International Journal of Electrical and Power Engineering 7 (3): 60-64, 2013
ISSN: 1990-7958

Improvement of Lolp Calculation Accuracy with Three-and-More-State Reliability Descriptions

Andras Istvan Fazekas and Eva V. Nagy
[1]Department of Energy Engineering, Budapest University of Technology and Economics,
MVM Hungarian Power Companies Ltd., Budapest, Hungary
[2]Department of Differential Equations,
Budapest University of Technology and Economics, Budapest, Hungary

Abstract: Markov models may be used for three-and-more-state reliability descriptions of power plants. Researchers, demonstrate the potential improvement in system reliability calculation accuracy (such as LOLP) with multi-state power plant reliability descriptions.

Key words: Loss-of-load probability calculations, power generation reliability, power system reliability, reliability modelling of power plant units with extraction condensing and back pressure turbine, power plant unit reliability description

INTRODUCTION

The three-or-more-state reliability description has particular significance for a power plant system with a large number of power plant units cogenerating heat and power. This is typically the case in the formerly socialist countries of Eastern Europe where most power plants deliver heat for heating purposes. This means that power plants have two products: Power for the transmission network and heat (usually in a hot water medium) for district heating systems.

The maximum power output of cogeneration power plants, such as extraction condensing and back pressure power plants, is a function of instantaneous heat output. The maximum power output of an extraction condensation plant decreases and that of a back pressure plant increases with increasing heat output.

Consequently, extraction condensing and back pressure power plants may experience states other than random failure (forced outage), i.e., full loss of capacity because the current heat output, also limits the maximum electric capacity available to the system.

In current practice, the reliability modelling of power plant units which cogenerate heat and power involves a two-state reliability description. This is a considerable oversimplification of the real course of operation and limits the accuracy of system reliability calculations (such as LOLP).

The calculation procedure presented here is novel in its application of a three-and-more-state reliability description to extraction condensing and back pressure power plants. Application of the method leads to a considerable improvement of the accuracy of LOLP calculations.

ESSENTIAL FEATURES OF THE STATE SPACE DESCRIPTION

In the reliability modelling of both power plant units and systems of power plants (or power plant units), the state space description method is one of the most advanced calculation procedures in current use (Liu and Singh, 2010).

The state-space description method characterises a power plant unit by its defined operating states and the probability of being in each operating state (Ganor and Zahavi, 1989; Galloway *et al*., 1969; Dehghani and Nikravesh, 2008; Endrenyi, 1978).

For the purpose of the reliability description, the operation of a power plant unit may be regarded as known if the probability of the power plant unit being in each defined operating state may clearly be defined for any time (or period of time). This means that the distribution of probabilities of being in different defined operating states are known for each time (or period) (Billinton and Allan, 1984).

Typical defined operating states of power plant units are operating at full capacity, unavailable (failed), operating at derated capacity, in reserve and failed reserve (Billinton and Allan, 1992). An unambiguous description

Corresponding Author: Andras Istvan Fazekas, Department of Energy Engineering, Budapest University of Technology and Economics, MVM Hungarian Power Companies Ltd., Budapest, Hungary

requires precisely-set criteria for each operating state, so that the power plant in the model can definitely be classed in one of the pre-defined possible operating states in every case. Which operating states should be defined in a specific case is decided by the specific aim of the study, the function of the power plant units making up the system, the time horizon of the calculations and the availability and differentiation of the statistical database describing the reliability behaviour of the power plant units being modelled.

A reliability description of power plant units based on probability theory, therefore determines the distribution of the probability of being in each defined operating state at any time.

To formalise the mentioned earliar, let us defined each operating state by $U_1, U_2, ..., U_m$ and the probability of being in each operating state at time t by p_1 (t), p_2 (t), ..., p_m (t). Since this is a complete event system, the following relation applies for any time t:

$$\sum_{i=1}^{i=m} p_i(t) = 1 \quad (1)$$

The distribution of probabilities of being in the various operating states is given by the row vector:

$$\underline{p}(t) = [p_1(t), p_2(t), ..., p_m(t)] = [p_i(t)] \quad (2)$$

If, we know the initial instantaneous probability distribution (i.e., the row vector $\underline{p}(t=0)$, the question is how to determine the probability distributions for being in each operating state at any other time.

This question may be answered using Markov Models. There is insufficient space to discuss the theory of Markov Models here and we give only the relations which are most important for this study.

Reliability descriptions of power plant units always use discrete state Markov Models, since power plant operating states are always discrete (and of finite number). There is a difference in calculation procedures, however depending on whether the reliability description uses discrete time-parameter Markov chains or continuous time-parameter Markov processes. A discrete state space and discrete time-parameter Markov Model, i.e., a Markov chain, involves a time variable with discrete values and if the time parameter can take any value, we have a discrete state-space, continuous time-parameter Markov chain.

Researchers will now look at an application of discrete state-space, continuous time-parameter Markov processes. In general, a stochastic process may be regarded as a Markov process if the following equation holds (the Markov property):

$$P\Big[\big(U(t_n) = u_n\big)\Big|\big(U(t_{n-1}) = u_{n-1}\big), ..., \big(U(t_2) = u_2\big), \big(U(t_1) = u_1\big)\Big] = P\Big[\big(U(t_n) = u_n\big)\Big|\big(U(t_{n-1}) = u_{n-1}\big)\Big] \quad (3)$$

Where:

$U(t_i)$ = The state variable at time (the possible values of state variable $U(t_i)$ are the defined operating states $U_1, U_2, ..., U_m$)

u_i = The value of state variable $U(t_i)$ at time, i.e., the operating state occupied at time t_i, one of the operating states ($U_1, U_2, ..., U_m$)

The mentioned earliar relation states that a system state $U(t_n) = u_n$ at time t_n depends only on the operating state at time t_{n-1} immediately prior to time t_n and so does not depend on the operating states at the previous times of the stochastic process ($t_0, t_1, t_2, ..., t_{n-2}$) (Armstadter, 1971).

Specific calculations crucially require knowledge of the state transition probabilities. These are defined by the following relation:

$$\pi_{ij}(t, t+\Delta t) = P\big[U(t+\Delta t) = U_j \big| U(t) = U_i\big] \quad (4)$$

It is implicitly assumed in practical calculations that the value of π_{ij} (t, t+Δt) does not depend on t, only Δt. Such Markov processes are called homogenous Markov processes. Otherwise the mathematical apparatus becomes very complicated. The matrix giving every possible case of the state transition probabilities is a stochastic matrix:

$$\underline{\underline{\Theta}} = \begin{bmatrix} \pi_{11} & \pi_{12} & \cdots & \pi_{1m} \\ \pi_{21} & \pi_{22} & \cdots & \pi_{2m} \\ \cdots & \cdots & \cdots & \vdots \\ \pi_{m1} & \pi_{m1} & \vdots & \pi_{mm} \end{bmatrix} \quad (5)$$

i.e., a matrix whose every row adds up to 1 or:

$$\sum_{i=1}^{i=m} \pi_{ij} = 1 \quad (6)$$

The determination of this matrix is central to the practical calculations. If sufficient historical data is available to determine its elements, then the power plant unit may be described using a Markov Model. In the vast majority of cases, the number of defined operating states varies between three and five because in practice, the data required to define state transition probabilities for more operating states is not available.

Knowing the elements of the state transition matrix, we can define two very important non-negative, continuous functions:

$$a_j(t) = \lim_{\Delta t \to 0} \frac{\pi_{jj}(t,0) - \pi_{jj}(t, t+\Delta t)}{\Delta t} = \lim_{\Delta t \to 0} \frac{1 - \pi_{jj}(t, \Delta t)}{\Delta t} \quad (7)$$

And:

$$a_{ji}(t) = \lim_{\Delta t \to 0} \frac{\pi_{ji}(t,0) - \pi_{ji}(t+\Delta t)}{-\Delta t} = \lim_{\Delta t \to 0} \frac{\pi_{ji}(t+\Delta t)}{\Delta t} \quad (8)$$

The function defined in Eq. 8 denotes the transition intensity from some state U_j to another state U_i at an arbitrary time t. In the case of Markov processes, the transition intensities do not depend on the time t; they are constant. It follows that:

$$\pi_{jj}(\Delta t) + \pi_{\substack{ji \\ i \neq j}}(\Delta t) = 1 \quad (9)$$

And:

$$\begin{aligned} a_{jj} = -a_j &= \lim_{\Delta t \to 0} - \frac{\left(1 - \pi_{jj}(t, t+\Delta t)\right)}{\Delta t} \\ &= \lim_{\Delta t \to 0} \frac{1 - \left(1 - \sum_{i \neq j}^{m} \pi_{ji}(\Delta t)\right)}{\Delta t} \\ &= \lim_{\Delta t \to 0} - \frac{1}{\Delta t}\left(\sum_{i \neq j}^{m} \pi_{ji}(\Delta t)\right) = -\sum_{i \neq j}^{m} a_{ji} \end{aligned} \quad (10)$$

The state transition intensities may be used to determine the transition intensity matrix:

$$\underline{\underline{A}} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \cdots & \cdots & \cdots & \vdots \\ a_{m1} & a_{m1} & \vdots & a_{mm} \end{bmatrix} \quad (11)$$

In the transition intensity matrix, $a_{jj} = -a_j$. It is true for elements of the transition intensity matrix that:

$$\sum_{j=1}^{j=m} a_{ij} = 0, \ (1 \leq i \leq m) \quad (12)$$

The transition probabilities and transition intensities satisfy the following relations:

$$a_{ij}\Delta t = \pi_{ij}, a_{ji}\Delta t = \pi_{ji}, a_{jj}\Delta t = \pi_{jj} \quad (13)$$

Taking these together, we can write the relation:

$$p_j(t+\Delta t) = p_j(t)\left[1 - \sum_{i \neq j}^{m} a_{ji}\Delta t\right] + \sum_{i \neq j}^{m} p_i(t) a_{ij}\Delta t \quad (14)$$

Without giving the detailed derivation, this may be transformed to yield a system of differential equations which gives the absolute probability of being in any defined state U_j (j = 1, 2, ..., m) at any time t:

$$\frac{dp_j(t)}{dt} = \sum_{i=1}^{m} p_i(t) a_{ij}, \ j = 1,2,\ldots,m \quad (15)$$

It is assumed that the initial distribution (t = 0) is:

$$p_j(t) = \alpha_j(t), \ j = 1,2,\ldots,m \quad (16)$$

Equation 15 may be written in the form:

$$\frac{d\underline{p}(t)}{dt} = \underline{p}(t) * \underline{\underline{A}}(\Delta t) \quad (17)$$

When the transition probabilities and transition intensities are known, it is possible to write the matrix differential Eq. 17 which may be solved in the way known from linear algebra (Roberts, 1964; Verma *et al*., 2010).

For the calculation of LOLP probabilities, it is important to determine the long-term state probabilities (Billinton, 1982), i.e., the value of vector:

$$\underline{p}(t) = [p_1(t), p_2(t), \ldots, p_m(t)] = [p_i(t)] \quad (18)$$

In the case that $t \to \infty$.

System configuration calculations: Where there are a finite number of system elements and a finite number of possible operating states, the system has a finite, determined number of total system states. For a specific power plant unit, the set of all possible operating states is called the state space (Cepin, 2011). For a power plant system, the set containing all possible system configuration is called the configuration space. The state space and configuration space can in principle be expressed graphically with state space diagrams of a complexity depending on the level of differentiation of the reliability description and the number of system elements and possible operating states. Where there are a large number of system elements and operating states, the diagram becomes awkward and unclear.

ADVANTAGES OF A MORE DIFFERENTIATED RELIABILITY DESCRIPTION OF POWER PLANT UNITS

A simple example can convey how much more differentiated the reliability description becomes for both power plant units and the power plant system through the use of a three-or-more-state reliability description.

We will consider an illustrative example which for the sake of clarity, consists of a system of only two power plant units. Table 1 and 2 give the main characteristics of the two power plant units of the example (probability of being in each operating state, power capacity when in each operating state) for a two-state and a four-state reliability description.

For the two-state reliability description, two operating states are defined for each power plant unit (operating at full capacity and unavailable). The four-state reliability description takes account of the reduced capacity which the power plant unit has available for the power system because of the variation in heat output. The power plant units in the example are extraction condensation units.

Table 3 shows the discrete probability distribution of the electric capacity available to the system from the two power plant units in the example for the two-and-four-state reliability descriptions.

Assuming that the power demand has the same distribution over the year in the two cases, the LOLP may be determined for the two and four-state reliability descriptions. The equation which defines LOLP is (Hall *et al*., 1968):

$$\vartheta = \sum_j P(C = C_j) P(L > C_j) \quad (19)$$

Table 1: Power plant unit characteristics in the two-state reliability description

Operating state	X1	X2
Unit U1		
Electric power capacity (MW)	220	0
Probability of being in operating state (-)	0,94	0,06
Unit U2		
Electric power capacity (MW)	400	0
Probability of being in operating state (-)	0,95	0,05

Table 2: Power plant unit characteristics in the four-state reliability description

Operating state	X1	X2	X3	X4
Unit U1				
Electric power capacity (MW)	220	180	140	0
Probability of being in operating state (-)	0,45	0,24	0,25	0,06
Unit U2				
Electric power capacity (MW)	400	350	290	0
Probability of being in operating state (-)	0,41	0,18	0,36	0,05

Where:

C (t) = The available electric capacity of the power plant system at time (MW)

C_j = A specific value of the available electric capacity of the power plant system (MW)

L (t) = The system load at time (MW)

The LOLP values for the example are given in Table 4. The four-state reliability description is clearly a more differentiated description of the real operating states and so allows the LOLP to be determined with greater accuracy. It follows without, further explanation that the second case gives a much more accurate figure for the probability distribution of the electric capacity available to the system and hence, a more accurate figure for LOLP.

Variation of LOLP value/result of comparative studies: The vast majority of electric power plant units in Hungary, also generate heat which they deliver to district heating systems they are connected to. That is what has created the need for a more differentiated (four-state) model of power plant unit reliability.

Comparative studies have been carried out on a specific power plant system to show the extent and sign of change in the LOLP values obtained from a four-state, rather than the hitherto-customary two-state reliability description of the power plant units in the system. The calculations involved a large volume of input data and without going into the details, the result is that the LOLP value given by the four-state reliability description is 29% higher than that give by the two-state description.

Table 3: Discrete probability distribution of power plant system electric capacity

Two-state reliability description		Four-state reliability description	
Electric capacity of power plant system (MW)	Probability of occurrence (-)	Electric capacity of power plant system (MW)	Probability of occurrence (-)
0	0,00300000	0	0,00300000
140		140	0,01250000
180		180	0,01200000
220	0,04700000	220	0,02250000
290		290	0,02160000
350		350	0,01080000
400	0,05700000	400	0,02460000
430		430	0,09000000
470		470	0,08640000
490		490	0,04500000
510		510	0,16200000
530		530	0,04320000
540		540	0,10250000
570		570	0,08100000
580		580	0,09840000
620	0,89300000	620	0,18450000

Table 4: LOLP value for two-and-four-state reliability descriptions

Two-state reliability description	Four-state reliability description
LOLP value	
0,085076923	0,228238462

DISCUSSION OF CALCULATION RESULTS

The calculation results have confirmed our former results (Fazekas and Nagy, 2010, 2011). However, it is important to emphasize the following repeatedly.

The earliar mentioned improvement in accuracy applies only to one specific calculation configuration. There remains the fundamental question of how much the newly-developed calculation procedure improves the accuracy of calculations in general. Owing to the nature of the problem, the accuracy improvement in any particular improvement depends on many factors but the main ones are:

- The number of power plant units modelled using the more differentiated reliability description relative to the number in the whole power plant system
- The proportion of the installed power capacity of the power plant system contributed by power plant units modelled using the more differentiated reliability description
- The reliability characteristics of the power plant units modelled using the more differentiated reliability description
- The time scale of the calculation

CONCLUSION

Further experiences have found improvements of between 10 and 30%. The most striking accuracy improvements have been found where there were large temperature changes during the period studied.

The new computation procedure is applicable to the three-and-more-state reliability description of extraction condensing and back-pressure power plant units where the heat output is predominantly generated for heating purposes, i.e., heat output is proportional to daily average ambient temperature. The reason for such a strict constraint on the area of application is that the probability distribution of maximum available power capacity for such power plant units may be derived from the probability distribution of the daily average ambient temperature. This is because the three-and-more-state reliability description assumes a knowledge of the probability of occupation of each defined operating state. This also, means that the new computation procedure is applicable in all cases when there is a means for determining the probability distribution of the defined operating states.

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