

## Scalar Stability Criteria for Interconnected Nonlinear Power Systems

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**Abstract:** This study establishes criteria for stability of nonlinear zero-input interconnected power systems using an arrangement of scalar blocks where the input-output pair for each block is strictly proper rational real or transfer function. The structure of the multivariable system as an interconnection of scalar feedback blocks provides a basis for obtaining new conditions for stability of zero-input interconnected nonlinear power systems.

**Key words:** Scalar feedback blocks, nonlinear stability, zero-input interconnected power systems, liapunov function

### INTRODUCTION

A modern electric power plant is a large scale, distributed and highly nonlinear (Liu *et al.*, 2007), complex multimachine interconnection (Wang *et al.*, 1993) with voltage and frequency dependent loads. Control of the system is required to maintain reasonably uniform frequency, divide the load between generators and control the tie-line interchange schedules. Overall control (Nicholson and Sterling, 1972) is based on a combination of manual intervention, fast continuously-acting feedback control of local regulating loops and an essentially slower-acting optimized generator scheduling based on predicted load disturbances. The nonlinearity (Lu *et al.*, 2006) arises essentially by reason of physical laws governing the operation of the components and it originates either inherently or intentionally (Gray and Nakhia, 1981). A typical example is an on/off time-delay servo employed as a protective device in speed control systems. The characteristics of the servo are such that there is always some lost motion of the armature in moving from one contact to the other. The extent of the movement over which neither contact is reached gives rise to a dead zone over which the output is zero. Since the servo must operate the shaft of a motor to overcome the coulomb friction, the differential equation incorporating the nonlinearity is described by a second order relation with inertia  $J$ , viscous damping  $F$  and torque per unit error  $K$  as follows:

$$J \frac{d^2 \theta_0}{dt^2} \pm F + K \theta_0 = K \theta_1 \quad (1)$$

which is reducible to the form

$$\pm F = F \operatorname{sgn} \theta_0 \quad (2)$$

where,  $\theta_0$  is the output and  $\theta_1$  the input. The magnitude of the corrective action is directly dependent on the size of the error signal. The dead zone (Gao and Wang, 2004; Gorecki *et al.*, 1989) is used to ensure that as long as the error magnitude is less than some defined value there is no regulative action.

Analytical methods do exist (Xu and Yu, 2006; Marshall and Salehi, 1982) which enable the dead zone for given signal magnitudes to be predicted in the event of limit cycle oscillations in uncontrolled conditions. In linear time-invariant multivariable systems (Obinabo and Anyasi, 2007; Mayne, 1973), stability is relatively simple to determine. A number of necessary and sufficient conditions has been established which yielded unequivocal information concerning the system stability.

The objective of this study, has been to establish conditions for stability of zero-input interconnected nonlinear power systems, using an arrangement of scalar blocks where the input-output pair for each block is strictly proper rational real or transfer function.

### FORMULATION OF THE PROBLEM

Consider a nonlinear machine interconnected power system with inputs  $r_1, \dots, r_n$  and outputs  $y_1, \dots, y_n$  characterized by the relations

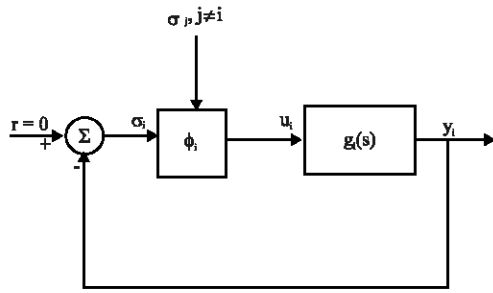


Fig. 1: A block diagram representation of the nonlinear zero-input power system

$$\begin{aligned} u_i(s) &= \phi_i(\sigma_1, \dots, \sigma_n), i = 1, \dots, n \\ \sigma_i(s) &= R_i(s) - Y_i(s), i = 1, \dots, n \\ Y_i(s) &= g_i(s)u_i(s), i = 1, \dots, n \end{aligned} \quad (3)$$

where,  $u_i$  and  $\sigma_i$  are auxiliary variables,  $i = 1, \dots, n$ ; capital letters denote transforms of the variable. Here,  $\phi_i: \mathbb{R}^n \rightarrow \mathbb{R}^1$  are continuous functions for  $i = 1, \dots, n$  and  $g_i(s)$ ,  $i = 1, \dots, n$  are strictly proper rational transfer functions. To exclude trivial loops we require no  $g_i(s) = 0$  identically,  $i = 1, \dots, n$ . A block diagram representation for (3) is shown in Fig. 1 which is a general representation for the nonlinear system and for which a state realization is easily obtained, given appropriate matrices  $A_i$ ,  $b$  and  $c^T$ ,  $i = 1, \dots, n$ , as:

$$\begin{aligned} \frac{dx_i}{dt} &= A_i x_i + b_i v_i \\ &= A_i x_i + b_i \phi(u_i - c_i^T x_1, \dots, u_n - c_n^T x_n) \\ y_i &= c_i^T x_i, \quad i = 1, \dots, n \end{aligned} \quad (4)$$

where,  $x_i = [x_{i1}, x_{i2}, \dots, x_{in}]^T \in \mathbb{R}^n$  is a state vector with all  $x$ , assumed to be available for measurement. In order for (4) to be controllable, it is required that  $b \neq 0$  for  $x$  in a certain controllability region  $U \subset \mathbb{R}_n$ .

The nonlinearity is described by the relation and  $u_i(s) = \phi_i(\sigma_1, \dots, \sigma_n)$ ,  $i = 1, \dots, n$ , and  $g(s)$  is, in general, expressed as follows:

$$\frac{Y(s)}{U(s)} = \frac{\beta_1 s^{n-1} + \beta_2 s^{n-2} + \dots + \beta_n}{a_0 s^n + a_1 s^{n-1} + \dots + a_n} \quad (5)$$

Now if  $x_i = y$ , it can be shown that

$$x_i = x_{i+1} + b_i u, \quad i = 1, \dots, n-1 \quad (6)$$

$$x_n = -\sum_{i=1}^n a_{n-i+1} x_i + b_n u \quad (7)$$

Where,

$$\begin{aligned} b_i &= \beta_i - \sum_{j=1}^{i-1} a_{i-j} b_j, \quad i = 1, \dots, n, \\ \beta_0 &= b_0 = 0 \text{ and } \alpha_0 = 1 \end{aligned}$$

Applying this to the problem on hand gives.

$$\begin{aligned} x_1 &= y \\ b_1 &= 1 \\ b_2 &= z-1 \end{aligned}$$

Considering the nonlinear element defined by  $u = -x_1 \phi(-x_1)$ , the state equations become

$$x_1 = x_2 - x_1 \phi(-x_1) \quad (8)$$

$$x_2 = -x_2 - (z-1)x_1 \phi(-x_1) \quad (9)$$

For convenience of notation we will write  $\phi$  for  $\phi(-x_1)$ .

### STABILITY CRITERIA FOR THE NONLINEAR STATE MODEL

If a system is stable in state space then a Liapunov function  $V$  and its gradient  $\nabla V$  exist 9Nwokah (1976). Given this gradient, both  $V$  and  $d/dt \{V\}$  were calculated as follows:

$$\frac{d}{dt}(V) = \sum_{i=1}^n \frac{\partial V}{\partial x_i} \frac{dx_i}{dt} \quad (10)$$

where for  $dx_i/dt$ , we inserted the system equations. This was then expressed as:

$$\frac{d}{dt}\{V\} = (\nabla V)^t \frac{dx}{dt}$$

To find  $V$  from  $\nabla V$  we defined a line integral

$$V = \int_0^x (\nabla V)^t dx \quad (11)$$

to an arbitrary point in the state space, provided

$$\frac{\partial \nabla V_i}{\partial x_j} = \frac{\partial \nabla V_j}{\partial x_i}, \text{ for all } i, j = \dots, n \quad (12)$$

From Eq. 10

$$\begin{aligned} V = & (a_{11}x_1 + a_{12}x_2)(x_2 - x_1\phi) + \\ & (a_{21}x_1 + 2x_2)(-x_3 - (z-1)x_1\phi) \\ & (a_{11} - a_{21} - a_{12}\phi - 2(z-1)\phi)x_1x_2 - \\ & \phi(a_{11} + a_{21}(z-1))x_1^2 - (2 - a_{12})x_2^2 \end{aligned}$$

To satisfy (12)

$$\frac{\partial \nabla V_1}{\partial x_2} = \frac{\partial \nabla V_2}{\partial x_2} \rightarrow a_{12} = a_{21}$$

Where, the coefficients  $\alpha_{ij}$  were computed from the vector  $\nabla V$  Eq. 13.

$$\nabla V = \begin{bmatrix} \nabla V_1 \\ \vdots \\ \nabla V_n \end{bmatrix} = \begin{bmatrix} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n \\ \vdots \\ a_{n1}x_1 + \dots + a_{nm}x_n \end{bmatrix} \quad (13)$$

$$a_{ij} = a_{ji} + a_{ij2}(x_1)$$

For  $V$  to be positive definite in the neighbourhood of the origin,  $\alpha_{ij}$  must always be positive. Also if the limit

$$\lim_{\|x\| \rightarrow \infty} V(x) \rightarrow \infty$$

were to be satisfied, then  $\alpha_{ij2}(x_1)$ , if it exists, must be an even function of  $x_1$  and greater than zero for large  $x_1$ . The coefficients  $\alpha_{ij}$  were chosen so as to make  $d/dt\{V\}$  at least negative semi-definite.

From Eq. 13 we obtained

$$\nabla V = \begin{bmatrix} \nabla V_1 \\ \vdots \\ \nabla V_2 \end{bmatrix} = \begin{bmatrix} a_{11}x_1 + a_{12}x_2 \\ \vdots \\ \alpha_{21}x_1 + 2x_2 \end{bmatrix} \quad (14)$$

Putting  $\alpha_{12} = \alpha_{21} = 0$  and  $\alpha_{11} = 2(z-1)\phi$ , the expression from  $d/dt\{T\}$  becomes

$$\frac{d}{dt}\{V\} = -2(z-1)\phi^2x_1^2 - 2x_2^2$$

which is obviously negative definite if  $z > 1$ . We now have

$$\nabla V = \begin{bmatrix} 2(z-1)\phi x_1 \\ \vdots \\ 2x_2 \end{bmatrix} \quad (15)$$

Form Eq. 11

$$\begin{aligned} V = & \int_{(\alpha, 0)}^{(x_1, 0)} (2(z-1)\phi x_1^1) dx_1^1 + \int_{(x_1, 0)}^{(x_1, x_2)} 2x_2^1 dx_2^1 \\ & = 2(z-1) \int_0^{x_1} (\phi x_1^1) dx_1^1 + x_2^2 \end{aligned} \quad (16)$$

Now for  $d/dt(V) < 0$  and  $V > 0$ , it is sufficient that

$$z > 1$$

and

$$\int_0^{x_1} \phi x_1^1 dx_1^1 > 0, \text{ for all } x. \quad (17)$$

Furthermore, the condition  $\lim_{\|x\| \rightarrow \infty} V(x) \rightarrow \infty$  is obviously  $\lim V(x)$  satisfied.

We now develop conditions for stability of the interconnected nonlinear power system based on the representation (3). If  $u(t) = 0, t \geq 0, i = 1, \dots, n$  and if we require  $\phi_i(0, \dots, 0) = 0, i = 1, \dots, n$ , it is clear that for the initial state zero we have  $y_i(t) = 0, t \geq 0$  and  $i = 1, \dots, n$ . If, in addition, for any initial state there is  $M > 0$  such that  $\langle M, t = 0$  and  $\lim_{t \rightarrow \infty} y_i(t) = 0, i = 1, \dots, n$ , then (3) is said to be stable.

**Theorem:** Suppose  $p_i < q_i$  and  $1 + q_i g_i(s) / 1 + p_i g_i(s)$  is positive real for  $i = 1, \dots, n$ . The closed-loop system represented by (1) is stable if there exist multipliers  $\lambda_i > 0, i = 1, \dots, n$  such that

$$\sum_{i=1}^n \lambda_i (q_i e_i - \phi_i(e_1, \dots, e_n)) - p_i e_i > 0 \quad \text{for all } (e_1, \dots, e_n) \neq 0 \quad (18)$$

**Proof:** As in (4) let

$$\begin{aligned} \frac{dx_1}{dt} &= A_1 x_1 + b_1 v_1 \\ y_i &= c_i^T x_i \end{aligned}$$

be a minimal realization for  $g_i(s)$ ,  $i = 1, \dots, m$ . It can be shown that there are  $\mu_i > 0$ ,  $v_i > 0$  such that for  $t \geq 0$  and  $i = 1, \dots, n$

$$\mu_i |x_i(t)|^2 \leq v_i |x_i(0)|^2 + \left(\frac{1}{q_i - p_i}\right)^2 \int_0^t (v_t + p_t y_t)(q_t y_t + v_t) dt \quad (19)$$

Thus if  $\lambda_i > 0$ ,  $i = 1, \dots, n$  then

$$\sum_t \lambda_t \mu_t (q_t - p_t)^2 |x_t(t)|^2 \leq \sum_t \lambda_t v_t (q_t - p_t)^2 |x_t(0)|^2 + \sum_t \int_0^t (v_t + p_t y_t)(q_t y_t + v_t) dt, t \geq 0 \quad (20)$$

But since  $\mu_i(t) = 0$ ,  $t \geq 0$ ,  $i = 1, \dots, n$  we have

$$v_i = \phi_i(e_1, \dots, e_n), y_i = -e_i, i = 1, \dots, n \quad (21)$$

And thus using (18)

$$\sum_i \lambda_i \int_0^t (v_i + p_i y_i)(q_i y_i + v_i) dt - \sum_i \lambda_i (\phi_i - p_i e_i)(q_i e_i - \phi_i) \leq 0, t \geq 0 \quad (22)$$

Consequently

$$\sum_i \lambda_i \mu_i (q_i - p_i)^2 |x_i(t)|^2 \leq \sum_i \lambda_i v_i (q_i - p_i)^2 |x_i(0)|^2, t \geq 0 \quad (23)$$

So that for some  $M > 0$

$$|e_i(t)| \leq M, t \geq 0, i = 1, \dots, n \quad (24)$$

Thus for some  $N > 0$  we also have

$$\int_0^t \sum_i \lambda_i (\phi_i - p_i e_i)(q_i e_i - \phi_i) dt \leq N, t \geq 0. \quad (25)$$

Under the stated assumption, it follows that

$$\lim_{t \rightarrow \infty} e_i(t) = 0, i = 1, \dots, n \quad (26)$$

Consequently

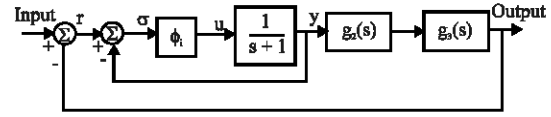


Fig. 2: An interconnection of two power generating plants

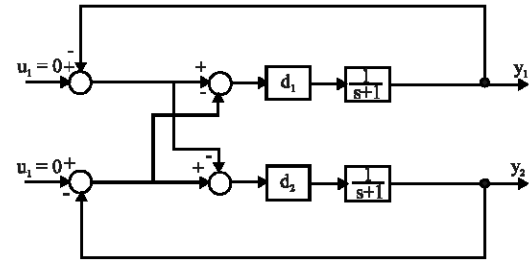


Fig. 3: An equivalent zero-input scalar representation of the generating plants

$$|y_i(t)| < M \quad (27)$$

$$\lim_{t \rightarrow \infty} y_i(t) = 0, i = 1, \dots, n.$$

which completes the proof.

**Example 1:** Consider an interconnection of two power generating plants with the nonlinear element and feedback loops (Fig. 2). An equivalent zero-input scalar representation is obtained in Fig. 3.

In the notation of (3) we have

$$g_1(s) = g_2(s) = \frac{1}{s+1}$$

and

$$\phi_1(e_1, e_2) = d_1(e_1 - e_2)$$

$$\phi_2(e_1, e_2) = d_2(e_2 - e_1)$$

An examination using the resulting simplified form of the theorem indicates that the closed loop system is stable if for all  $\sigma_1 \neq 0$ ,  $\sigma_2 \neq 0$ ,

$$\frac{d_1}{\sigma_1} > -1, \frac{d_2}{\sigma_2} > -1,$$

$$\left(\frac{d_1}{\sigma_1} + 1\right)\left(\frac{d_2}{\sigma_2} + 1\right) > \frac{1}{4} \left(\frac{d_1}{\sigma_1} + \frac{d_2}{\sigma_2}\right)^2,$$

Where,  $\sigma_1 = e_1 - e_2$  and  $\sigma_2 = e_2 - e_1$ .

**Example 2:** A representation of a multimachine interconnection is considered in Fig. 4; it is of one form (3) with

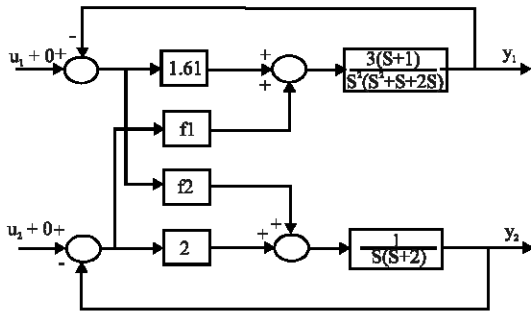


Fig. 4: A representation of a multimachine interconnection

$$g_1(s) = \frac{3(s+1)}{s^2(s^2+s+25)}$$

$$g_2(s) = \frac{1}{s(s+2)}$$

and

$$\phi_1(e_1, e_2) = 1.61e_1 + d_1(e_2)$$

$$\phi_2(e_1, e_2) = d_2(e_1) + 2e_2$$

For stability, we chose  $\alpha_1 = 1$ ,  $\beta_1 = 2.22$  and  $\beta_2 = 0$ ,  $\beta_2 = 4$ . On the basis of the theorem, the closed-loop system is stable if

$$|f_1(e_2)| \leq k_1 |e_2|, |f_2(e_1)| \leq k_2 |e_1|$$

hold for all  $e_1$  and  $e_2$  and if  $k_1 k_2 < 1.22$ .

### CONCLUSION

A procedure for the design of a stable nonlinear interconnected (composite) power system is established here. For the continuous autonomous systems  $g_i(s)$ ,  $i = 1, \dots, n$  the parameters  $\alpha_i$  and  $\beta_i$  for each  $i = 1, \dots, n$  were then chosen in a manner similar to Obinabo and Anyasi (2007). On the basis of  $\alpha_i, \beta_i$ ,  $i = 1, \dots, n$  so obtained, we chose the functions  $\phi_1, \dots, \phi_n$  subject to the constraints imposed on their selection so that inequality (18) was satisfied. A trial and error approach was employed since selection of  $\alpha_i, \beta_i$ ,  $i = 1, \dots, n$  was not unique.

The approach also shows an evaluation of the effects of any uncertainties in  $g_i(s)$ ,  $i = 1, \dots, n$  and  $\phi_i$ ,  $i = 1, \dots, n$ . In particular, it is possible to assess the effects of failure of certain components directly from examination of inequality (18) expressed in the following equivalent form

$$\sum_{i=1}^m \lambda_i \left| \left( \frac{q_i - p_i}{2} \right) e_i \right|^2 > \sum_{i=1}^m \lambda_i^2 \left| \phi_i \left( e_1, \dots, e_m - \left( \frac{p_i + q_i}{2} \right) e_i \right) \right|^2 \quad (28)$$

for all  $(e_1, \dots, e_m) \neq 0$ . It is the inequality (18), or the equivalent inequality (28), which presents the difficulties of verification in the application of the theorem.

The inequality (18) is however, satisfied if for each  $(e_1, \dots, e_n)$  with  $e_i \neq 0$

$$a_i e_i < e_i \phi_i(e_1, \dots, e_m) < \beta_i e_i, \quad i = 1, \dots, n$$

In terms of the slopes of the nonlinear functions, the inequality (18) is considered as follows. Suppose that,

$$\phi_i(e_1, \dots, e_m) = \Phi_i(\sigma_i), \quad i = 1, \dots, n$$

Where:

$$\sigma_i = \sum_{j=1}^n k_{ij} e_j, \quad i = 1, \dots, n, \text{ so that}$$

$$\phi_i(e_1, \dots, e_n) = \sum_{j=1}^n \left( \frac{\phi_i(\alpha_j)}{\alpha_j} k_{ij} \right) e_j, \quad i = 1, \dots, n.$$

It is easily shown that the inequality (18) is satisfied if and only if for each  $\sigma_i \neq 0$ ,  $i = 1, \dots, n$  the matrix  $[\Lambda_{ij}]$  is positive definite where

$$\Lambda_{ij} = \lambda_i (q_i - k_{ij} \frac{\phi_i}{\sigma_i}) (k_{ij} \frac{\phi_i}{\sigma_i} - p_i) - \sum_{j=1}^n \lambda_j k_{ij}^2 \left( \frac{\phi_j}{\sigma_j} \right)^2$$

$$\Lambda_{ij} = \Lambda_{ji} = \frac{1}{2} \{ \lambda_i k_{ij} \frac{\phi_i}{\sigma_i} [q_i - k_{ij} \frac{\phi_i}{\sigma_i}] - (k_{ij} \frac{\phi_i}{\sigma_i} - p_i) \}$$

$$+ \lambda_j k_{ji} \frac{\phi_j}{\sigma_j} [(q_j - k_{jj} \frac{\phi_j}{\sigma_j}) - (k_{jj} \frac{\phi_j}{\sigma_j} - p_j)] \}$$

$$- \sum_{\substack{r=1 \\ r \neq i}}^n \lambda_r k_{ri} k_{ij} \left( \frac{\phi_r}{\sigma_r} \right)^2, \quad i \neq j.$$

The condition that  $[\Lambda_{ij}]$  is positive definite constitutes a set of constraints on the slopes

$$\frac{\phi_1}{\sigma_1}, \dots, \frac{\phi_n}{\sigma_n}$$

which guarantee stability of nonlinear interconnected systems of the form (3).

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