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Background Inverse Scattering from Approximately Rough Periodic Surfaces

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Abstract: The background scattering of plane electromagnetic waves by arbitrarily periodic shaped surfaces is investigated. The scattered waves will be assumed to propagate in discrete Floquet modes. Electromagnetic fields are solved for first using the method of separation of variables and then expressed in a very compact form by introducing the modified spherical vector wave functions. The scattered waves are obtained using an exact method or a simplified model based on Rayleigh's hypothesis. For the general treatment, we express the field of the scattered waves in terms of the unknown value of the field and its normal derivative at the boundary, by making use of the Green's functions. The proposed formula allows mathematically exact calculation of the near-field, in the context of scalar wave theory.

Key words: Scattering, Rayleigh hypothesis, floquet modes, neuman problem, dirichlet problem

INTRODUCTION

The Scattering of Electromagnetic (EM) waves by periodic surfaces has receiving considerable attention for a long time (Millar, 1971; Kroger and Kretschmann, 1970; Bennett and Mattsson, 1999; Ogilvy, 1991; Beckmann and Spizzichino, 1963; Beckmann, 1967; Nieto and Garcia, 1981; Marx and Vorburger, 1990; Ishimaru *et al*., 2000; Rappaport and El-Shenawee, 2000). Knowledge of the scattered fields is required in many areas, such as investigations of the scattering of light by small chemical and biological particles (Kroger and Kretschmann, 1970) and the scattering of microwaves by raindrops (Nieto and Garcia, 1981). During the past several decades, researchers in the areas of applied electromagnetism and underwater acoustics have been searching for rigorous and efficient models for mathematically describing the problem of EM and acoustic wave propagation over rough surfaces as well as the scattering of those waves by such surfaces (Bennett and Mattsson, 1999).

Modeling low grazing-angle EM wave scattering from periodic rough surfaces is a technically-challenging problem. Quite a few methods have been used in the analysis of different scattering problems (Bennett and Mattsson, 1999). The popular Parabolic Wave Equation (PWE) approximation model had been developed to describe accurately the situations where the EM field or, at least, the predominant part of it is propagating in one direction, i.e. the forward direction (Ogilvy, 1991). In this case, the specular scattered field is the predominant field and scattering away from the forward (specular) direction is very small. Two main formalisms of the PWE model described above; namely the PWE/split step approach and the PWE/Volterra approach (Wong and Bray, 1988). The so called PWE/split step model, can be used in both homogeneous and slowly varying inhomogeneous media (Beckmann and Spizzichino, 1963). In this case, the PWE is solved as an initial-value problem given an initial field distribution on a vertical plane. On one hand, the PWE model approximately accounts for the forward interactions only. On the other hand, the PWE model does not account for other types of surface field interactions.

The second PWE propagation and scattering model, to be called the PWE/Volterra model, is also derived by setting up the boundary integral equation counterpart of the PWE, which is of the Volterra type, is set up and solved for the surface current induced by an incident field on a rough surface in homogeneous media. This formalism of the PWE model was proven to be more accurate than the PWE/split-step approach in general (Beckmann and Spizzichino, 1963).

The Boundary Integral Equation (BIE) model was developed to take into account for all kinds of surface field interactions and the problem of propagation over a rough surface as well as the rough surface scattering problem (Nasir and Chew, 1994). In this model, an integral equation of the Fredholm type, which governs the current induced on the rough surface by all kinds of surface field interactions, is derived on the basis of Helmholtz equation. This equation is solved for these surface currents using numerical methods and the resulting currents are then used in radiation integrals involving the appropriate propagators to calculate the scattered field everywhere in the medium. Distinct integral equations are set up for the TE and TM cases.

The applicability of the BIE model in a certain medium is tied to the feasibility of determining the Green's function (propagator) of that medium. In homogeneous media, this modeling is straightforward since the homogeneous media propagator (Green's function) is well known (Chan *et al*., 1991). In inhomogeneous media, the Green's function is not readily known and hence, the BIE approach is not straightforward.

To study more complex scattering problems, a matrix formulation, which could be classified as an integral equation method, was introduced early by Waterman (Millar, 1971). This method, using a transition matrix to relate the incident field and the scattered field, is usually called T-matrix (transition matrix) method. A good application of the T-matrix method using the vector wave functions was implemented by Bennett and Mattsson, (1999). The differential scattering characteristics of closed three-dimensional arbitrarily shaped dielectric objects were investigated. Recently, Li *et al*. (1990) extended the method and obtained the dyadic Green's functions for this scattering problem.

The T-matrix method is ideally suited for analyzing the EM scattering by nonspherical particles. Although, the method is applicable to arbitrarily-shaped particles, it has been applied almost exclusively to axisymmetric particles, i.e., bodies-of-revolution (Rappaport and El-Shenwee, 2000; Wong and Bray, 1988; Beckmann and Sizzichino, 1963).

A similar procedure based on the T-matrix method was devised by Lakhtakia to study the EM response of nonspherical chiral objects exposed to an incident field (Ogilvy, 1991). The scattering problems involving anisotropic obstacles are mostly restricted to planar (or stratified) structures (Ko and Mittra, 1993; Balanis, 1989), cylinders (Beckmann and Spizzichino, 1963; Awadallah, 1998; Tappert, 1977; Toporkov *et al*., 1998), or spheres (Beckmann, 1967).

The scattering of plane EM waves by periodic (or grating) structures has been studied extensively by either integral equation methods or variational approaches (Bao *et al*., 1995). The techniques used in these investigations rely upon the periodicity of the structure and that of the incident wave, which allow the reduction of the problem to one in a single periodic cell bounded by a finite part of the boundary.

The problem of scattering of waves from periodic surfaces has been of interest to physicists, engineers and applied mathematicians for many years because of its large number of applications in optics, acoustics, radio-wave propagation and radar techniques. Chandler-Wilde and Ross (1995) established a uniqueness result for the Dirichlet problem for the Helmholtz equation in an arbitrary unbounded domain for the case when the imaginary part of the wave vector, $\vec{k}$, is positive, i.e., Im $\vec{k}>0$. In Chandler-Wilde and Ross (1996) the same authors proposed an integral equation formulation for the Dirichlet problem in two-dimension and proved, by standard operator perturbation arguments, in the case when the whole boundary is both Lyapunov and a small perturbation of a flat boundary, that the integral equation is uniquely solvable in the space of bounded and continuous functions and hence that, for a variety of incident fields including an incident plane wave, the boundary value problem for the scattered field has a solution.

In this study, the problem under consideration is a boundary value problem, in which the EM vector wave equation (partial differential equation) derived from the Maxwell's equation. The natural surfaces under consideration include rough ocean surfaces and rough terrain. Scattering of plane EM waves at the periodically rough surfaces is formulated as a boundary-value problem and solved using the method (Rayleigh, 1907). This method of approach is more appropriate than the normal distribution method of approach in this case. This is because the scattering is initially denser than the latest scattering. The Rayleigh hypothesis gives valid results for smooth boundaries with corrugations that are not too deep (Lakhtakia and Depine, 1993; Kazandjian, 1996). The limit of validity of methods based on the Rayleigh hypothesis has been investigated for impenetrable gratings (Millar, 1971; Hill and Celli, 1978); gratings made of isotropic penetrable materials (Popov and Mashev, 1987), electrically uniaxial materials (Depine and Gigli, 1994) or anisotropic absorbers (Inchaussandague *et al*., 2003) and gyroelectromagnetic index-matched, periodically corrugated interfaces (Gigli and Inchaussandague, 2004).

The scattering problem in a 3D was formulated the setting with the Dirichlet boundary condition, but the method can also be used for the Neumann and Robin boundary conditions (Semion and Alexander, 2006).

THEORETICAL BACKGROUND

The problem under consideration here is a boundary value problem, in which the EM field vectors (the electric field $\vec{E}$ and the magnetic field $\vec{H}$) are governed by a coupled vector wave equation (partial differential equation) derived from the Maxwell's equation, Fig. 1. There are two boundary conditions to be satisfied by the fields; namely the one satisfied on the periodic surface and the one satisfied at infinity. The latter condition is usually referred to as the radiation condition. In homogeneous or slowly varying nonmagnetic media, the coupled vector wave equation decouples and reduces to the well-known Helmholtz equation (partial differential equation of the hyperbolic type) governing the individual

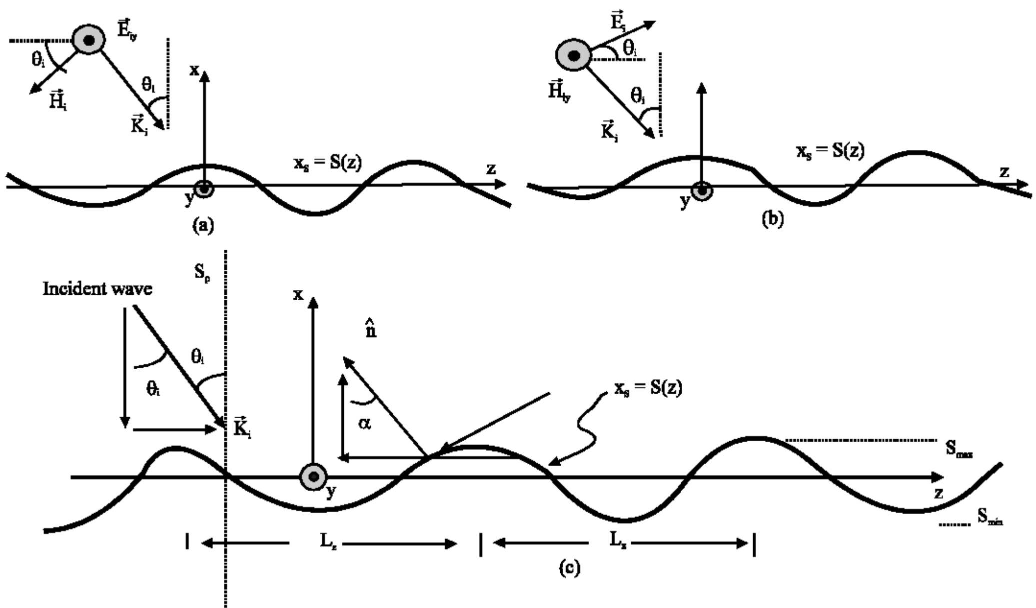

Fig. 1: Geometry of the problem of propagation over a one-dimensional approximately periodic surface. (a) Horizontally polarized (TE) wave, (b) vertically polarized (TM) wave and (c) geometrical representation of the surface including the incident direction. In this figure, $\vec{K}_i$ represents a non-attenuated wave vector of the incident wave direction of propagation, $x_S = S(z)$ represents the one-dimensional surface and S_p is the vertical plane, on which the initial field

scalar components of the field vectors. In many practical situations, the surfaces of interest have one-dimensional roughness, i.e., roughness that is confined to a plane, say the xz-plane, in a Cartesian coordinate system (Fig. 1). This is frequently the case for the ocean surface when the wind is blowing predominantly in one direction. For this particular scenario and without loss of generality, the general EM problem with arbitrary polarization may be decomposed into two decoupled problems, namely the TE and TM problems as illustrated in Fig. 1. In this case, the relevant Helmholtz equation is written for the E_y component in the TE case and for the H_y component in the TM case. These equations are then solved subjected to the relevant boundary conditions mentioned above.

The approximations can be used to consider the scattering of a plane wave from a periodic surface as shown in Fig. 1. Here and hereafter, we shall use as $\hat{x}, \hat{y}$ and $\hat{z}$ the unit Cartesian vectors;

$$\vec{k}_i = |K_i|\hat{k} = -k_{ix}\,\hat{x} + k_{iz}\,\hat{z}$$

as a wave vector where:

$$|K_i| = \frac{\omega}{c}$$

is the wave number in incident region,

$$\hat{k} = -\cos\theta_i\,\hat{x} + \sin\theta_i\,\hat{z}$$

is a unit vector along the direction of propagation and c the speed of light in vacuum; and E_0 and H_0 (to be defined later) are complex amplitudes depending on the polarization of the incident plane wave. Because of the periodicity of the surface, the scattered waves will propagate in discrete Floquet modes (Chicone, 1999). For this, the modes with longitudinal wave numbers larger than ω/c will be slow waves. This is because

$$k_{xn} = \sqrt{\left(\frac{\omega}{c}\right)^2 - k_{zn}^2} \text{ and } k_{zn} > \frac{\omega}{c}$$

for k_{xn} to be imaginary as will be discussed in details later. The scattered waves may be detected using an exact method or a simplified one based on Rayleigh's hypothesis (Rayleigh, 1907). The main difference is that treatment of the scattered waves in the region $S_{min}<x<S_{max}$ presents a special scattering problem as the scattered waves there propagate up and down. For $x>S_{max}$, the scattered waves move only upwards.

According to Rayleigh's hypothesis, if S_{max}-S_{min} is sufficiently smaller than L_g, where, L_z is the approximated period along the z-direction shown in Fig. 1, we may omit the scattered waves, which propagate downwards in the region $S_{min}<x<S_{max}$. Then, we match the boundary conditions along the actual boundary. The Rayleigh hypothesis is not valid for all periodic surfaces Lakhtakia and Depine (1993). It has been shown (Lakhtakia and Depine, 1993; Kazandjian, 1996) that it is valid for a sinusoidal surface, if the maximum slope of the surface is <0.448 (Gigli and Inchaussandague, 2004), that can be satisfied by the following conditions:

$$S_{max} - S_{min} < \frac{0.448}{\pi} L_z = 0.1426 L_z \quad (1)$$

For the general treatment, we express the field of the scattered waves in terms of the (unknown) value of the field and its normal derivative at the boundary, by use of the Green's functions (Chan *et al*., 1991) (Huygens' principle). For $x<S_{min}$ (extended boundary), the incident wave and the downward propagating waves must give zero total field (extinction theorem) (Ricardo and Lakhtakia, 2005). From this relation we specify the Floquet harmonics (Ricardo and Lakhtakia, 2005) of the field and its normal derivative at the boundary. Then, we calculate the scattered field for $x>S_{max}$, which is due to the upward propagating discrete Floquet modes with known values at the boundary.

Geometrical model and physical considerations: The following assumptions will be based on the geometrical model of Fig. 1. First, we will continue considering that there is no variation a long y: $\partial/\partial y = 0$. Then, consider the following setup shown in Fig. 1 (c) shown before, where

$$\hat{n} = \cos\alpha\,\hat{x} - \sin\alpha\,\hat{z} \quad (2)$$

representing a unit vector normal to the surface, pointing away from the conductor and

$$dL = \sqrt{dx^2 + dz^2} = \sqrt{1 + \left(\frac{dS}{dz}\right)^2}\,dz$$

as infinitesimal length along the boundary. The unit vector can be rewritten as:

$$\hat{n} = \frac{\hat{x} - \frac{dS}{dz}\hat{z}}{\sqrt{1 + \left(\frac{dS}{dz}\right)^2}} = \frac{\hat{x} - \tan\alpha\,\hat{z}}{\sqrt{1 + \tan^2\alpha}} \quad (3)$$

While, the wave vector of the incident wave as specified before becomes

$$\vec{k}_i = -k_{ix}\,\hat{x} + k_{iz}\,\hat{z} = \frac{\omega}{c}\left(-\cos\theta_i\,\hat{x} + \sin\theta_i\,\hat{z}\right) \quad (4)$$

Floquet modes: After the determination of the Floquet expansion coefficients, the diffraction efficiencies for the propagating plane wave components of the reflected or refracted fields can be calculated as the ratio between the diffracted and the incident intensities (Ricardo and Lakhtakia, 2005). For the reflected Floquet harmonics (Lakhtakia and Depine, 1993), Let:

$$E_y(x, z, t) = \underline{E}_y(x, z)e^{-j\omega t} \quad (5)$$

If the period of the boundary is L_z, then:

$$\underline{E}_y(x, z + L_z) = e^{ik_{iz}L_z}\underline{E}_y(x, z) \quad (6)$$

Or:

$$\underline{E}_y(x, z + L_z)e^{-ik_{iz}(z+L_z)} = \underline{E}_y(x,z)e^{-ik_{iz}z} \quad (7)$$

Therefore $\underline{E}_y(z)e^{-ik_{iz}z}$, is periodic with period L_z and thus by Fourier series can be written as:

$$\underline{E}_y(z)e^{-ik_{iz}z} = \sum_{n=-\infty}^{\infty} E_{yn}e^{i\left(k_{iz} + n\frac{2\pi}{L_z}\right)z} = \sum_{n=-\infty}^{\infty} E_{yn}e^{ik_{zn}z} \quad (8)$$

Where:

$$k_{zn} = k_{iz} + n\frac{2\pi}{L_z} \quad (9)$$

And thus for the x component:

$$k_{xn} = \sqrt{\frac{\omega^2}{c^2} - k_{zn}^2} \quad (10)$$

RAYLEIGH METHOD AND HYPOTHESIS

For that purpose, we invoke the Rayleigh hypothesis (Rayleigh, 1907; Christiansen and Kleinman, 1996; Ramm and Gutman, 2004) that is, we assume that the electric field of the incident wave is

$$\underline{E}_{iy} = E_0 e^{ik_{iz}z - ik_{ix}x} \quad (11)$$

and Scattered field is

$$\underline{E}_{sy} = E_0 \sum_{n=-\infty}^{\infty} R_n^D e^{ik_{zn}z + ik_{xn}x} \quad (12)$$

In the case of the magnetic field, the incident wave can be written as (Gregg, 2002; Giorgio and Gasca, 1995):

$$\underline{H}_{iy} = H_0 e^{ik_{iz}z - ik_{ix}x} \tag{13}$$

And the scattered field:

$$\underline{H}_{sy} = H_0 \sum_{n=-\infty}^{\infty} R_n^N e^{ik_{zn}z + ik_{xn}x} \tag{14}$$

Incident waves and scattered wave expansions Eq. 11-14, which are strictly valid outside the corrugation region. With these definitions, we are able to write the following Floquet expansions that represent rigorously the electromagnetic fields in the region $S_{min}<x<S_{max}$ (Lakhtakia and Depine, 1993).
The family of functions

$$\left\{e^{ik_{zn}z}, \quad |n| < \infty\right\}$$

constitute the so-called Rayleigh basis. Introducing expansions Eq. 11-14 into the boundary conditions and thereafter projecting the resulting equations into the Rayleigh basis, we obtain a matrix equation for the diffraction amplitudes in terms of E_0 and H_0. Such hypothesis is generally applicable to TE and TM modes. We shall consider both modes in our analysis.

Dirichlet problem (TE): We consider the two dimensional Dirichlet boundary-value problem for the Helmholtz equation in a non-locally perturbed half-plane (Axler *et al*., 2004; Ebenfelt and Viscardi, 2004). This problem models the time-harmonic EM scattering by a one-dimensional, infinite, smooth, perfectly conducting surface; the same problem arises in acoustic scattering by a sound-soft rough surface where the total field vanishes; the same problem models two-dimensional electromagnetic scattering by a perfectly conducting, infinite, smooth surface in the transverse magnetic polarization case (DeSanto *et al*., 1998). The Dirichlet problem Eq. (11-12) (TE mode with $E_z = 0$ as shown in Fig. 1a) boundary surface was considered as:

$$x_S = S(z) = -S_0 \cos\left(2\pi \frac{z}{L_z}\right) \tag{15}$$

With:

$$S_0 < 0.0713 L_z \tag{16}$$

and k_{zn} and k_{xn} are as given before and without including the downward moving modes with as $e^{-ik_{xn}x}$ Rayleigh hypothesis states (Lakhtakia and Depine, 1993).

To determine the unknown coefficient R^D_n, we match the boundary conditions on the surface $X_s = S(z)$ as:

$$\underline{E}_y = (\underline{E}_{iy} + \underline{E}_{sy})\Big|_{x=x_S=S(z)} = 0 \tag{17}$$

That is:

$$e^{-ik_{ix}S(z)} + \sum_{n=-\infty}^{\infty} R_n^D e^{in\frac{2\pi}{L_z}z} e^{i\sqrt{\frac{\omega^2}{c^2} - k_{zn}^2}\,S(z)} = 0 \tag{18}$$

By letting

$$2\pi \frac{z}{L_z} = w$$

where, z then becomes as:

$$z = L_z \frac{w}{2\pi}$$

and if we multiply both sides of the above equation by

$$e^{-im\frac{2\pi}{L_z}z}$$

then by integrating over L_z:

$$-\frac{1}{2\pi}\int_0^{2\pi} e^{-ik_{ix}S\left(L_z \frac{w}{2\pi}\right) - imw} dw = \frac{1}{2\pi}\sum_{n=-\infty}^{\infty} R_n^D \int_0^{2\pi} e^{i(n-m)w} e^{i\sqrt{\frac{\omega^2}{c^2} - k_{zn}^2}\,S\left(L_z \frac{w}{2\pi}\right)} d\omega \tag{19}$$

Let:

$$K_{mn}^D = \frac{1}{2\pi}\int_0^{2\pi} e^{i(n-m)w} e^{i\sqrt{\frac{\omega^2}{c^2} - k_{zn}^2}\,S\left(L_z \frac{w}{2\pi}\right)} \tag{20}$$

$$A_m^D = -\frac{1}{2\pi}\int_0^{2\pi} e^{-imw} e^{-ik_{ix}S\left(L_z \frac{w}{2\pi}\right)} dw \tag{21}$$

Then:

$$\sum_{n=-\infty}^{\infty} K_{mn}^D R_n^D = A_m^D \tag{22}$$

or

$$\left[K_{mn}^D\right]_{mxn} \left[R_n^D\right]_{nx1} = \left[A_m^D\right]_{mx1} \tag{23}$$

The above equation may be truncated. The solution will be:

$$\left[R_n^D\right]_{nx1} = \left[K_{mn}^D\right]_{nxm}^{-1} \left[A_m^D\right]_{mx1} \tag{24}$$

Now, if we assume that the surface is sinusoidal with S_{max} - S_{min} = S_0:

$$x_S = S(z) = -S_0 \cos\left(2\pi \frac{z}{L_z}\right) = -S_0 \cos(w) \quad (25)$$

The integrals for K^D_{mn} and A^D_m are of the form:

$$\frac{1}{2\pi}\int_0^{2\pi} e^{imw \pm iu\cos(w)} dw = i^{\pm|m|} J_{|m|}(u) \quad (26)$$

Therefore,

$$A^D_m = -\frac{1}{2\pi}\int_0^{2\pi} e^{-imw + ik_{ix}S_0\cos(w)} dw = -i^{|m|} J_{|m|}(k_{ix}S_0) \quad (27)$$

And:

$$K^D_{mn} = \frac{1}{2\pi}\int_0^{2\pi} e^{i(n-m)w - ik_{xn}S_0\cos(w)} dw = i^{-|m-n|} J_{|m-n|}(k_{xn}S_0) \quad (28)$$

For slow waves with $k_{xn} = i|k_{xn}|$, then

$$J_{|m-n|}(k_{xn}S_0) = i^{|m-n|} I_{|m-n|}\left(|k_{xn}|S_0\right) \quad (29)$$

$$K^D_{mn} = I_{|m-n|}\left(|k_{xn}|S_0\right) \quad (30)$$

Accordingly, the incident wave becomes:

$$E_{iy} = E_0 e^{ik_{iz}z - ik_{ix}x} e^{-i\omega t} \quad (31)$$

Moreover, the scattered waves becomes:

$$E_{sy} = E_0 \sum_{n=-\infty}^{\infty} R^D_n e^{ik_{zn}z + ik_{xn}x} e^{-i\omega t} \quad (32)$$

Where:

$$k_{zn} = k_{iz} + n\frac{2\pi}{L_z} \text{ and } k_{xn} = \sqrt{\frac{\omega^2}{c^2} - k_{zn}^2}$$

With the solution as specified by:

$$\left[R^D_n\right]_{nx1} = \left[K^D_{mn}\right]^{-1}_{nxm} \left[A^D_m\right]_{mx1} \quad (33)$$

and A^D_m, K^D_{mn} as specified in Eq. 27-30.

Neuman problem (TM): The TM mode problem can be derived by considering the following Neuman Problem boundary surface:

$$x_S = S(z) = -S_0 \cos\left(2\pi \frac{z}{L_z}\right)$$

with: $S_0 < 0.0713L_z$.

For the TM problem specified in Eq. (13) and (14), the boundary condition for this case can be specified as:

$$\vec{E}\times\hat{n} = 0 \sim \left(\nabla\times\hat{H}\right)\times\hat{n} = \left[(\nabla H_y)\times\hat{y}\right]\times\hat{n} = \frac{\partial H_y}{\partial n}\hat{y} \quad (34)$$

that is:

$$\left.\frac{\partial H_y}{\partial n}\right|_{x=x_S=S(z)} = \hat{n}\bullet\nabla \underline{H}_y\Big|_{x=x_S=S(z)} = 0 \quad (35)$$

or:

$$\left(\hat{x} - \frac{dS}{dz}\hat{z}\right)\bullet\left(\hat{x}\frac{\partial}{\partial x} + \hat{z}\frac{\partial}{\partial z}\right)\underline{H}_y\Big|_{x=x_S} = \left(\frac{\partial}{\partial x} - \frac{\partial S}{\partial z}\frac{\partial}{\partial z}\right)\underline{H}_y\Big|_{x=xS} = 0 \quad (36)$$

which can be rewritten as:

$$-ik_{ix}e^{-ik_{ix}S(z)} + \sum_{n=-\infty}^{\infty} ik_{xn}R^N_n e^{in\frac{2\pi}{L_z}z} e^{ik_{xn}S(z)} - \frac{dS}{dz}\left[ik_{iz}e^{-ik_{ix}S(z)} + \sum_{n=-\infty}^{\infty} ik_{zn}R^N_n e^{in\frac{2\pi}{L_z}z} e^{ik_{xn}S(z)}\right] = 0 \quad (37)$$

or as can be simplified to:

$$\left(k_{ix} + \frac{dS}{dz}k_{iz}\right)e^{-ik_{ix}S(z)} = \sum_{n=-\infty}^{\infty}\left(k_{xn} - \frac{dS}{dz}k_{zn}\right)R^N_n e^{in\frac{2\pi}{L_z}z} e^{ik_{xn}S(z)} \quad (38)$$

By multiplying both sides by

$$e^{-im\frac{2\pi}{L_z}z}$$

then by integrating over L_z and make the changes as for the Dirichlet case, we can write this as:

$$\left[K^N_{mn}\right]_{mxn}\left[R^N_n\right]_{nx1} = \left[A^N_m\right]_{mx1} \quad (39)$$

The above equation may be truncated and the solution will be:

$$\left[R_n^N\right]_{nx1} = \left[K_{mn}^N\right]_{nxm}^{-1}\left[A_m^N\right]_{mx1} \quad (40)$$

where:

$$A_m^N = -\frac{1}{2\pi}\int_0^{2\pi}\left(k_{ix} + \frac{2\pi}{L_z}\frac{dS}{dw}k_{iz}\right)^* e^{-imw}e^{-ik_{ix}S\left(L_z\frac{w}{2\pi}\right)}dw \quad (41)$$

$$K_{mn}^N = \frac{1}{2\pi}\int_0^{2\pi}\left(k_{xn} - \frac{2\pi}{L_Z}\frac{dS}{dz}k_{zn}\right)^* e^{i(n-m)w}e^{ik_{xn}S\left(L_Z\frac{w}{2\pi}\right)}dw \quad (42)$$

By applying these results for the sinusoidal surface with

$$x_S = S(z) = -S_0\cos\left(2\pi\frac{z}{L_z}\right) = -S_0\cos(w) \quad (43)$$

$$\frac{dS}{dw} = S_0\sin(w)$$

In addition, using the identity

$$\frac{1}{2\pi}\int_0^{2\pi}e^{imw\pm iu\cos(w)}dw = i^{\pm|m|}J_{|m|}(u)$$

as before and by deriving the other equality as follows:

$$\frac{1}{2\pi}\int_0^{2\pi}\sin w e^{imw\pm iu\cos(w)}dw = \frac{1}{2\pi}\int_0^{2\pi}\frac{e^{iw}-e^{-iw}}{i2}e^{imw\pm iu\cos(w)}dw \quad (44)$$

which can be written as:

$$\frac{1}{2\pi}\int_0^{2\pi}\sin w e^{imw\pm iu\cos(w)}dw = -\frac{i}{2}\left[i^{\pm|m+1|}J_{|m+1|}(u) - i^{\pm|m-1|}J_{|m-1|}(u)\right] \quad (45)$$

Therefore,

$$A_m^N = k_{ix}i^{|m|}J_{|m|}(k_{ix}S_0) - i\frac{\pi S_0}{L_z}k_{iz}\begin{bmatrix} i^{|m+1|}J_{|m+1|}(k_{ix}S_0) - \\ i^{|m-1|}J_{|m-1|}(k_{ix}S_0)\end{bmatrix} \quad (46)$$

$$K_{mn}^N = k_{xn}i^{|m-n|}J_{|m-n|}(k_{ix}S_0) + i\frac{\pi S_0}{L_z}k_{zn}\begin{bmatrix} i^{-|m-n+1|}J_{|m-n+1|}(k_{xn}S_0) - \\ i^{-|m-n-1|}J_{|m-n-1|}(k_{xn}S_0)\end{bmatrix} \quad (47)$$

For slow waves with $k_{xn} = i|k_{xn}|$, then

$$J_{|m-n\pm 1|}(k_{xn}S_0) = i^{|m-n\pm 1|}I_{|m-n\pm 1|}\left(|k_{xn}|S_0\right) \quad (48)$$

And:

$$K_{mn}^N = i|k_{xn}|I_{|m-n|}\left(|k_{xn}|S_0\right) + i\frac{\pi S_0}{L_Z}k_{zn}\begin{bmatrix} J_{|m-n+1|}\left(|k_{xn}|S_0\right) - \\ J_{|m-n-1|}\left(|k_{xn}|S_0\right)\end{bmatrix} \quad (49)$$

Therefore, the incident wave becomes:

$$H_{iy} = H_0e^{ik_{iz}z - ik_{ix}x}e^{-i\omega t} \quad (50)$$

Moreover, the scattered waves becomes:

$$H_{sy} = H_0\sum_{n=-\infty}^{\infty}R_n^N e^{ik_{zn}z + ik_{xn}x}e^{-i\omega t} \quad (51)$$

Where, k_{zn} and k_{xn} are the same as for Dirichlet Problem (TE) and for R^N_n, the solution has the form:

$$\left[R_n^N\right]_{nx1} = \left[K_{mn}^N\right]_{nxm}^{-1}\left[A_m^N\right]_{mx1} \quad (52)$$

With: A^N_m, K^N_{mn} as specified in Eq. 46-49.

CONCLUSION

A model for investigating the EM scattering by an arbitrarily periodic surface has been developed. Before dealing with the scattering problem, EM fields in the medium are solved first using the method of separation of variables and then formulated in terms of the spherical vector wave functions in a very compact form. A procedure based on Rayleigh hypothesis is then applied in the analysis of the scattering problem. The scattered field and the incident field are related by imposing the boundary conditions and making use of the equivalence principle. The scattered fields are derived by imposing both Dirichlet (TE) and Neuman (TM) conditions for the vector wave functions. The important point in this study is that the derived formula allows easy computational calculations of diffracted fields near the aperture.

In another incoming study, the combined effect of atmospheric conditions and surface roughness on the propagation and scattering problem will be included. Such type of research is in progress

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