nonlinear systems amenable to virtual linearization. Asymptotic stability is guaranteed even if the perfect model following conditions do not hold, provided that some sufficient conditions are satisfied. Simulations show the scheme to be capable of effectively controlling certain nonlinear systems.

Key words: Model reference adaptive control, nonlinear system, time varying systems, robustness, stability

INTRODUCTION

The simple adaptive control approach to direct MRAC of multi-input multi-output plants was first proposed by Sobel *et al.* (1982). This approach uses a control structure which is a linear combination of feedforward of the model states and inputs and feedback of the error between plant and model outputs. This class of algorithms requires neither full state access nor satisfaction of the perfect model following conditions. Asymptotic stability is ensured provided that the plant is Almost Strictly Positive Real (ASPR): that is, for a plant represented by (A, B, C) there exists a feedback gain matrix $\tilde{K}_e$ (not needed for implementation) such that

$$Z(s) = C(sI - A + B\tilde{K}_e C)^{-1}B$$

is strictly positive real. Barkana (1987) extended the original algorithm (which required the plant to satisfy the ASPR condition), to a class of plants which violates this condition. This approach involved designing a supplementary feed forward filter to be included in parallel with the original plant resulting in a new augmented plant which had to satisfy the same strictly positive real condition, unfortunately, the tracking error was not the true difference between the plant and the model outputs since it included the contribution of the supplementary feedforward filter. Thus, the approach was susceptible to a steady state error. Neat *et al.* (1992) suggested the incorporation of the feedforward filter of Barkana (1987) into the reference model's output as well as the plant's output in a manner so as to yield asymptotic tracking. Barkana (1991, 2005a) gives more studies about the ASPR condition and the convergence of the adaptive gains.

This study gives a new algorithm in order to overcome the ASPR condition for a class of systems. We design a direct adaptive controller that is indeed robust with respect to significant parameter uncertainty and unmodeled dynamics.

In this study, the Command Generator Tracker (CGT) concept (Sobet *et al.*, 1982) has been proven to be a very useful tool in the development of Model Reference Adaptive Control (MRAC) algorithms (Barkana, 1987).

In this study, the time varying CGT concept (Neat *et al.*, 1992) is developed, thus allowing the application of MRAC to linear time varying systems and to nonlinear systems which can be operated upon using virtual linearization.

CGT CONCEPT FOR TIME VARYING SYSTEMS

The linear varying system can be described by the following equations (Broussard and Obrien, 1979):

$$\begin{cases} \dot{x}_p(t) = A_p(t)\,x_p(t) + B_p(t)\,u_p(t) \\ y_p(t) = C_p\,x_p(t) \end{cases} \tag{1}$$

Where, $x_p(t)$ is the (n×1) state vector, $u_p(t)$ is the (m×1) control vector, $y_p(t)$ is the (q×1) plant output vector and A_p, B_p are matrices with appropriate dimensions. The linear invariantmodel to be followed is described by:

$$\begin{cases} \dot{x}_m(t) = A_m x_m(t) + B_m u_m(t) \\ y_m(t) = C_m\,x_m(t) \end{cases} \tag{2}$$

Where, $x_m(t)$ is the (n_m×1) state vector, $u_m(t)$ is the (m_m×1) control vector, $y_m(t)$ is the (q×1) plant output vector and A_m, B_m are matrices with appropriate dimensions.

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If the perfect model following is achieved, that means $y_p(t) = y_m(t)$ for $t \geq t_0$, then the resulting trajectories of control and the state are noted respectively $u^*_p(t)$ and $x^*_p(t)$ by definition, the ideal variables must satisfy:

$$C_p\ x^*_p(t) = C_m\ x_m(t) \tag{3}$$

$$\dot{x}^*_p(t) = A_p(t)\,x^*_p(t) + B_p(t)\,u^*_p(t) \tag{4}$$

$$\begin{bmatrix} x^*_p(t) \\ u^*_p(t) \end{bmatrix} = \begin{bmatrix} S_{11}(t) & S_{12}(t) \\ S_{21}(t) & S_{22}(t) \end{bmatrix} . \begin{bmatrix} x_m(t) \\ u_m(t) \end{bmatrix} \tag{5}$$

Where the $S_{ij}(t)$ matrices are appropriately dimensioned time varying matrices.

Provided that $u_m = 0$, the $S_{ij}(t)$ matrices will have to satisfy the following set of Eq. 5:

$$A_p(t)\,S_{11}(t) + B_p(t)\,S_{21}(t) = \dot{S}_{11}(t) + S_{11}(t)A_m \tag{6}$$

$$A_p(t)\,S_{12}(t) + B_p(t)\,S_{22}(t) = \dot{S}_{12}(t) + S_{11}(t)B_m \tag{7}$$

$$C_p S_{11}(t) = C_m \tag{8}$$

$$C_p S_{12}(t) = 0 \tag{9}$$

The Eq. 6-9 can be writen as:

$$\begin{bmatrix} A_p(t) & B_p(t) \\ C_p & 0 \end{bmatrix} \times \begin{bmatrix} S_{11}(t) & S_{12}(t) \\ S_{21}(t) & S_{22}(t) \end{bmatrix} = \begin{bmatrix} \dot{S}_{11}(t) + S_{11}(t)A_m & \dot{S}_{12}(t) + S_{11}(t)B_m \\ C_m & 0 \end{bmatrix} \tag{10}$$

One suppose that m = q, then if the matrix

$$\begin{bmatrix} A_p(t) & B_p(t) \\ C_p & 0 \end{bmatrix}$$

is invertible, Eq. 10 gives:

$$\begin{bmatrix} S_{11}(t) & S_{12}(t) \\ S_{21}(t) & S_{22}(t) \end{bmatrix} = \begin{bmatrix} \Omega_{11}(t) & \Omega_{12}(t) \\ \Omega_{21}(t) & \Omega_{22}(t) \end{bmatrix} \times \begin{bmatrix} \dot{S}_{11}(t) + S_{11}(t)A_m & \dot{S}_{12}(t) + S_{11}(t)B_m \\ C_m & 0 \end{bmatrix} \tag{11}$$

Where:

$$\begin{bmatrix} \Omega_{11}(t) & \Omega_{12}(t) \\ \Omega_{21}(t) & \Omega_{22}(t) \end{bmatrix} = \begin{bmatrix} A_p(t) & B_p(t) \\ C_p & 0 \end{bmatrix}^{-1} \tag{12}$$

Using Eq. 11, the differential system to be solved is given by:

$$\Omega_{11}(t)\,\dot{S}_{11}(t) = S_{11}(t) - \Omega_{11}(t)\,S_{11}(t)A_m - \Omega_{12}(t)C_m \tag{13}$$

$$\Omega_{11}(t)\,\dot{S}_{12}(t) = S_{12}(t) - \Omega_{11}(t)\,S_{11}(t)B_m \tag{14}$$

$$S_{21}(t) = \Omega_{21}(t)\,\dot{S}_{11}(t) + \Omega_{21}(t)\,S_{11}(t)A_m + \Omega_{22}(t)C_m \tag{15}$$

$$S_{22}(t) = \Omega_{21}(t)\,\dot{S}_{12}(t) + \Omega_{21}(t)\,S_{11}(t)B_m \tag{16}$$

We see that equation $S_{21}(t)$ and $S_{22}(t)$ depend of $S_{11}(t)$ and $S_{12}(t)$ by using (15) and (16), then, just Eq. 13 and 14 must be solved.

MRAC ALGORITHM

The MRAC problem will be solved for the following process equations:

$$\begin{cases} \dot{x}_p(t) = A_p(t)\,x_p(t) + B_p(t)\,u_p(t) \\ y_p(t) = C_p\ x_p(t) \end{cases} \tag{17}$$

The objective is to find a control u(t) such that the plant out put $y_p(t)$ follows the output $y_m(t)$ of the reference model:

$$\begin{cases} \dot{x}_m(t) = A_m x_m(t) + B_m u_m(t) \\ y_m(t) = C_m\ x_m(t) \end{cases} \tag{18}$$

To facilitate the development of the control law, the time varying CGT concept is presented here.
The control law is chosen of the form

$$u(t) = K_e(t)(y_m(t) - y(t)) + K_x(t)x_m(t) + K_u(t)u_m(t) \tag{19}$$

or

$$u = K.r \tag{20}$$

Where: $K = (K_e, K_x, K_u)$ and

$$r^T = ((y_m - y)^T, x_m^T, u_m^T)$$

K(t) is generated according to the following adaptive rule:

$$K(t) = K_I(t) + K_p(t) \tag{21}$$

$$\dot{K}_I(t) = C_p(y_m(t) - y(t))r^T T \tag{22}$$

$$K_p(t) = C(y_m(t) - y(t))r^T\bar{T} \quad (23)$$

with the error defined as:

$$e(t) = x^*(t) - x(t) \quad (24)$$

The error dynamics will be:

$$\dot{e}(t) = \dot{x}^*(t) - \dot{x}(t) \quad (25)$$

Using Eq. 17, 4, 5 and 21 we obtain:

$$\begin{aligned} e(t) &= A_p(t)e(t) + B_p(t)[S_{21}(t)x_m(t) + \\ &S_{22}(t)u_m(t) - K_p(t)r(t) - K_I(t)r(t)] \end{aligned} \quad (26)$$

STABILITY ANALYSIS

Stability will be analyzed using a Lyapunov approach. Let the Lyapunov function be Kaufman *et al.* (1998):

$$\begin{aligned} V(t) &= e^T(t)P(t)e(t) + TR[S(K_I(t) - \tilde{K}(t))T^{-1} \\ &\times (K_I(t) - \tilde{K}(t))^T S^T] \end{aligned} \quad (27)$$

TR: Trace of the matrix

T and P(t) are respectively constant and time varying positive definite symmetric matrices, S is a non-singular matrix and $\tilde{K}$ is partitioned in the same manner as K, i.e.,

$$\tilde{K}(t) = [\tilde{K}_e(t), \tilde{K}_x(t), \tilde{K}_u(t)] \quad (28)$$

and is assumed to be such that:

$$\dot{\tilde{K}} = C_p e(t) r^T(t) T_1(t) \quad (29)$$

With $T_1(t)$ a time varying matrix.

After some algebraic manipulations, the time derivative of V becomes:

$$\begin{aligned} \dot{V} &= e^T(t)[\dot{P}(t) + P(t)(A(t) - B(t)\tilde{K}_e^{'}(t)C_p) \\ &+ (A_p(t) - B_p(t)\tilde{K}_e^{'}(t)C_p)^T P(t)]e(t) \\ &- 2e^T(t)P(t)B_p(t)(S^TS)^{-1}B_p{}^T P(t)e(t)r^T(t)\bar{T}r(t) \end{aligned} \quad (30)$$

This expression arises if the output matrix satisfies

$$C_p = (S^TS)^{-1}B_p{}^T(t)P(t) \quad (31)$$

And if some

$$\tilde{K}_x^{'}(t), \quad \tilde{K}_u^{'}(t)$$

are chosen so that:

$$\tilde{K}_x^{'}(t) = S_{21}(t) \text{ and } \tilde{K}_u^{'}(t) = S_{22}(t)$$

This derivative will be negative definite in e if:

- T is positive definite
- Tis positive semi definite
- $C = (S^TS)^{-1}B_p{}^T(t)P(t)$
- $\dot{P}(t) + P(t)(A_p(t) - B_p(t)\tilde{K}_e^{'}(t)C_p) + (A_p(t) - B_p(t)\tilde{K}_e^{'}(t)C_p)^T P(t)$

is negative definite for some $\tilde{K}_e^{'}(t)$ not needed form implementation.

Note that the matrices

$$\tilde{K}_x^{'}(t), \tilde{K}_u^{'}(t), \tilde{K}_e^{'}(t), \ P(t)$$

as well as the $S_{ij}(t)$ matrices are not needed for the implementation of the control law.

TREATMENT OF NONLINEAR SYSTEMS

The class of nonlinear systems which can be operated upon using the virtual linearization procedure can be controlled via the MRAC scheme of this study. The procedure is best explained through an example:

Let

$$\dot{x}(t) = g_i(x,u,t) + h_i(x,u,t) + P_i(x,u,t), i = 1,2 \quad (32)$$

And write

$$\dot{x} = (g_i / x_1)x_1 + (h_i / x_2)x_2 + (P_i / u)u \quad (33)$$

Where:

$$x_1 \xrightarrow{\lim} 0 \ |g_i / x_1| \langle \infty$$
$$x_2 \xrightarrow{\lim} 0 \ |h_i / x_2| \langle \infty$$
$$u \xrightarrow{\lim} 0 \ |P_i / u| \langle \infty$$

Then the linearized system will be:

$$\dot{x}(t) = A_p(t)x(t) + B_p(t)u(t)$$

Where:

$$A_p(t) = \begin{bmatrix} \frac{g_1}{x_1} & \frac{h_1}{x_2} \\ \frac{g_2}{x_1} & \frac{h_2}{x_2} \end{bmatrix}, B(t) = \begin{bmatrix} P_1 / u \\ P_2 / u \end{bmatrix}$$

Note that this way of rewriting the system does nothing but rearrange the terms in each equation so that when x and u are specified, the systems appears to be linear.

ILLUSTRATIVE EXAMPLES

Example 1: MRAC for a linear system

Let a SISO system of transfer function

$$G(s)=\frac{2(s+4)}{s^2-3s-2}$$

Its representation in state form is given by:

$$\dot{x}_p(t)=A_p x_p(t)+B_p u_p(t)$$
$$y_p(t)=C_p x_p(t)$$

Where:

$$A_p=\begin{bmatrix}0 & 1\\ 2 & 3\end{bmatrix},\ B_p=\begin{bmatrix}0\\1\end{bmatrix},\ C_p=[8\quad 4]$$

The transfer function of the model is given by

$$G_m(s)=\frac{1}{0.2s+1}$$

That means

$$\dot{x}_m(t)=A_m x_m(t)+B_m u_m(t)$$
$$y_m(t)=C_m x_m(t)$$

Where:

$$A_m=[-5],\ B_m=[5],\ C_m=[1]$$

The input to the model is a wave signal of amplitude ±1 and a period of 60 sec.The application of the MRAC using the CGT concept leads to a asymptoticaly stable error.

Figure 1 gives the outputs of the system and the model which we can see the good following at steady state, the control signal is given in Fig. 2 which we see that it have an acceptable values.

Example 2: Nonlinear third order system
The equations for the plant are:

$$\dot{x}_1=(1-x_1^2-0.088)x_1-0.877x_1+0.47x_1^2$$
$$+3.846x_1^3-0.215u+0.28ux_1^2+46u^2x_1+0.63u^3$$
$$\dot{x}_2=x_3$$
$$\dot{x}_3=-0.396x_3-4.208x_1-0.47x_1^2-3.564x_1^3$$
$$-20.967u+6.265ux_1^2+46u^2x_1+61.4u^3$$
$$y(t)=x_1(t)$$

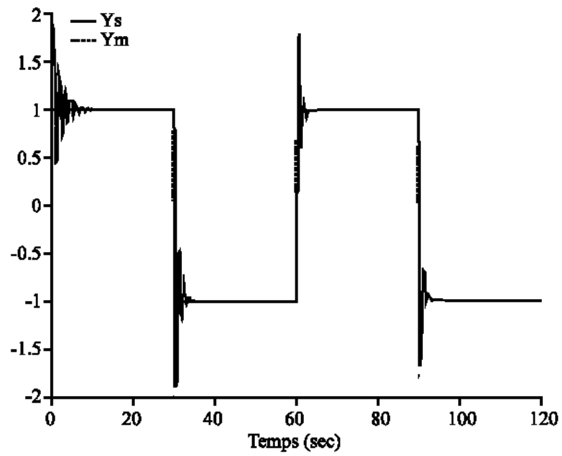

Fig. 1: Outputs of the system and the model

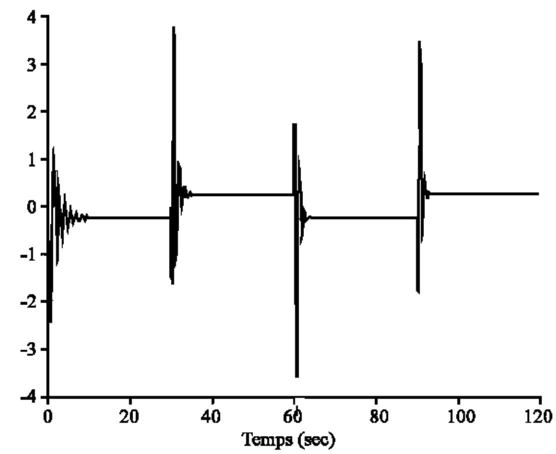

Fig. 2: Command signal

The model equations are:

$$\dot{x}_m(t)=\begin{bmatrix}-0.5 & 0 & 1\\ 0 & 0 & 1\\ -9.87 & 0 & -1.7\end{bmatrix}x_m(t)+\begin{bmatrix}1\\1\\1\end{bmatrix}u_m$$
$$y_m(t)=(1,0,0)x_m(t)$$

The simulation was carried out for the initial conditions:

$$x(0)=x_m(0)=[0\ 0\ 0]^T,\ T=\bar{T}\,I_5,\ K_I(0)=0$$

The simulation results shown in Fig. 3 indicates that the plant output does following asymptotically the model output.

Figure 4 shows the command signal which is bounded and smooth.

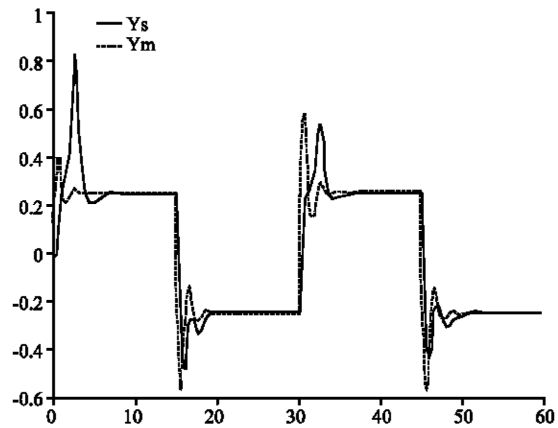

Fig. 3: Outputs of the system and the model

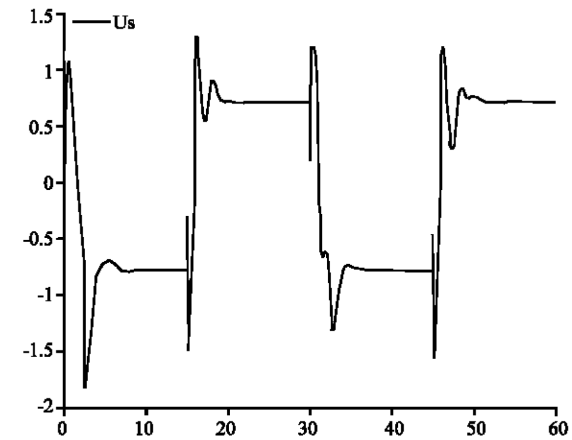

Fig. 4: Command signal

CONCLUSION

In this study, one presents at first the concept of CGT for nonlinear systems then, an extension of adaptive MRAC to nonlinear systems was developed by using the procedure of virtual linearization, a simulation on a third order nonlinear system was tested, leading to an error asymptotically stable between the system and the reference model. This study will make it possible the MRAC to be applied to the nonlinear systems and even with the systems strong linearties which is the case of the majority of the industrial systems.

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