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# Solution of the Transient Electro-Magneto-Thermal Problems Fed by Voltage in Induction Heating Applications

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**Abstract:** The aim of this study is to treat the electro-magneto-thermal coupling applied to a induction heating system. The voltage feeding makes the present study very interesting especially in the transient thermal regime. The most delicate study of this problem means the treatment at the same time of four physics aspects: the electrical equation of the inductor, the electromagnetic equation in the load, the equation of the thermal in transient or permanent regime and sometimes one may add the fluid flowing equation. The non-linearity and complexity of the equations system thus obtained require the use of numerical methods to solve it. A software tool on the base of the finite difference method coupled with a semi-analytic one has been developed. We show that, the evolution of the dissipation active power and its effects on the temperature and the impedance along the time of heats.

**Key words:** Induction heating, voltage, time, power, temperature

## INTRODUCTION

In order to realize representation model of the studied device, a precise knowledge of the spatial and temporal distribution of electric and magnetic fields and the temperature is required. This knowledge is obtained by the resolution of the partial derivatives equations describing the physical phenomena. The electromagnetic model gives to the thermal one its source, which is the Joule effect of the Foucault currents. On the base of the temperature field thus obtained, the new values of electromagnetic coefficients are computed. So, this cycle is restarted again until convergence of a number of criteria. In the solid state of load, we consider that the transport term has no sense and we study the problem of no stationary conduction. In the case of the fluid load for which the heating phenomenon is very fast regarding to the other time delays of the system, we consider the stationary regime (Vinsard, 1990; Mekideche, 1993).

Several researchs have been published concerning the magneto-thermal coupling in feeding by current to know Bleuvin (1984) and Feliachi and Devely (1991) that presented a Direct Coupling Method (DCM) of these phenomena. But the device feeding by current source remains always an incomplete model, because it excludes the true electromagnetic coupling between the inductor and the load. To give an approach of a more real coupling, a model fed by voltage is developed in collaboration with L2ES laboratory of Belfort (French). This model is applied to the axisymmetric devices. It permits to determine the most sensitive points; the total impedance, the power density transmitted to the load and the temperature of a sinusoidal voltage supply.

## MATERIALS AND METHODS

**Mathematical model:** For the device, the inductor shows the energy source that includes a set of turns on, which we apply a sinusoidal voltage. The proposed model consists in formulating the Maxwell's equations and the Ohm's law for each elementary turn for a given meshing. So, we obtain a voltage equation in term of vector potential as:

$$\text{Rot}\vec{E}=\frac{\partial\vec{\beta}}{\partial t} \quad (1)$$

$$\text{Rot}\vec{B}=\mu\vec{J} \quad (2)$$

$$\text{Div}\vec{B}=0 \quad (3)$$

$$\vec{B}=\text{rot}\vec{A} \quad (4)$$

$$\vec{J}=\sigma\left(\text{gr}\vec{\text{a}}\text{du}+\frac{\partial\vec{A}}{\partial t}\right) \quad (5)$$

Where:

$\vec{E}, \vec{B}$ = The electrical magnetic fields
$\vec{J}$ = The current density vector
$\vec{A}$ = The vector magnetic potential
$\sigma$ = The conductivity
$\mu$ = The permeability

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Knowing that in axisymmetric case and in cylindrical coordinates:

$$\vec{grad}u = \frac{\partial u}{\partial r}\vec{e}_r + \frac{1}{r}\frac{\partial u}{\partial \theta}\vec{e}_\theta = -\frac{u}{2\pi r}\vec{e}_\theta$$

$$\int \vec{A}d\vec{l} = \iint rot\vec{A}ds = \phi = A.2\pi r$$

$$\frac{\partial A}{\partial t} = \frac{1}{2\pi r}\frac{d\phi}{dt} = i\omega A$$

We obtain the final expression of the voltage, for every elementary turn: according the current density, the magnetic vector potential, the resistivity and the radius with regard to the axis. It is given by the Eq. 6:

$$u = 2\pi r\left(\frac{1}{\sigma}J + i\omega A\right) \tag{6}$$

with, ω is the voltage pulsation and $i^2 = -1$

Moreover, we may establish the expression of the vector magnetic potential creates by a spindly turn (Durand, 1968):

$$A = A_{ij,kl} = \frac{\mu}{2\pi}\iiint_v \frac{J_{kl}}{|r_{ij} - r_{kl}|}dv \tag{7}$$

We notice that Eq. (7) gives the magnetic vector potential in a point (i, j) produced by an elementary current located at the node (k, l). The formula can be generalized by adding all the effects of the current loops.

The electromagnetic phenomena appearing in electrical engineering devices (particularity in the load) are described by Maxwell's equations and the three relations of the considered region. By assuming the load without movement, the space charge density and the currents of displacement may be neglected. The formulation through the magnetic vector potential reduces considerably the system size and the hypothesis of Coulomb's gauge becomes naturally verified (Mekideche, 1993) and we obtain the Eq. 8 and 9:

$$Rot\left(\frac{1}{\mu}rot\vec{A}\right) + \sigma\frac{\partial \vec{A}}{\partial t} = 0 \tag{8}$$

$$div\vec{A} = 0 \tag{9}$$

By choosing an adequate cylindrical coordinates, the gradient of the scalar potential vanish and magnetic vector potential depends only to $\vec{e}_\theta$ direction. So, the Eq. 8 becomes:

$$\frac{\partial^2 A}{\partial r^2} + \frac{\partial^2 A}{\partial z^2} + \frac{1}{r}\frac{\partial A}{\partial r} - \frac{A}{r^2} = \mu\sigma_c\frac{\partial A}{\partial t} \tag{10}$$

For thermal equation in the load, considering Fourier's law by applying the first thermodynamics principle in an elementary volume (V), the equation of the conduction with term of velocity may be given in its general shape:

$$\rho C_p\frac{\partial T}{\partial t} = div\left(K\vec{grad}(T)\right) - \rho C_p\vec{v}\vec{grad} + P \tag{11}$$

Where,

| | | |
|---|---|---|
| ρ | = | The mass density |
| $C_p$ | = | Specific heat |
| K | = | Thermal conductivity |
| va | = | Velocity of displacement of the volume with regard to the thermal sources |
| Pa | = | Field source |
| T | = | The searched field of temperature |

In the case of the solid loads, the term of velocity is non-existent and the equation of the transient regime will be as Eq. 12:

$$\rho C_p\frac{\partial T}{\partial t} = div(K\vec{grad}(T)) + P \tag{12}$$

The power density generated by the Joule's effect of Foucault's currents in the heated load is given by the Eq. 13:

$$P = \frac{1}{2}\sigma\omega^2 AA^* \tag{13}$$

With

$A^*$ = The complex conjugated of A

**Numerical models:** For the working convenience, we firstly treat separately each bloc of the studied system and each phenomenon. Afterwards, we combine the different parts to get the final computation code.

**Numerical model of the inductor:** Considering an inductor containing $N_s$ turns, each one is subdivided in n elementary turns. In a discretization point k, the Eq. 6 has the following form Eq. 14:

$$U_k = 2\pi r_{ik}\left(\rho J_{ik} + i\omega\sum_{i=1}^{n}\sum_{j=1}^{N_s}A_{ik,ij}\right) \tag{14}$$

The expression of the vector magnetic potential between two nodes is given by Mostaghimi and Boulos (1988) and Bleuvin (1984):

$$A_{ij,kl} = \frac{\mu}{2\pi}I_{kl}G(K)\sqrt{\frac{r_{kl}}{r_{ij}}} \tag{15}$$

By adding all the effects of the current loops of all the turns, the Eq. 23 may be generalized as Eq. 16:

$$A_{ij,kl} = \frac{\mu}{2\pi}\sum_{l=1}^{N_s}\sum_{k=1}^{n}\sqrt{\frac{r_{kl}}{r_{ij}}}\ J_{kl}S_{kl}G_{ij,kl}(k) \tag{16}$$

With,

$$k = \left[\frac{4.r_{kl}r_{ij}}{\left(r_{kl}+r_{ij}\right)^2+\left(z_{kl}-z_{ij}\right)^2}\right]^{\frac{1}{2}}$$

$$G(k) = \frac{\left(2-k^2\right)F(k)-2.E(k)}{k}$$

$$F(k) = \int_0^{\frac{\pi}{2}}\frac{1}{\sqrt{1-k^2\sin^2(\theta)}}d\theta$$

is the elliptic integral of first kind

$$E(k) = \int_0^{\frac{\pi}{2}}\sqrt{1-k^2\sin^2(\theta)}\ d\theta$$

is the elliptic integral of second kind

For reasons of programming complexity of, we take two interpolations of the following interpolations of the functions E(k) and F(k) (Stegun, 1972; Mekideche, 1993):

$$E\,(k) = 1+0.4630151\times M_1+0.1077812\times M_2+ (0.2452727\times M_1+0.0412496\times M_2)\times C$$

$$F(k) = 1.3862944+0.1119723\times M_1+0.0725296\times M_2+ (0.5+0.1213478\times M_1+0.0288729\ M_2)\times C$$

With,

$$M_1 = 1\text{-}k^2, M_2 = M_1^2 \text{ and } C = \mathrm{Log}(\frac{1}{M_1})$$

We notice that there are ($N_s\times n$) current unknowns and we shall have the same equations number in order to be able to resolve the problem. Thus, we add the current conservation law between all the turns:

$$\sum_{i=1}^{n}J_{i1} = \sum_{i=1}^{n}J_{i2} = \cdots = \sum_{i=1}^{n}J_{iN_s} \tag{17}$$

Afterwards, we express the total voltage of the inductor by adding the all contributions of the main turns:

$$u = \sum_{j=1}^{Ns}u_j \tag{18}$$

There are two approaches to solve this algebraic equation system: the direct approach by substitution of Eq. 16 in 14 and iterative one. We choose the second method for following reasons (Bleuvin, 1984):

- Its simplicity of application, even for the big size systems
- More convenient in the no linear case
- May be easily applied to the case of coupling with other equations (the thermal and/or the fluid flowing problems)
- Offers the possibility to solve the problem block by block (turn by turn at the inductor on one hand, and turns-load separately on the other hand)

The inconvenience of this approach consists on the dependence of the calculation time on the unknowns number. This Coupled Circuit Method (CCM) is easily applicable especially in the loaded case. So, we have just to add the induced currents effects on the inductor that is to say the coupling element in the Eq. 16 that becomes:

$$A_{\alpha,\beta} = \frac{\mu}{2\pi}\sum_{l=1}^{N_s}\sum_{k=1}^{n}\sqrt{\frac{r_{k,l}}{r_{\alpha,\beta}}}\ J_{k,l}S_{k,l}G_{kl,\alpha\beta}(k) + \frac{\mu}{2\pi}\sum_{i=1}^{M}\sum_{j=1}^{N}\sqrt{\frac{r_{i,j}}{r_{\alpha,\beta}}}\ J_{i,j}S_{i,j}G_{ij,\alpha\beta}(k) \tag{19}$$

In Eq. 19, the first term is the same of Eq. 16 (without load case) and the second term represents the coupling element expressing the load presence.

**Electromagnetic model in the load:** We remark that the use of the CCM can increase the time of calculation especially when the number of unknowns is high. In addition, the calculation of the vector potential in the air is not interesting and the problem resolution is not necessary. Thus, we propose to use another method to solve complete electromagnetic equations in both the regions, the inductor and the load. In this research, we use the mixed methodology: The Finite-Difference Method (FDM) inside the load and the CCM in the inductor and the load borders. The use of FDM in the load alloys to write the Eq. 17 at any point (i, j) of the domain:

$$A_{i,j} = C_1\left[\frac{A_{i+1,j}+A_{i-1,j}}{h_1^2}+\frac{A_{i,j+1}+A_{i,j-1}}{h_2^2}+ \frac{1}{2r_{i,j}h_1}\left(A_{i+1,j}-A_{i-1,j}\right)\right] \tag{20}$$

With, $C_1 = C_1\,(r_{i,j}, h_1, h_2, \sigma, \omega, \mu)$

We notice that the MCC allows solving the problem of limits conditions by setting the condition A = 0 on the symmetry axis. So, the rest of borders will be calculated by using Eq. 20.

**Thermal model:** With the same way of the electromagnetic modeling in axisymmetric case the Eq. 12 may be simplified in cylindrical coordinates as Eq. 21:

$$\rho C_p \frac{\partial T}{\partial t} = \frac{\partial}{\partial r}(k\frac{\partial T}{\partial r}) + \frac{K}{r}\frac{\partial T}{\partial r} + \frac{\partial}{\partial z}(K\frac{\partial T}{\partial z}) + P \qquad (21)$$

The computation of the temperature $T^{n+1}$ at the instant t + Δt and in the iteration (n+1), can be evaluated by using the explicit or the implicit methods. In the present study, we use implicit method, which doesn't has problem of instability. Then, we may choose the space and the time steps without additional constraint. This method is so unconditionally stable and duct to results more precise than those obtained by the explicit method. Thus, the conduction equation for every interns point (i, j) is given by the Eq. 22:

$$T_{ij}^{n+1} = \frac{1}{a_1 + a_2 + a_3}\left\{\begin{array}{l} a_1 T_{ij}^{n+1} + a_4 T_{i+1,j}^{n+1} + a_5 T_{i-1,j}^{n+1} + \\ a_6 T_{i,j+1}^{n+1} + a_7 T_{i,j-1}^{n+1} + P_{ij} \end{array}\right\} \qquad (22)$$

With $a_k$ are the positive constants.

The Eq. 23 must be solved with the following general boundary conditions at the surface of the workpiece:

$$-k\left(\partial T/\partial n\right) = h_c(T_s - T_a) + c_r(T_s^4 - T_a^4) \qquad (23)$$

Where,

s, a = Denote surface and ambient temperature
$h_c$, $c_r$ = Convection and radiation loss coefficients
n = The outward normal to the surface

Finally in order to evaluate the thermal conductivity at the node interfaces, we use a more correct approach that consists in considering the adjacent elements as two materials of different thermal conductivities (Patankar, 1989).

## RESULTS AND DISCUSSION

In all heating problems, one tries to know the induced distribution of current densities allowing characterizing the heat dissipation in the piece from the knowledge of the inductor and load characteristics (nature, form, voltage feeding).

Figure 1 shows general diagram of the inductor-load device. The cylindrical billet of steel is represented in an axis system (r, θ, z) in axisymmetric case; the dimensions are given in mm. The conditions to the thermal boundaries and the physical properties of the load have been taken by Bleuvin (1984) and the simulation data of the inductor are:

- Voltage of feeding: u = 25 V
- Working frequency: f = 1 kHz

**Radial evolution of the temperature:** Figure 2 shows that at 10 sec (the starting of the heater), the temperature increases quickly at the external surface of the billet. Thus, a great gradient of the temperature appears between the revolution axis and the external surface. At the time, when the most heats outside points attain the transition temperature, the sources density decrease and the entropy of the device increase. On the other hand, the maximal temperatures variation is low and the temperature

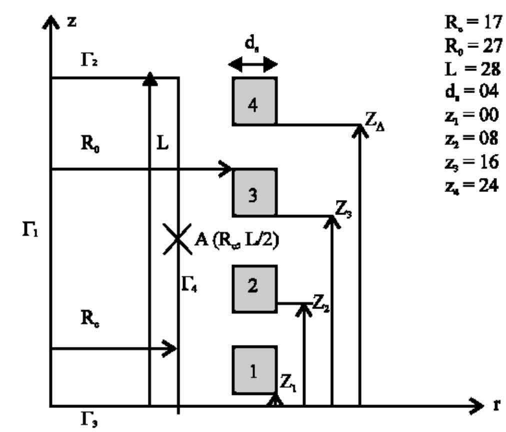


Fig. 1: Diagram general inductor-load

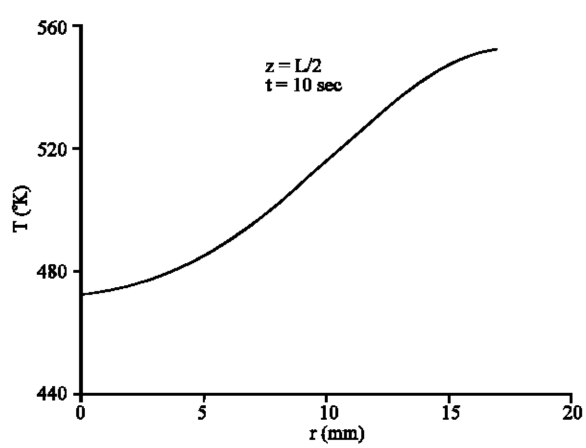


Fig. 2: Radial evolution of the temperature

gradient is smaller more and more (Fig. 3-5). At the end of the third phase of the heating, we notice an inversion of the thermal gradient. The thermal losses increase according to the temperature and the most heats points are now located in the zone of the billet center (Fig. 6).

**Evolution of the dissipated power:** At the heating beginning (about 10 sec), the dissipated power is not sensitive by the decrease of the electrical conductivity (Fig. 7).

Moreover, according to the physical data of the load and the work frequency, it can increase as well as decrease. In present case, the resistivity increasing with respect to the temperature causes a considerable fall of the power until the moment, when the most external points attain the curie temperature.

Finally, during the last phase of the heating, sources of energies evolve slowly and the total power stabilizes to a value relatively weak with regard to the one calculated to the initial temperature (27°C).

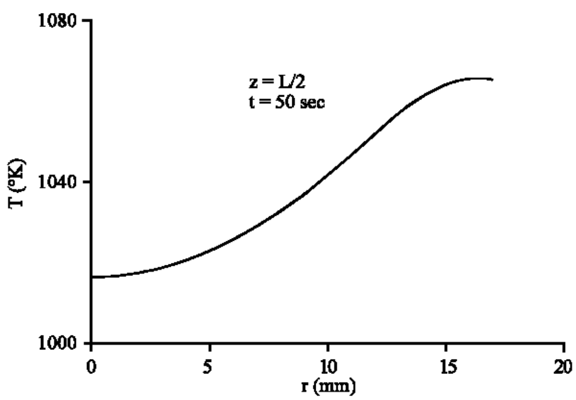


Fig. 3: Radial evolution of the temperature

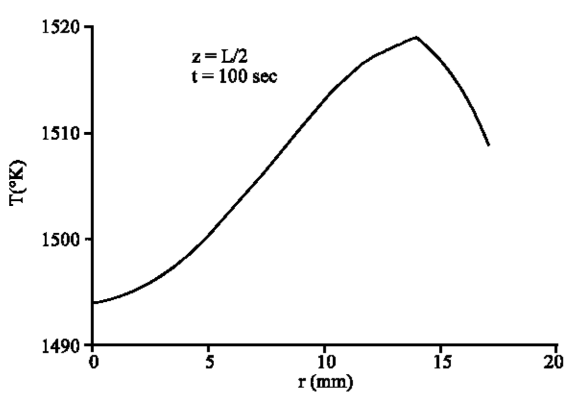


Fig. 4: Radial evolution of the temperature

**Behaviour of impedance:** In Table 1, we show the variation of impedance by the effect of the temperature. A first comparison concerning the value of impedance between presence and without load at t = 0 (without magneto-thermal coupling). We remark also the increase of the reactance value, but the value of

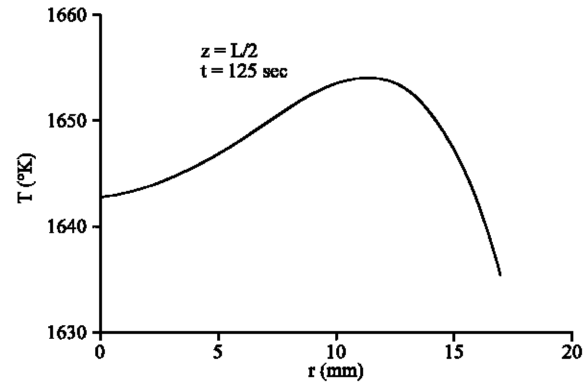


Fig. 5: Radial evolution of the temperature

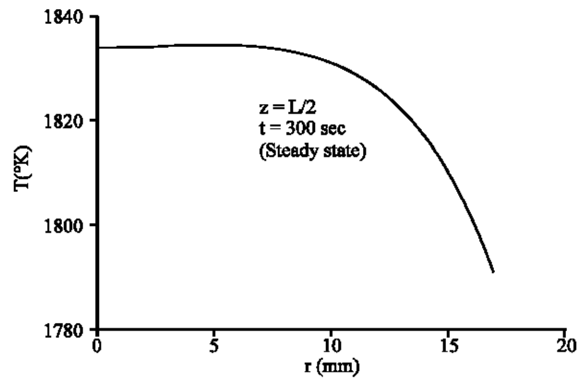


Fig. 6: Radial evolution of the temperature

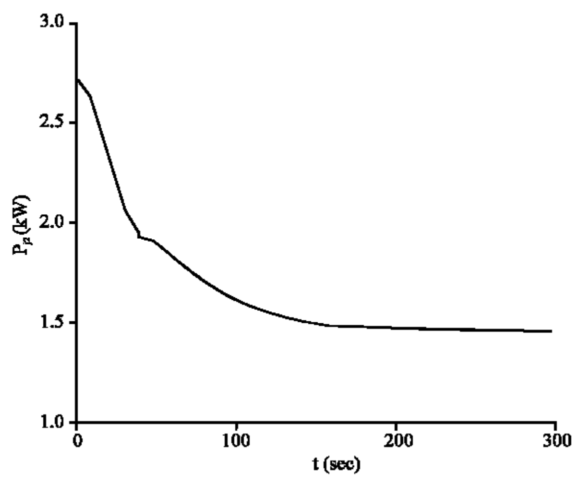


Fig. 7: Evolution of the dissipated active power

Table 1: The variation of impedance

| Impedance (mΩ) | Without load | With load | Time (sec) |
|---|---|---|---|
| Z = R + iX | 4.562 + i5.248 | 4.983 + i4.698 | 0 |
| - | - | 4.970 + i4.916 | 10 |
| - | - | 4.946 + i5.031 | 20 |
| - | - | 4.880 + i5.117 | 40 |
| - | - | 4.830 + i5.171 | 100 |
| - | - | 4.805 + i5.195 | >300 |

resistance decrease. So, it's necessary to improve the power factor, during the heating process, by using a pack of capacitors.

## CONCLUSION

An electromagnetic model and a thermal one as well as a methodology for the solution of induction heating coupled non-linear electro-magneto-thermal problems. The originality of the proposed model is its ability to take into account the induced reaction in the inductor and the magneto-thermal coupling in the case of voltage feeding. The developed computation code has been tested at the sensible model points as well as their coupling. A comparative study has been realized on the magneto-thermal in the current feeding case, with present model (Delage and Ernst, 1984; Bleuvin, 1984; Mekideche, 1993). So, the validity of present software tool has been checked out. Thus, it can be used to study most complex configurations and in the case of a voltage feeding. The model constitutes a powerful mean for the designer for command and optimal inductor design.

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