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The Construction of Rational Transfer Function Matrices for Inverse Multimachine Power Systems with Defined Input-Output Parameters

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Abstract: A method for computing rational transfer function matrices for linear constant multimachine power systems defined by the relation $\underline{y}$ (s) = G (s) $\underline{x}$ (s), $\underline{x}$ (s) = $\underline{u}$ (s) - F (s) $\underline{y}$ (s) directly from the Markov parameters is presented, where,

$$G(s) \triangleq (g_{kl}(s))$$

represents the plant and

$$F(s) \triangleq \text{diag}\left(f_1(s),\ldots,f_n(s)\right)$$

the controller. Since, computational modeling provides a potentially powerful way of integrating structural properties of dynamic systems given in state space, an algorithm is described which enables transfer function matrices of constant multimachine power systems to be obtained by inversion. The result of this study confers several advantages over the other methods reported in the literature and can be applied directly to differential systems with input-output parameters without having to derive state space realization.

Key words: Transfer function matrices, linear constant multivariable systems, Markov parameters

INTRODUCTION

Coupled dynamic systems described by multiple input-output configurations occur essentially in modern power plants where typically a change in any cause or input, give rise to many effects or output changes. Extensive studies of deterministic and stochastic processes (Ji and Kim, 2007; Kalman *et al*., 1969; Rosenbrock, 1970; MacFarlane, 1974) report structural decomposition of linear multivariable systems which can be represented by transfer function matrices using spectral factorization (Meyer, 1990; Fernando and Nicholson, 1982; Whalley, 1978). The importance of this problem has necessitated many research studies some of which have been focused on determining the internal structure of the interactive process (Kerckhoffs *et al*., 2006; Kalman *et al*., 1969) often characterized by a matrix of rational functions in the Laplace transform variables and used to synthesize the Wiener-Hopf optimum filter in communications theory, or compensating networks in control engineering directly from the insight gained from analysis of such systems.

Use of simplified transfer function matrices in multivariable control system design (Bengiamin and Chan, 1982) offers considerable savings in computation time and analysis and leads to much effort towards reduction of the order of a mathematical model in some systematic manners. One of the powerful synthesis techniques for the scalar case which also provides much insight into the problem on hand is the root-locus technique (Nwokah, 1983). A single variable root locus method was originally developed and later extended to the multivariable formulations with a single gain (Owens, 1978; MacFarlane, 1980) and multiple gain (Byrnes, 1981; Nwokah, 1981) parameters multiplying the open-loop transfer matrix G (s). It is shown that for a transfer function matrix of order n, a separately adjustable parameter a_k, k = 1, ..., n constructed as a feedback loop across a multivariable system yields an asymptotic behaviour of the closed-loop system under simultaneous variation of the adjustable parameter such that as $a_k \rightarrow \infty$ for k = 1, ..., n, the finite poles of G (s) either terminate or remain in the left hand of the complex frequency plane.

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The use of scalar feedback loops constructed over multivariable systems (Obinabo, 2008) enables the controller dynamics to be chosen in such a manner to yield the desired performance and sensitivity of the system to large parameter uncertainty.

Since, each input-output pair in any given situation generally leads to a different root locus family, sufficient conditions are desired under which the matrix of Markov parameters of the open-loop transfer function matrix will guarantee acceptable performance for every gain matrix. The results presented in this study, provide a good basis for operating the multimachine power plant, choosing an input-output pairing configuration that will guarantee certain types of closed-loop asymptotic behaviour as p is varied from 0 to ∞. An algorithm is described which enables transfer functions of linear multimachine power systems to be obtained by inversion.

MATHEMATICAL PRELIMINARIES

Definition: Given a formal power series

$$f(x) = C_0 + C_1x + C_2x^2 + ... \quad (1)$$

the rational function $p_m(x)/Q_m(x)$, where, $p_m(x)$ and $Q_m(x)$ are polynomials of degree m and n, respectively, symbolically denoted by (m, n), is said to be a Pade approximant of f (x), if and only if the power series expansion of (m, n) is identical with that of f (x) up to and including terms of orders s^{m+n}. Let the function to be approximated be defined by Eq. (1) and let the approximant be defined by:

$$\frac{P_m(x)}{Q_n(x)} = \frac{a_0 + a_1x + ... + a_mx^m}{b_0 + b_1x + ... + b_{n-1}x^{n-1} + x^n} \quad (2)$$

where, the power series expansion of Eq. (2) is to agree with that of Eq. (1) as far as and including, the term in s^{m+n}.

Consider a linear multimachine power system with zero initial conditions described by the relations given:

$$\underline{x}(s) = \underline{u}(s) - F(s)\,\underline{y}(s) \quad (3)$$

$$\underline{y}(s) = G(s)\,\underline{x}(s) \quad (4)$$

where, $\underline{u}(s)$, $\underline{x}(s)$ and $\underline{y}(s)$ are Laplace transforms of $\underline{u}$, $\underline{x}$ and $\underline{y}$, defined by the transfer function matrix H (s) that relates the function $\underline{y}$ to $\underline{u}$ (i.e., $\underline{y}(s) = H(s)\underline{u}(s)$) as:

$$H(s) = G(s)\,[1 + F(s)G(s)]^{-1} \quad (5)$$

If G (s) is nonsingular, we obtain an alternative relation:

$$\hat{H}(s) = F(s) + \hat{G}(s) \quad (6)$$

Where,

$$\hat{G}(s) \underline{\underline{\Delta}} (\hat{g}_{kl}(s)) = G(s)^{-1} \quad (7)$$

$$\hat{H}(s) \underline{\underline{\Delta}} (\hat{h}_{kl}(s)) = H(s)^{-1} \quad (8)$$

The system Eq. (3) and (4) is embedded in a feedback configuration (Fig. 1) with the square linear open-loop system S (A, B, C) and described by the relations:

$$\begin{aligned} \dot{\underline{x}} &= A\underline{x} + B\underline{u} \\ \underline{y} &= C\underline{x} \end{aligned} \quad (9)$$

where, $\underline{x}$, $\underline{u}$ and $\underline{y}$ are the n-, m- and m- dimensional state-, input-and output-vectors, respectively and A, B and C are constant matrices of appropriate dimensions. The function $\underline{r}$ is an m- dimensional reference input-vector, K is a constant nonsingular feedback controller matrix, while k is a variable scalar gain.

In the single parameter multivariable root locus problems (MacFarlane, 1980), the open-loop transfer function matrix of the system with n inputs and n outputs is assumed to be of the form:

$$Q(s) = pQ_0(s) \quad (10)$$

where, p is a scalar parameter (Obinabo, 2008) and the matrix Q (s) is usually taken to include both the effects of any precompenstor, actuator and sensor dynamics as well as the plant itself. The root locus is then obtained by plotting the roots of the characteristic equation:

$$\det(I + Q(s)) = 0 \quad (11)$$

in the s plane as p is varied from 0 to ∞. To allow for different gains in the various loops, the function Q (s) in Eq. (10) is replaced by:

$$Q(s) = pAQ_0(s) \quad (12)$$

where, A is a constant diagonal matrix

$$A = \text{diag}(a_0, ..., a_n) \quad (13)$$

with $a_k > 0$ for k = 1, ..., n, Eq. (11) then becomes

$$\det(1 + pAQ_0(s)) = 0 \quad (14)$$

whose roots for a particular selection of the matrix A yields one member of a root locus family.

Theorem 1: If the function f (n), given in Eq. (1), reduces to a rational function $p_m(x)/q_m(x)$ given in Eq. (2), then the following recursive relationship must hold:

$$C_K = -\frac{1}{b_0}\sum_{j=1}^{K} b_j C_{K-j},\ k>m \quad (15)$$

Proof: It should be noted that in the statement of the theorem b_n is normalized to unity and that:

$$C_k = 0,\ k<0 \quad (16)$$

The proof of the theorem is very simple and relation Eq. (15) is shown to hold simply by considering the power series expansion of the given rational function. Thus let,

$$\frac{P_m(x)}{Q_n(x)} = C_0 + C_1 x + C_2 x^2 + \cdots \quad (17)$$

Then using Eq. (2) we get:

$$C_0 = \frac{a_0}{b_0} \quad (18)$$

and in general

$$C_K = \frac{1}{b_0}\left[a_K - \sum_{K-1}^{K} b_j C_{k-j}\right], \quad k>0 \quad (19)$$

with $a_k = 0$, $k > m$
Hence Eq. (19) reduces to,

$$C_K = -\frac{1}{b_0}\sum_{K-1}^{K} b_j C_{K-j}, \quad k>m \quad (20)$$

which proves the theorem.

Now consider the problem when the power series expansion for f (x) is given by Eq. (1) and we would like to obtain the two polynomials:

$$P_m(x) = \sum_{i=0}^{m} a_i x^i \quad (21)$$

and

$$Q_n(x) = \sum_{i=0}^{n} b_i x^i, \quad b_n = 1 \quad (22)$$

such that

$$f(x) = \frac{P_m(x)}{Q_n(x)} \quad (23)$$

assuming that $m \le n$ let:

$$C_{K,K} = \begin{bmatrix} C_2 & C_3 & C_4 & \cdots & C_{K+1} \\ C_3 & C_4 & C_5 & \cdots & C_{K+2} \\ C_4 & C_5 & C_6 & \cdots & C_{K+3} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ C_{K+1} & C_{K+2} & C_{K+3} & \cdots & C_{2K} \end{bmatrix} \quad (24)$$

We can determine the value of n using the following theorem.

Theorem 2: If Eq. (23) is satisfied, then there exists a unique n such that:

$$\det C_{n,n} \neq 0 \quad (25)$$

$$\det C_{n+a,n+a} = 0 \quad (26)$$

Thus having determined the value of n by theorem (2), the coefficients of the denominator of Eq. (2) are given by:

$$C_{n,n}\underline{b} = -\underline{C} \quad (27)$$

Where,

$$\underline{b} = (b_{n+1}, b_{n-2}, \ldots, b_0) \quad (28)$$

$$\underline{c} = (c_1, c_2, \ldots, c_n) \quad (29)$$

The coefficients of $P_m(x)$ were determined using the first (m+1) of the set of the linear equations which define the given problem.

REPRESENTATION OF THE INVERSE MULTIMACHINE POWER SYSTEM

Given a power series expansion of a linear constant multimachine power system which is reducible to a transfer function and it is known that the order of the numerator does not exceed that of the denominator, then an algorithm is desired to determine the transfer function. The transfer function so determined will be in a reduced form and will consequently not have any common zeros in the numerator and denominator. The inverse of the system is described as that whose input vector is the vector $\underline{y}$ of the relation $\underline{y} = Cx(t)$ and the output vector the vector $\underline{u}$ of the relation $\dot{\underline{x}}(t) = A\underline{x}(t) + B\underline{x}(t)$. The poles and zeros are described by the configuration of Fig. 1, where, $A_1 = NAM$, $A_2 = V_1AU$, $B_1 = NAU_1$, $C_1 = V_1AM$, $B_2 = V_1B$, $C_2 = CU_1$.

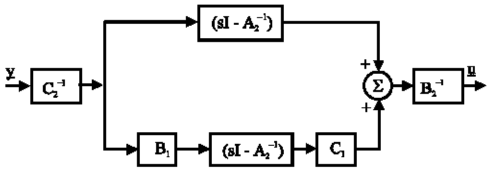

Fig. 1: Representation of the inverse multimachine electric power system with the input vector $\underline{y}$ and the output vector $\underline{u}$

It is verified (Nwokah, 1983) that certain relation exists between the angles of the loci that correspond to the finite zeros in S (A, B, C) and the angle of departure, from the poles, of the root loci in the inverse system. The angle of approach is also shown to equal the angle of departure of the root locus from the pole of the system as shown in Fig. 2, where r is an m-dimensional reference input-vector, K is a constant nonsingular feedback controller matrix and k is a variable scalar gain.

The characteristics polynomial of the above closed-loop system is given by:

$$\begin{aligned}\Delta_C(s,k)\underline{\underline{\Delta}}|SI-A_C| &= |SI_n - A + K\,BKC| \\ &= |SI-A| = |I_m + kC(SI-A)^{-1}BK| \\ &= |SI-A| = |I_m + kG(S)K|\end{aligned} \tag{30}$$

where,

A_c = A -kBKC = The closed loop state matrix.
G (s) =C (sI - A)$^{-1}$ B = The open loop transfer function matrix.

In order to prevent D (s) from losing rank identically (independently of s), which corresponds to the case of depended control action and/or outputs, it is henceforth assumed that B and C, the input-and output-coupling matrices have full rank. For a given K, the solution of the equation

$$\Delta_C\,(s, k) = 0 \tag{31}$$

for s in terms of k determines implicitly the dependence of the closed-loop poles, p_i, i = 1, ..., n, on the feedback gain k.

Let the problem be described generally by the following linear time-invariant dynamical system:

$$\begin{aligned}\dot{\underline{x}}(t) &= A\underline{x}(t) + B\underline{u}(t) \\ y(t) &= C\underline{x}(t) + D\underline{u}(t)\end{aligned} \tag{32}$$

where, $\underline{u}$ is the m-dimensional input vector, y is the r-dimensional output vector and $\underline{x}$ is the n-dimensional state vector. The matrices A, B, C and D have dimensions compatible with $\underline{x}$, y and $\underline{u}$.

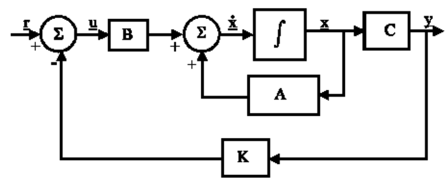

Fig. 2: The proper part of the inverse system (Fig. 1) is represented here as the feedforward path

A procedure whereby a stable multimachine power system can be designed using the theorem has been established. First, it is required to obtain Nyquist contours and this is possible if the transfer function matrices can be obtained. Thus, by taking Laplace transforms Eq. (32) it can shown that:

$$\begin{aligned}y(s) &= \left(C(sI-A)^{-1}B + D\right)\underline{u}(s) \\ &= T(s)\underline{u}(s)\end{aligned} \tag{33}$$

Where,

$$T(s) = \left(C(sI-A)^{-1}B + D\right) \tag{34}$$

T (s) is a rational transfer function matrix of dimension (r×m). Various methods are available for computing T (s) and they involve computation of the characteristic matrix $(sI + A)^{-1}$ using Fadeeva's algorithm (Gantmacher, 1959) and then using Eq. (34) to compute T (s) by first determining det (sI + A), which gives the common denominator of each entry of T (s), while the numerator of each entry is given in terms of the Markov parameters of the system (Shamash, 1972). In this study, we compute T (s) directly from the Markov parameters of the system. The approach has advantage of giving each entry of T (s) in reduced form and also it is directly applicable to system described by input/output data. Eq. 34 may be rewritten in the form:

$$\begin{aligned}T(s) &= C(sI-A)^{-1}B + D \\ &= D + C(s) = \\ &C\,(sI + s^{-2}\;\;A + s^{-3}A^2A^2 + \cdots)B \\ &= D + CBs_{-1} + CABs^{-2} \\ &+ CA^2Bs^{-3} + \cdots\end{aligned} \tag{35}$$

$$= D + y_0s^{-1} + sy_1s^{-2} + ... \tag{36}$$

Where,

$$y_1 = CA^iB, + y_1s^{-2} + ... \tag{37}$$

They's are the so-called parameters of the systems. In the case when m = r = 1, D and y_1 (i = 0, 1, ...) are constant scalars and T (s) is simply an infinite series in negative powers of s. Since, the system Eq. (32) is finite in dimension, the power series for T (s) is equivalent to a rational transfer function. Thus, let

$$T(s) = C_0 + C_1 s^{-1} + C_2 s^{-2} + ... \quad (38)$$

Where,

$$C_0 = D \quad (39)$$

$$C_1 = y_{i+1},\ i = 1, 2, ... \quad (40)$$

Then it is easily seen that by letting $z = s^{-1}$, Eq. (38) is similar to Eq. (1) and thus Fadeeva's algorithm (Gantmacher, 1959) was used to construct the unique rational transfer function for T (s).

Example 1: Consider a linear constant multimachine power system for which the system equation arose naturally in the matrix form $\dot{\underline{x}}(t) = A\underline{x}(t) + \underline{u}(t)$, with

$$A = \begin{bmatrix} 0 & -2 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix},$$

$$C = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -2 & 2 \end{bmatrix}, D = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

The transfer function is obtained as:

$$T(s) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} s^{-1} \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix} s^{-2} + \begin{bmatrix} -4 & 0 \\ 0 & -2 \end{bmatrix} s^{-3}$$
$$+ \begin{bmatrix} 0 & -4 \\ 4 & 0 \end{bmatrix} s^{-4} + \begin{bmatrix} 8 & 0 \\ 0 & 4 \end{bmatrix} s^{-5} + \begin{bmatrix} 0 & 8 \\ -8 & 0 \end{bmatrix} s^{-6} ...$$
$$= \begin{bmatrix} t_{11}(s) & t_{12}(s) \\ t_{21}(s) & t_{22}(s) \end{bmatrix}$$

The Markov parameters were computed as follows:

$$y_k = C(A^{k-1})B$$

$$y_{k+1} = C(A(A^{k-1}B)),\ k \le 0$$

Using theorem 2 we obtain:

$$t_{11}(s) = 1 + \frac{2s}{s^2+2},\ t_{12}(s) = -t_{21}(s) = \frac{2}{s^2+2},$$
$$t_{22}(s) = \frac{2s^2+2}{s(s^2+2)} + 1$$

Therefore,

$$T(s) = \begin{bmatrix} \dfrac{s^2+2s+2}{s^2+2} & \dfrac{2}{s^2+2} \\ \dfrac{2}{s^2+2} & \dfrac{s^3+2s^2+2s+2}{s(s^2+2)} \end{bmatrix}$$

Example 2: For the multimachine electric power system S (A, B, C) defined (Fig. 1 and 2) by the matrices:

$$A = \begin{bmatrix} -4 & 7 & -1 & 13 \\ 0 & 3 & 0 & 2 \\ 4 & 7 & -4 & 4 \\ 0 & -1 & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 2 & 0 \\ -2 & 0 \end{bmatrix},$$

$$C = \begin{bmatrix} 0 & -5 & 2 & -2 \\ -4 & 7 & 0 & 3 \end{bmatrix}, \quad K = 1$$

It can be shown easily that

$$CBK = \begin{bmatrix} 3 & 0 \\ -11 & 8 \end{bmatrix} \text{and}$$

$$BKC = \begin{bmatrix} -4 & 7 & 0 & 3 \\ 0 & -5 & 2 & -2 \\ 0 & -10 & 4 & -4 \\ 0 & 10 & -4 & 4 \end{bmatrix}.$$

With N chosen as:

$$N = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & -2 & 1 & 0 \end{bmatrix}$$

the matrix M that also satisfies NM = I is then found to be

$$M = \frac{1}{12} \begin{bmatrix} 11 & -13 \\ 8 & -16 \\ 16 & -20 \\ -4 & 20 \end{bmatrix}.$$

The system resulting from the NAM is shown to have two complex zeros at $z_1 = 1 + i$ and $z_2 = 1 + i$. These results have been rigorously determined and they satisfy the conditions laid out in the theorems.

CONCLUSION

The transfer function matrices of linear multimachine power systems were computed directly form the Markov parameters by inversion using the algorithm due to Fadeeva. The study shows that by letting $z = s^{-1}$, Eq. (38) is reduced to a form similar to Eq. (1) giving each entry of T (s) in reduced form. The advantage inherent in this approach, is that each entry of T (s) will be easily applied to systems described by their input-output parameters without having to derive a state space realization. For systems described by a set of constant differential equations:

$$L(s)y = M(s)\underline{u} \tag{41}$$

where,

L (s) and M (s): Are (r×r) and (r×m) polynomial matrices derived for Eq. (41) is obtained as:

$$y(s) = L(s)^{-1}M(s)\,\underline{u}(s) = T(s)\underline{u}(s) \tag{42}$$

Where,

$$T(s)=L(s)^{-1}M(s) \tag{43}$$

The transfer function T (s) was obtained by inverting L (s) and post multiplying by M (s). More directly, the Markov parameters or the time moments of Eq. (41) were obtained as outlined by Shamash (1972) and then the method of Gantmacher (1959) was used to compute the transfer function matrices.

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