

## Optimising Reactive Compensation in Power System

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**Abstract:** This research presents a comprehensive method of optimizing the reactive compensation used in power systems to obtain acceptable voltage profiles during periods of abnormal system loads and foreseeable contingencies. The system equations which are non-linear are first approximated to a linear form and then Linear Programming Technique is applied to obtain an optimal solution. An iterative procedure is then used to obtain results of acceptable accuracy. The main features of this research are not only that both inductive and capacitive compensation is optimized but also the bus bars where compensation is applied can be selected to suit the users operating constraints in the case of Independent Power Plant (IPP). Linear Programming Model was used for the determination of the minimum reactive power, required at designated buses during reactive compensation. Complete load flow analysis was done using Power System Analysis Tool Box (PSAT) a matlab base computer software which establishes both normal and contingency bus voltage profiles.

**Key words:** Optimisation, transmission, compensation, bus, load, reactive, simulation, linear, Nigeria

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### INTRODUCTION

An essential feature in the planning of a power system is the provision of reactive compensation for the control of voltage profile. The computational aspects of reactive compensation in an electric energy system consists of two main jobs namely; load flow analysis and the determination of minimum reactive power required at designated buses, subject to certain constraint (Ike, 2008). During periods of heavy loads and at very light loads, some of the voltages in the system could fall outside acceptable limits.

Also, during contingencies such as loss of a major transmission line or a generating unit, the bus-bar voltages could attain unacceptable values. The need for proper optimization (best or most favorable) of both inductive and capacitive reactive compensation is readily high lighted when considering supply networks like that of Nigeria where the local primary transmission network is entirely by high voltage cables.

The reactive generation exceeds that of the load especially during light loads and shunt reactors (in transmission stations) are switched in to prevent excessive voltage rise (Ike, 2008).

Ideally, the voltage level at all the buses should be constant at all times. To achieve this under all operating conditions require that the reactive power at all buses be controllable and also Tap-Changing Under Load (TCUL)

transformers be used at every bus to adjust the voltage level. This is difficult and expensive to implement. In practice, type-two buses are provided with variable reactive powers and a number of crucial buses are then equipped with TCUL transformers. Hence, the possibility of using TCUL transformers to improve the system's voltage profile is first explored before any nodal reactive compensation is attempted.

Also, tolerance limits are given for the voltage levels for all the buses except the swing bus where the voltage magnitude is fixed and remains constant. In the event of contingencies like failure of system's generators or transmission lines, the system is equipped to adjust and thus ensures that the bus voltages are still within specified limits (Nagrath and Kothari, 1994).

### MATERIALS AND METHODS

**Description of the solution technique:** Figure 1 shows the various stages involved in the proposed solution technique and a brief description of the function of each stage is as follows: In stage one, the AC, load flow program using the Gauss-Siedel Interactive Technique is used to determine the normal bus voltage levels of the system. In stage two, for a specified contingency (e.g., loss of a transmission line), the new bus admittance matrix is formed. Load pattern is fed in as data into the program. The prevailing bus voltage levels during the contingency

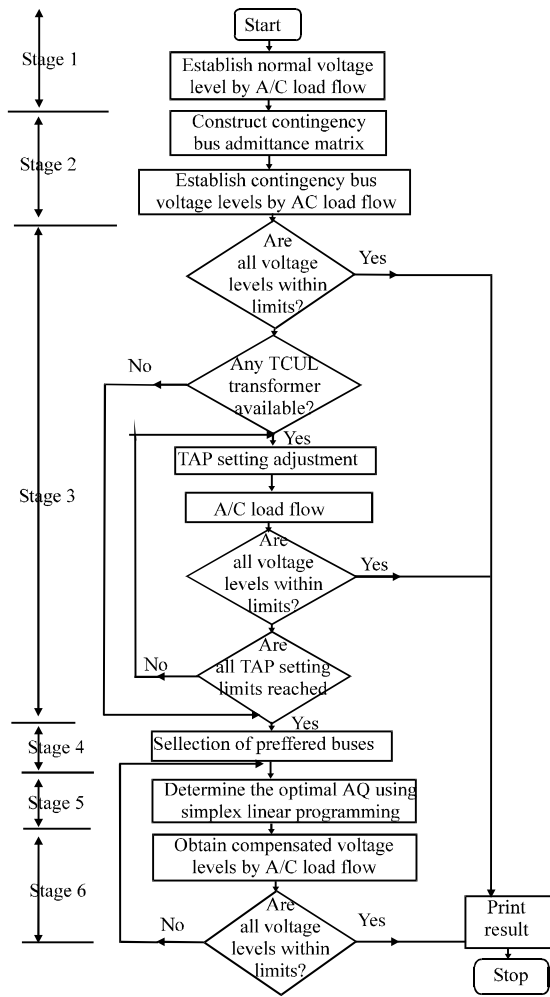


Fig. 1: Flow chart of the solution technique

or abnormal load conditions are calculated by a re-run of the AC, load flow program. In stage three, a decision is then made as to whether the contingency bus voltage levels are acceptable or not. If the levels are acceptable then researchers print results and stop. But if the levels are not within specified acceptable bounds and the bus is equipped with TCUL transformers existing in the network to meet the specified voltage constraints. It enters the successive stages only when all the constraints are not satisfied either owing to absence of TCUL transformers or attainment of tap setting limit on the available transformers.

In stage four, type-two buses at which reactive compensation can not be applied due to economic or by using a large weighting factor in the cost function to be minimized. This allows the user the flexibility of varying the location of the reactive compensation and also to

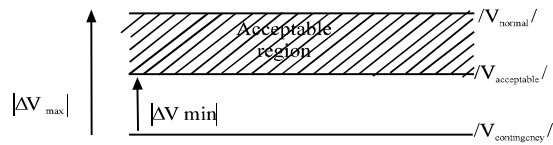


Fig. 2: Representation of voltage boundary limits

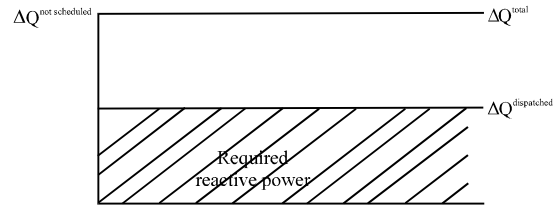


Fig. 3: Reactive power dispatched

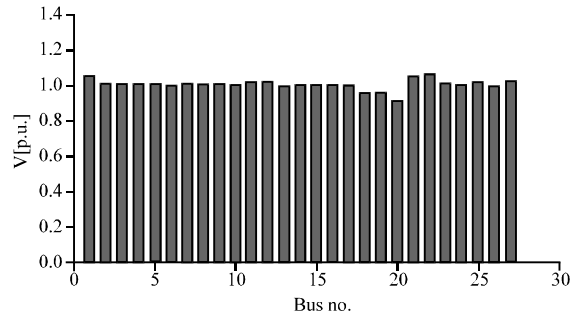


Fig. 4: Voltage profile at base case load

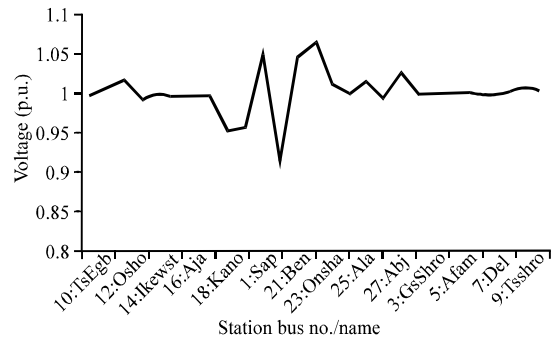


Fig. 5: System bus voltage profile

obtain a realistic solution for systems having location constraints. Consequently in stage five, a re-course is made to the modification of the reactive power at preferred type-two buses. Here, the simplex method of linear programming is used to obtain the minimum additional reactive power necessary to restore the voltages to lie within the preset bounds.

In stage six since, the procedure in linear programming is only approximate, the AC, load flow obtained solution is acceptable (Fig. 1-6). From the foregoing, it is evident that the computational aspect of

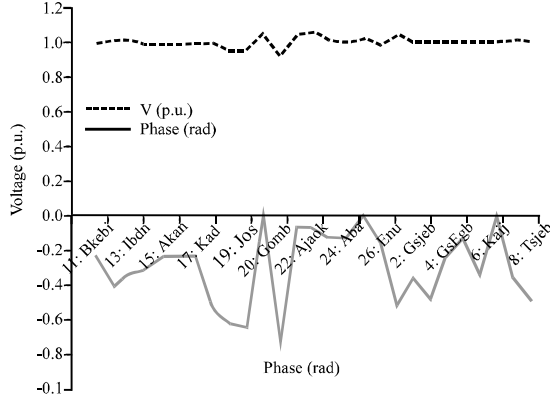


Fig. 6: System bus voltage profile and angle (rad)

reactive compensation in an electric power system consists of load flow analysis and simplex linear programming.

### RESULTS AND DISCUSSION

**Linear programming analysis/problem formulation:** The major purpose of reactive power correction study is to determine the minimum amount of reactive power compensation required in a power system to obtain acceptable voltage profiles during contingencies such as critical outage cases. The system equations which are non-linear are first linearized (approximated to a linear form) based on the approximation that for small changes, the relationship between voltage and reactive power is linear (Ike, 2008; Choi *et al.*, 1981). Thus:

$$|\Delta V_i| \approx \sum_{j=1}^n X_{ij} \Delta Q_j \quad (1)$$

Where:

- $|\Delta V_i|$  = The change in voltage magnitude required to bring the voltage at bus i within the level
- $\Delta Q_i$  = The change in reactive power at bus i needed to correct the voltage
- $X_{ij}$  = The reactive linking buses i and j. Here j has been taken to include all the n buses

The objective function to be minimized is the amount of reactive power added into the system, i.e.:

$$H = \sum_{j=1}^n C_j \Delta Q_j \quad (2)$$

Where:

- H = Minimum corrective reactive power to be added to the system

- $C_j$  = Weighting factor at bus j. It takes into consideration the economics of varying the reactive power at the bus concerned
- n = Total number of bus where reactive compensation is to be considered

$\Delta Q_i$  is as explained in Eq. 1 before. The objective function H therefore, provides a measure of the cost involved in the reactive compensation during any contingency. If reactive compensation is undesirable or difficult to implement physically at bus K then  $C_k$  may be assigned a relatively large value (say 100) while each  $C_j$  ( $j \neq k$ ) is assigned a much smaller (say unity). This is to ensure that  $\Delta Q_k$  will assume a negligible value in the final solution of the linear programming search.

**Constraints:** Normally, the corrective reactive power  $\Delta Q_i$  dispatched should be such that the resulting changes in the bus voltage should be greater than the minimum voltage increase  $|V_{\min,i}|$  and at the same time less than the maximum voltage increase  $|V_{\max,i}|$  for all i. This is translated into mathematical form (Li *et al.*, 2005; Baran *et al.*, 2001) and shown in Fig. 2:

$$\sum_{j=1}^n X_{ij} \Delta Q_j^{\text{dispatched}} \geq |\Delta V_{\min,i}| = |V_{Hi} - V_{con,i}| \quad (3)$$

and:

$$\sum_{j=1}^n X_{ij} \Delta Q_j^{\text{dispatched}} \leq |\Delta V_{\max,i}| = |V_{con,i} - V_{Li}| \quad (4)$$

with:

$$\Delta Q_j^{\text{dispatched}} \geq 0 \quad (5)$$

for capacitive compensation. In the above case, it is evident that capacitive reactive power compensation is required to raise the bus voltage to an acceptable level. The minimum voltage increase need is given by:

$$|\Delta V_{\min}| = |V_{\text{acceptable}}| - |V_{\text{contingency}}| \quad (6)$$

$$|\Delta V_{\max}| = |V_{\text{normal}}| - |V_{\text{contingency}}| \quad (7)$$

**Building the constraint matrix by simplex method:** An appropriate way of displaying all the information required in a case study of this nature is by the use of a Simplex table. The table can be regarded as a representation of the detached coefficients of both the variables in the constraint equations and of the objective function to be minimized.

**Slack variables:** To fit the form of the Simplex solution, all inequalities must be changed or linearised to equalities. This is done by adding slack or dummy variables

Table 1: The 100% base case power flow result

Bus	V [p.u.]	Phase [rad]	P gen [p.u.]	Q gen [p.u.]	P load [p.u.]	Q load [p.u.]
10:TsEgb	0.99789	-0.22866	0.0000	0.00000	0.80000	0.60000
11:Bkebi	1.00750	-0.39716	0.0000	0.00000	0.56400	0.42300
12:Osho	1.01530	-0.33268	0.0000	0.00000	1.44900	1.08680
13:Ib dn	0.99426	-0.30519	0.0000	0.00000	1.33000	0.97500
14:Ikewst	0.99821	-0.22981	0.0000	0.00000	3.32000	2.49000
15:Akan	0.99584	-0.23243	0.0000	0.00000	2.28000	1.71000
16:Aja	0.99709	-0.22963	0.0000	0.00000	0.95000	0.71250
17:Kad	0.99586	-0.52668	0.0000	0.00000	3.00000	0.90000
18:Kano	0.95112	-0.61994	0.0000	0.00000	1.30000	0.80000
19:Jos	0.95774	-0.63605	0.0000	0.00000	0.96000	0.25000
1:Sap	1.05000	0.00000	16.3278	2.67360	0.00000	0.00000
20:Gomb	0.91494	-0.71471	0.0000	0.00000	0.78445	0.46991
21:Ben	1.04840	-0.05700	0.0000	0.00000	1.24200	0.93300
22:Ajaok	1.06400	-0.06754	0.0000	0.00000	0.65000	0.48750
23:Onsha	1.00970	-0.12223	0.0000	0.00000	0.86000	0.64500
24:Aba	1.00020	-0.12216	0.0000	0.00000	0.74800	0.42000
25:Ala	1.01460	0.00760	0.0000	0.00000	0.20000	0.15000
26:Enu	0.99258	-0.14485	0.0000	0.00000	0.90000	0.67500
27:Abj	1.02590	-0.50974	0.0000	0.00000	1.30000	0.60000
2:GsJeb	1.00000	-0.35276	0.9000	-4.87160	0.00000	0.00000
3:GsShro	1.00000	-0.48091	1.5000	-3.31630	0.00000	0.00000
4:GsEgb	1.00000	-0.21901	2.2000	-0.27288	0.00000	0.00000
5:Afam	1.00000	-0.11673	0.7200	-0.17531	0.00000	0.00000
6:Kaij	0.99800	-0.34338	2.3000	-3.10960	0.00000	0.00000
7:Del	1.00000	0.01244	0.7000	-4.67470	0.00000	0.00000
8:TsJeb	1.00700	-0.35499	0.0000	0.00000	0.07900	0.05930
9:Tsshro	1.00460	-0.48373	0.0000	0.00000	1.09600	0.82200

Minimum voltage limit violation at bus 20: Gomb [ $V_{min} = 0.94$ ]

complete with appropriate values (as necessary) to each constraint. These slack variables not only make the constraints equalities but they also provide hypothetical produced variables for Table 1 (Stagg and El-Abid, 1968; Kreyszig, 2001). Normally all constraints will take one of the following forms:

$$\begin{aligned} A+B &\leq 100 \\ C+D &= 100 \\ E+F &\geq 100 \end{aligned}$$

The presence of both upper and lower constraints in any particular equation will always necessitate two equations in Table 1 e.g.,  $10 \leq A \leq 90$  will result the following two equations:

$$\begin{aligned} A &\leq 90 \\ A &\geq 10 \end{aligned}$$

For the 1st type, a slack variable with zero value (or cost) is added thus, changing the constraint from:

$$A + B \leq 100 \text{ to } S_1 + A + B = 100$$

For the 2nd type, a slack variable is also added but an extremely high cost is assigned to it (say 100,000 for cost minimizing problems or -100,000 for profit maximizing problems). This is to make certain that the slack variable will not appear in the optimum solution and that the equality  $C + D = 100$  will be enforced. Thus, the constraint  $C + D = 100$  becomes:

$$\int_2^{100,000} +C + D = 100$$

(or -100,000 for profit maximizing problems)

For the 3rd type, two slack variables are added. The first slack variable will have a high cost associated with it and will be the one ensuring that the amount of E and F will be at least 100. The second slack variable will have a zero cost and -1 coefficient, thereby permitting  $E+F$  to be  $>100$ . Thus, the constraint  $E+F \geq 100$  becomes:

$$\int_3^{100,000} - \int_4^0 +E + F = 100$$

(or -100,000 for profit maximizing problems)

Now, refer to Eq. 8a, b and 9a, b and note that they contain both upper and lower constraints and since the presence of both upper and lower limits in any particular equation will always necessitate two equations in Table 1, consequently Eq. 3 becomes:

$$X_{ij} \Delta Q_j \leq |\Delta V_{max} i| \quad (8a)$$

$$|V_{Hi} - V_{con} i| = |\Delta V_{max} i| \quad (8b)$$

and Eq. 9a, b becomes:

$$X_{ij}\Delta Q_j \geq |\Delta V_{min}, i| \quad (9a) \quad \text{and:}$$

$$|V_{con}, i - V_{Li}| = |\Delta V_{min}, i| \quad (9b)$$

Therefore, adding the necessary slack variables with appropriate values to the constraint Eq. 5, 8a, b and 9a, b yields the following:

$$\left. \begin{aligned} \int_1^{100,000} - \int_2^0 + Q_j &= 0 \dots (Eq. 5) \\ \int_3^0 + X_{ij}\Delta Q_j &= |\Delta V_{max}, i| \dots (Eq. 8a) \\ \int_4^{100,000} + |V_{Hi} - V_{con}, i| &= |\Delta V_{max}, i| \dots (Eq. 8b) \\ \int_5^{100,000} - \int_6^0 + X_{ij}\Delta Q_j &= |\Delta V_{min}, i| \dots (Eq. 9a) \\ \int_7^{100,000} + |V_{con}, i - V_{Li}| &= |\Delta V_{min}, i| \dots (Eq. 9b) \end{aligned} \right\} \quad (10)$$

In a power system, inductive as well as capacitive reactive power may be required to meet the voltage constraints. Hence, by simply defining a new term  $\Delta Q_j^{total}$  which is the total additional amount of corrective reactive power required at bus j during a given contingency, researchers obtain a solution which optimizes the capacitive reactive power and adds inductance necessary to obtain a feasible solution., i.e.:

$$\Delta Q_j^{total} = \Delta Q_j^{dispatched} + \Delta Q_j^{notscheduled} \quad (11)$$

Equation 11 is shown graphically in Fig. 3. With this new formulation which includes capacitors as well as inductors, the objective function is modified to become:

$$H = \sum_{j=1}^n C_j Q_j^{total} - \sum_{j=1}^n \epsilon \Delta Q_j^{notscheduled} \quad (12)$$

Subject to the constraints:

$$\Delta Q_j^{total} \geq 0 \quad (13)$$

$$\Delta Q_j^{notscheduled} \geq 0 \quad (14)$$

$$\sum_{j=1}^n X_{ij} (\Delta Q_j^{total} - \Delta Q_j^{notscheduled}) \geq |V_{min}| \quad (15)$$

$$\sum_{j=1}^n X_{ij} (\Delta Q_j^{total} - \Delta Q_j^{notscheduled}) \leq |V_{max}| \quad (16)$$

Note that when  $\Delta Q_j^{total} > \Delta Q_j^{notscheduled}$  capacitive reactive power is dispatched and when  $\Delta Q_j^{total} < \Delta Q_j^{notscheduled}$  inductive power is dispatched.

### Minimization of both capacitive and inductive reactive power:

In order to minimize both the capacitive and inductive reactive power at the same time, the objective function to be minimized is the amount of reactive power (both capacitive and inductive) added into the system, i.e.:

$$H = \sum_{j=1}^m C_j \Delta Q_j \quad (17)$$

Where:

m = The total number of buses where reactive compensation is to be considered

$C_j$  = The weighted factor at bus j

The main advantage of this formulation over others is that here  $\Delta Q_j$  is neither capacitive nor inductive but it is treated purely as a decision variable in the linear programming stage of the solution procedure. To distinguish whether  $\Delta Q_j$  should be added to (i.e.,  $\Delta Q_j$  is capacitive) or subtracted from (i.e.,  $\Delta Q_j$  is inductive), the net reactive power at bus j depends on the voltage level at bus j.

Therefore, after the linear programming stage, an AC load flow is performed and the voltage at bus j is then compared with its prescribed limits. Should it fall below its lower limit  $V_L$  (i.e., a case of under voltage) then  $\Delta Q_j$  is capacitive and its value is therefore added to the net reactive power injected at bus j.

Conversely, if the voltage level exceeds its upper limit  $V_H$  (i.e., a case of over voltage) then  $\Delta Q_j$  is inductive and is subtracted from the net reactive power at bus j. Normally, the voltage constraints are:

$$\sum_{j=1}^m X_{ij} \Delta Q_j \geq |\Delta V_{min}|$$

and:

$$\sum_{j=1}^m X_{ij} \Delta Q_j \leq |\Delta V_{max}|$$

with:

$$\Delta Q_j \geq 0 \text{ for all } j$$

But if a common upper and lower voltage bound is assumed for all busbars then the voltage constraints equation becomes:

$$\sum_{j=1}^m X_{ij} \Delta Q_j \geq |\Delta V_{\min}| \quad (18)$$

$$\Delta Q_j \geq 0 \text{ for all } j \quad (19)$$

Where, for capacitive reactive compensation:

$$\Delta V_{\min} = |V_L - V_{\text{contingency}}|$$

For inductive reactive compensation:

$$\Delta V_{\min} = |V_H - V_{\text{contingency}}|$$

$V_L$  and  $V_H$  are the lower and upper voltage bounds which are assumed common for all busbars.

**Power flow numeric simulation results:** Power flow study for transmission network system was simulated in Power System Analysis Toolbox (PSAT), a matlab based computer power system software using Newton-Raphson Iterative Methods to determine the various voltage levels at the base case load capacity. The base voltage used is 330 kV. The input data used in the simulation of the voltage profile are the various line loadings and generation as obtained from the Power Holding Company of Nigeria (PHCN) which represents the base case for the model in Table 1.

### CONCLUSION

The study presents the application of method of linear programming to optimizing the reactive compensation in a power system network for the purpose of identifying and enhancing voltage profile at the various

buses in the network where voltage profile is out of an acceptable limit. The power flow analysis was done with power system analysis toolbox, a Matlab based computer software which uses the Gauss-Siedel Interactive Technique to determine the normal bus voltage levels of the system. The modeled power system network voltage shown the various bus voltage in the network and also indentifies bus with minimum voltage limit violation at bus 20: Gomb [ $V_{\min} = 0.94$ ]. Reactive power compensation applied at this bus will return the system network voltage within acceptable limit.

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