

## Reliability Modelling of Combined Heat and Power Generating Power Plant Units

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**Abstract:** This study briefly sets out a 3 and more state reliability description of combined heat and power generating units (of extraction condensing and back-pressure steam turbine power plant units). The new procedure improves the accuracy of system-level reliability (Loss of Load Probability (LOLP)) calculations for power generation systems by giving a more differentiated (and thus more accurate) description of the available capacity from extraction condensing and back-pressure turbine power plants. Applying the 3 or more state reliability model for power plant units with extraction condensing and back pressure turbine and the deduction of the reliability calculation's input data form the novelty of the newly developed calculation method.

**Key words:** Loss of load probability, power plant unit, power generation system, power system reliability, heat output, Hungary

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### INTRODUCTION

For the purposes of calculating, the reliability of power generation systems, combined heat and power plants have hitherto always been modelled as must-run, quasi-must-run or aggregated power plant units. In many cases, the capacity of co-generated power plant units is simply subtracted from the system-level power demand. This implies a two-state reliability model i.e., incorporation of the available capacity of cogeneration power plant units without regard to capacity reduction due to heat output. The two-state reliability model states that the power plant unit is either operational with full capacity or non-operational with zero capacity. The two-state reliability description of power plant units only produces a result of satisfactory accuracy if the modelled power plant units have a high annual operating time are in operation for most of the year. It is clear without further explanation that the two-state reliability model gives a very rough approximation of the real operation of extraction condensing and back-pressure power plant units which co-generate heat and power. There are basically two reasons for this. Firstly, the annual utilisation of these power plant units, although considerably greater than that of peak-operation units is considerably less than that of base-load units. Secondly, cogeneration requires that for a certain portion of the operating period, extraction condensing and back-pressure steam turbine power plant units make less than

their nominal capacity available to the power system. Consequently, the two-state reliability description is not suitable for differentiated reliability modelling of cogeneration power plants. This situation is not remedied by an attempt to incorporate heat output-induced capacity loss using some kind of average figure for the whole period (heat output shortfall (MW)).

The differentiated reliability description (3 and more state reliability modelling) of extraction condensing and back-pressure power plant units is made possible by determining for each power plant unit, the distribution function for the probability of occupation of its operating states. A precondition for the 3 and more state reliability description of power plant units, therefore is knowledge of the distribution of the occupation probabilities of all defined operating states (during the period of study). The two-state reliability description assumes that the power plant unit exclusively occupies either the failed, zero capacity or operational at full capacity operating states. The discrete probability distribution function required for the two-state reliability description may be defined from the fault statistics of each power plant unit. The three and more state reliability description calls for statistical data which is not available in the great majority of cases. The control technology of extraction condensing and back-pressure power plant units, however is such that their current available maximum power capacity is a function of the average daily ambient temperature because the majority (80-90%) of these units operate as heat sources

for heating systems and this makes possible their three and more state reliability description. It should be stressed that the novelty content of the newly developed calculation method lies not in the fact that the instantaneous maximum available power capacity of these power plant units is a function of instantaneous heat output and therefore of average daily ambient temperature but in the recognition that this relationship permits their three and more state reliability description by virtue of the desired random variable of the maximum available power capacity ( $\chi_{LPPmax}$ ) being a transform of the random variable of the ambient mean air temperature ( $\zeta_{TK}$ ). This permits calculation of the probability of occupation of each defined operating state (in the test period).

**Determination of the discrete probability distribution of the maximum available power capacity in the case of extraction condensing and backpressure steam turbine power plant units:** Extraction condensing and back-pressure power plant units obey the relation:

$$L_{PPmax} = f(\dot{Q}(T_k)) \quad (1)$$

Here,  $\dot{Q}$  (MW) is the current heat output of the power plant unit. The ambient temperature and consequently, its daily average vary randomly in time. In the description of this random process, event space  $\Omega_{Ei}$  is filled by event elements  $E_i$ ;  $\Omega_{Ei} = \{E_1, E_2, \dots, E_n\}$ . Every event element  $E_i$  occurs when the daily mean ambient temperature ( $T_k$ ) falls in the range ( $T_{k,i-1}, T_{k,i}$ ) (Roberts, 1964). The probability distribution of the daily mean ambient temperature is known for each geographical point. This means that in a given geographical region in the average over a specific period, the probability of occurrence of each temperature i.e., for each possible value of the random variable  $\zeta_{TK}$ . The distribution function is defined as:

$$F_{\zeta_{TK}}(T_k) = P(\zeta_{TK} < T_k) \quad (2)$$

Extraction condensing and back-pressure steam-turbine power plant units operate as heat sources for heat consumers whose instantaneous heat demand changes in proportion to the daily average ambient temperature as shown in Eq. 1. The power plant unit heat output is defined by the relation:

$$\dot{Q}_i = f(T_{k,i}) \quad (3)$$

The maximum power output of extraction condensing and back-pressure power plant units is a function of current temperature i.e., obeys the relation:

$$L_{PPmax,i} = f(\dot{Q}_i) \quad (4)$$

Consequently, instantaneous available power capacity (maximum possible power output at the given heat output) varies with daily average ambient temperature as follows:

$$L_{PPmax,i} = f(T_{k,i}) \quad (5)$$

It is thus, possible to determine the related quantities  $T_{k,i}$ ,  $\dot{Q}_i$  and  $L_{PPmax,i}$  i.e., for the occurrence of random event  $E_i$ ; the resulting values of  $\dot{Q}_i$  and  $L_{PPmax,i}$  may be obtained. Analogously, the random variable  $\zeta_{TK}$  may be used to define the power plant heat output random variable  $\omega_Q$  and the power plant maximum power output (maximum available power capacity) random variable  $\chi_{LLPmax}$ . From relations to Eq. 1, 3, 4 and 5:

$$\omega_Q = \omega_Q(\zeta_{TK}) \quad (6)$$

$$\chi_{LPPmax} = \chi_{LPPmax}(\omega_Q) \quad (7)$$

and finally:

$$\chi_{LPPmax} = \chi_{LPPmax}(\zeta_{TK}) \quad (8)$$

This means that the power plant heat output random variable  $\omega_Q$  and the power plant maximum power output (maximum available power capacity) random variable  $\chi_{LLPmax}$  are both transforms of the daily average ambient temperature variable  $\zeta_{TK}$ .

The probability distribution of each random variable and its transform may be determined from the following theorem (Roberts, 1964; Hall *et al.*, 1968): if  $\xi$  is a discrete random variable whose possible values are the numbers  $x_1, x_2, \dots$  and  $y = r(x)$  is an arbitrary function, then the distribution of the random variable  $\eta = r(\xi)$  is defined by the probabilities:

$$P(\eta = y_k) = \sum_{r(x_i)=y_k} P(\xi = x_i), \quad (k=1,2,\dots) \quad (9)$$

Where  $y_1, y_2, \dots$  are non-equal values of  $r(x_1), r(x_2)$ . This follows from the fact that an event  $\eta = y_k$  occurs if and only if the value  $x$  taken by  $\xi$  is the value for which  $r(x_i) = y_k$  (Galloway *et al.*, 1969; Endrenyi, 1978; Armstadter, 1971). Clearly:

$$\sum_k P(\eta = y_k) = 1 \quad (10)$$

In general, therefore, it may be stated that:

$$P(\chi_{L_{PPmax}} = L_{PPmax,r}) = \sum_{L_{PPmax}(T_{k,i})=L_{PPmax,r}} P(\zeta_{TK} = T_{k,i}), \quad (r=1,2,\dots) \quad (11)$$

This statement may naturally also be expressed for the heat outputs, in which case it takes the form:

$$P(\omega_Q = \dot{Q}_r) = \sum_{\dot{Q}(T_{k,i})=\dot{Q}_r} P(\zeta_{TK} = T_{k,i}), \quad (r=1,2,\dots) \quad (12)$$

**Deduction of the discrete probability distribution of the maximum available power capacity:** Where  $T_k$  ( $^{\circ}\text{C}$ ) represents the daily average ambient temperature and  $t_k$  ( $^{\circ}\text{C}$ ) is an arbitrary temperature value,  $D(t_k)$  (h) gives the duration within a test period when the relation  $T_k \geq t_k$  holds i.e., the number of hours during the reference period when the daily average ambient temperature is greater than or equal to a specified temperature  $t_k$  ( $^{\circ}\text{C}$ ). The duration diagram of the daily average ambient temperature for the period under examination may be determined from meteorological statistical data for the given geographical location. The daily average ambient temperature duration diagram for a given period may be rendered into a daily average ambient temperature random variable via a multi-step transformation. The first step is to transform the time values on the abscissa into relative time values. The values on the abscissa of the resulting diagram may be viewed as probabilities that the daily average ambient temperature is greater than or equal to the corresponding daily average ambient temperature value i.e., the probability  $P(T_k \geq t_k)$ . The second transformation step is to exchange the abscissa and the ordinate (Endrenyi, 1978; Billinton and Allan, 1984). This puts probabilities on the ordinate and daily average ambient temperature values on the abscissa. The curve obtained from these two transformations may be viewed as the complementary curve of the distribution function (Billinton and Allan, 1992) of the random variable  $T_k$  i.e.,

$$d(t_k) = 1 - F_{TK}(t_k) \quad (13)$$

In this relation,

- $d(t_k)$  = The relative duration [-], when  $T_k \geq t_k$ ,  $(D(t_k)/\tau)$
- $F_{TK}(t_k)$  = The probability distribution function of random variable  $T_k$
- $\tau$  = Reference time interval (h)

It follows that the probability distribution function of random variable  $T_k$  is defined by the relation:

$$F_{TK}(t_k) = 1 - d(t_k) \quad (14)$$

And this gives the relative duration in which  $T_{k,i} < T_k$ . The relative duration may be viewed as a probability. Then, the relative duration is equivalent to the probability that  $T_{k,i} < T_k$  i.e.,

$$F_{TK}(t_k) = P(T_k < t_k) \quad (15)$$

$F_{TK}(t_k) = P(T_{k,i} < t_k)$  and so gives the probability that the daily average ambient temperature ( $T_{k,i}$  ( $^{\circ}\text{C}$ )) is smaller than  $t_k$ . The relation thus defined may be proved to satisfy the requirements of a distribution function.

**Reliability description of extraction condensing and back pressure steam turbine power plant units using state-space method:**

Reliability modelling of power plant units using a state-space description method involves characterising each power plant unit or power plant system (power plant units in the power plant system) with defined operating states and probabilities of transitions among these states. For reliability purposes, the states of a power plant unit or power plant system are given in terms of the probability distribution of the operating states which power plant units constituting the power plant system may occupy at a given time (Billinton and Allan, 1984, 1992; Billinton, 1982). The state of the system or operating state of the power plant unit at a given time is clearly defined if the probability distribution of possible operating states and system configurations are known for that time (Liu and Singh, 2010; Dehghani and Nikravesh, 2008). The operating states which are customarily defined include operational at full capacity, failed (non-operational), operational at reduced capacity in reserve failed in reserve, etc.

**Calculation of the availability and failure factors:**

For extraction condensing and back-pressure steam-turbine power plant units, the availability factor for each operational operating state ( $^{\text{th}}K_{oi}$  (-)) may be calculated as the time of occupation of each operational operating state as a proportion of the total reference period. This relative time may be determined from the probability distribution function of the maximum available power capacity and represents the probability that at any time the power plant unit occupies that operating state.

The availability factor for all of the power plant unit's operational operating states may be determined with the constraint that the periods of occupation of the operational and failed operating states add up to the total reference period. The failure factor ( $^{\text{th}}k_D$  (-)) may be calculated in the usual way (Billinton and Allan, 1984, 1992; Billinton, 1982). Extraction condensing and back-pressure steam-turbine power plant units obey the following relations:

$$\sum_{i=1}^{i=d-1} {}^d T_{Ui} + {}^d T_D = {}^d T \quad (16)$$

$${}^d K_{Ui} = \frac{{}^d T_{Ui}}{{}^d T} = \frac{{}^d T_{Ui}}{\left(\sum_{i=1}^{d-1} {}^d T_{Ui}\right) + {}^d T_D} = \quad (17)$$

$$P(L_{PP,max,Uia} \leq \chi_{LPP,max} < L_{PP,max,Uif}) = F_{\chi_{LPP,max}}(L_{PP,max,Uif}) - F_{\chi_{LPP,max}}(L_{PP,max,Uia})$$

$${}^d K_D = \frac{{}^d T_D}{{}^d T} = \frac{{}^d T_D}{\left(\sum_{i=1}^{d-1} {}^d T_{Ui}\right) + {}^d T_D} \quad (18)$$

In the foregoing:

- $d$  = The number of possible defined operating states (-)
- ${}^d T_{Ui}$  = The duration of occupation of the  $i$ th operational operating state (h)
- ${}^d K_{Ui}$  = The availability factor for the  $i$ th operational operating state (-)
- ${}^d T_D$  = The duration of occupation of the failed operating state (h)
- ${}^d K_D$  = The failure factor for the failed operating state (-)
- ${}^d T$  = The total duration of the reference period (-)
- $L_{PP,max,Uia}$  = The lower limit of the output range of the  $i$ th operational operating state (MW)
- $L_{PP,max,Uif}$  = The upper limit of the output range of the  $i$ th operational operating state (MW)

**Determination of the average power output:** A further important item of input data for LOLP computations is the average power capacity for each defined operational operating state ( $\overline{L_{PP,Uk}}$  (MW)). The principle of determination this average power output is as follows: the principle of calculation of the average capacity for each defined operational operating state is that the electrical energy actually output during occupation of a given capacity range should be equal to the energy output calculated with the average value.

The average power in the  $k$ th operational operating state ( $\overline{L_{PP,Uk}}$  (MW)) is the probability of occurrence-weighted sum of power values in the operating state:

$$\sum_{i \in G_{Uk}} P_i L_{PP,max,i}$$

divided by the sum of the probabilities of the power values:

$$\sum_{i \in G_{Uk}} P_i$$

## CONCLUSION

The question naturally arises as to how much the newly-developed procedure improves the accuracy of computation. Experience has shown improvement in accuracy of 10-30%. The greatest improvement is found in cases where there are considerable temperature swings during the test period.

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