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Modeling and Simulation of a Horizontally Moving Suspended Mass Pendulum Base using $H\infty$ Optimal Loop Shaping Controller with First and Second Order Desired Loop Shaping Functions

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Abstract: In this study, a horizontally moving suspended mass pendulum base is designed and controlled using robust control theory. $H\infty$ optimal loop shaping with first and second order desired loop shaping function controllers are used to improve the performance of the system using MATLAB/Simulink Toolbox. Comparison of the $H\infty$ optimal loop shaping with first and second order desired loop shaping function controllers for the proposed system have been done to track the desired angular position of the pendulum using step and sine wave input signals and a promising result has been obtained successfully.

INTRODUCTION

A pendulum with suspended mass is a system that has a mass suspended in its base and a mass suspended from a pivot, so that, it can swing freely. When the suspended mass in the base of the pendulum forced to move horizontally by applying a force, the pendulum become displaced sideways from its resting, equilibrium position, then the pendulum will be subjected to a restoring force due to gravity that will accelerate it back toward the equilibrium position[1].

The restoring force acting on the pendulum's mass causes it to oscillate about the equilibrium position, swinging back and forth. The time for one complete cycle, a left swing and a right swing is called the period. The period depends on the length of the pendulum and also to a slight degree on the amplitude, the width of the pendulum's swing.

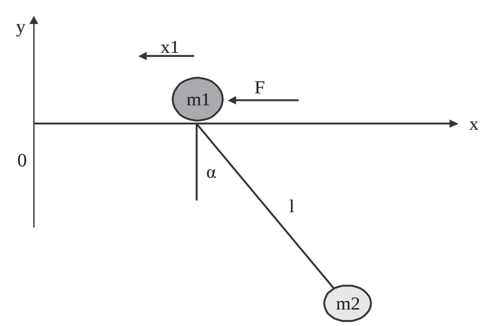

Fig. 1: Pendulum with suspended mass

MATERIALS AND METHODS

Mathematical modeling of the system: A system consists of two point masses, m 1 and m2, connected with a weightless rigid rod of length l (Fig. 1)[2]. The motion occurs in a gravity field and is considered to be in a plane, i.e., is considered in the coordinates x, y, t. The location of point of a mass m1 (suspension) is not fixed and can move along the axis x. The mathematical model of the system will be as follow:

The four functions of time of the system are x1(t), y1(t), x2 (t), y2(t), i.e., the Cartesian coordinates of the first and second points. The suspension cannot move vertically Y1 = 0. While the second is described by equation:

$$(x_1 - x_2)^2 + y_2^2 = l^2$$

We choose the generalized coordinates as $q_1(t) = x_1(t)$ and $q_2(t) = \alpha(t)$ where α is the angle between the vertical is and the axis of rod:

$$x_1 = q_1, x_2 = q_1 + l\sin q_2, y_2 = -l\cos q_2$$

The kinetic energy of system T = T1+T2 in coordinates q1, q2. For the suspension we have:

$$T_1 = \frac{m_1 v_1^2}{2} = \frac{m_1 v_{1x}^2}{2} = \frac{m_1 \dot{x}_1^2}{2} \qquad (1)$$

For the pendulum we obtain:

$$T_2 = \frac{m_2 v_2^2}{2} = \frac{m_2}{2}\left(v_{2x}^2 + v_{2y}^2\right) \qquad (2)$$

V2x and V2y simply simplified as:

$$v_{2x} = \dot{x}_1 + l\dot{\alpha}\cos\alpha$$
$$v_{2y} = l\dot{\alpha}\sin\alpha$$

Rewrite T2 as function of α and x1:

$$T_2 = \frac{m_2 \dot{x}_1^2}{2} + \frac{m_2}{2}\left(2l\dot{\alpha}\dot{x}_1\cos\alpha + l^2\dot{\alpha}^2\right) \qquad (3)$$

For the force of gravity F2 acting on the pendulum. For its projection we have:

$$F_{gx} = 0$$
$$F_{gy} = -m_2 g = -m_2 \frac{\partial \prod}{\partial y_2}$$

where, $\Pi(y_2) = m_2 g y_2$ is the potential energy of the pendulum. In coordinates q_1, q_2 $\Pi(y_2)$ is expressed by equation:

$$V(q_2) = -m_2 l g \cos\alpha \qquad (4)$$

In so far as the considered motion is potential, it is necessary to use the Lagrangian equations:

$$L = T - V = T_1 + T_2 - V$$

Or:

$$L = \frac{m_1 + m_2}{2}\dot{x}_1^2 + \frac{m_2 l}{2}\left(l\dot{\alpha}^2 + 2\dot{x}_1\dot{\alpha}\cos\alpha\right) + m_2 l g\cos\alpha \qquad (5)$$

Differentiating L by $q_1, \dot{q}_1, q_2, \dot{q}_2$ (recall that $q_1 = x_1, q_2 = \alpha$) we obtain:

$$\frac{\partial L}{\partial q_1} = 0,\ \frac{\partial L}{\partial \dot{q}_1} = (m_1 + m_2)\dot{x}_1 + m_2 l\dot{\alpha}\cos\alpha$$
$$\frac{\partial L}{\partial q_2} = -m_2 l\sin\alpha(\dot{x}_1\dot{\alpha} + g),\ \frac{\partial L}{\partial \dot{q}_2} = m_2 l(l\dot{\alpha} + \dot{x}_1\cos\alpha)$$

Then:

$$\frac{d}{dx}\left(\frac{\partial L}{\partial \dot{q}_1} - \frac{\partial L}{\partial q_1}\right) = F$$
$$\frac{d}{dx}\left(\frac{\partial L}{\partial \dot{q}_2} - \frac{\partial L}{\partial q_2}\right) = 0$$

Substituting the obtained expressions into Lagrangian equations and differentiating them by t, we come to two equations with respect to x_1 and α:

$$(m_1 + m_2)\ddot{x}_1 + m_2 l\ddot{\alpha}\cos\alpha - m_2 l\dot{\alpha}^2\sin\alpha = F \qquad (6)$$

$$\ddot{x}_1\cos\alpha + l\ddot{\alpha} + g\sin\alpha = 0 \qquad (7)$$

Linearizing the above equation as:

- cos α = 1
- sin α = α
- $\dot{\alpha}^2 = 0$

After linearization, Eq. 6 and 7 becomes:

$$(m_1 + m_2)\ddot{x}_1 + m_2 l\ddot{\alpha} = F \qquad (8)$$

$$\ddot{x}_1 + l\ddot{\alpha} + g\alpha = 0 \qquad (9)$$

Let:

$$z_1 = x_1,\ z_2 = \dot{x}_1, z_3 = \alpha, z_4 = \dot{\alpha}, z_5 = \ddot{\alpha}$$

Table 1: Parameters of the system

Parameters	Symbols	Values
Suspended mass	m_1	0.4 kg
Pendulum mass	m_2	0.2 kg
Rod length	l	0.4 m
Gravitational constant	g	10 m/sec^2

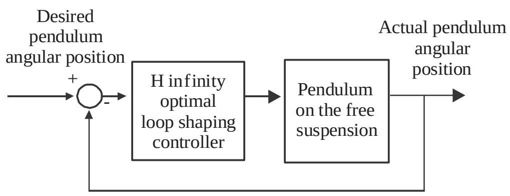

Fig. 2: Block diagram of the pendulum on the free suspension with H∞ optimal loop shaping controller

The state space representation of the system becomes:

$$\dot{z} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{m_2 l}{(m_1+m_2)} \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{1}{(m_1+m_2)} \\ 0 \\ 0 \\ 0 \end{pmatrix} F$$

$$y = (0\,0\,1\,0\,0)z$$

The parameters of the system are shown in Table 1: The state space representation of the system then becomes:

$$\dot{z} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.33 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1.67 \\ 0 \\ 0 \\ 0 \end{pmatrix} F$$

$$y = (0\,0\,1\,0\,0)z$$

Proposed controllers design

H∞ optimal loop shaping control: H∞ Optimal Loop Shaping Control computes a stabilizing H_∞ controller K for plant G to shape the sigma plot of the loop transfer function GK to have desired loop shape G_d with accuracy γ = GAM in the sense that if ω_0 is the 0 db crossover frequency of the sigma plot of $G_d(j\omega)$, then, roughly:

$$\underline{\sigma}(G(j\omega)K(j\omega)) \geq \frac{1}{\gamma}\underline{\sigma}(G_d(j\omega)) \text{ for all } \omega > \omega_0$$

$$\underline{\sigma}(G(j\omega)K(j\omega)) \geq \gamma\,\underline{\sigma}(G_d(j\omega)) \text{ for all } \omega > \omega_0$$

A MIMO stable min-phase shaping pre-filter W, the shaped plant G_s = GW, the controller for the shaped plant K_s = WK as well as the frequency range $\{\omega_{min}, \omega_{max}\}$ over which the loop shaping is achieved. The block diagram of the pendulum on the free suspension with H Optimal Loop Shaping Controller is shown in Fig. 2.

In this study, the plant has been desired loop shaped with a first order and second order system. For the first order, the desired loop shaping function is:

$$G_{d1} = \frac{1}{s+1}$$

And the H∞ optimal loop shaping controller becomes:

$$K_1(s) = \frac{5.369s^5 + 4.409s^4 + 9.094s^3 + 1.764s^2 + 1.399s + 3.415}{s^6 + 1.639s^5 + 1.007s^4 + 2.751s^3 + 2.820s^2 + 2.818s + 6.955}$$

For the second order, the desired loop shaping function is:

$$G_{d2} = \frac{1}{s^2 + 2s + 6}$$

And the H∞ optimal loop shaping controller becomes:

$$K_2(s) = \frac{7.314s^7 + 3.221s^6 + 2.645s^5 + 5.459s^4 + 1.123s^3 + 2.353s^2 + 6.077s + 1.222}{s^8 + 1.639s^7 + 1.007s^6 + 2.753s^5 + 2.826s^4 + 1.153s^3 + 2.867s^2 + 3.523s + 8.939}$$

RESULTS AND DISCUSSION

Here, in this study, the investigation of the open loop response and the closed loop response with the proposed controller have been done[3, 4]. Finally, the comparison of the system with the proposed controllers for a first order and a second order desired loop shaping design have been done.

Open loop response of the pendulum: The open loop response of the system for a 0.1 Newton suspended mass force simulation is shown in Fig. 3. The pendulum angular position angle increases for the 0.1 N input.

Comparison of the step response of pendulum with suspended mass using H∞ optimal loop shaping controller with first and second order desired loop shaping function controllers: The simulation result of the step response of pendulum with suspended mass using H∞ optimal loop shaping controller with first and second order desired loop shaping function is shown in Fig. 4. The data of the rise time, percentage overshoot, settling time and peak value is shown in Table 2.

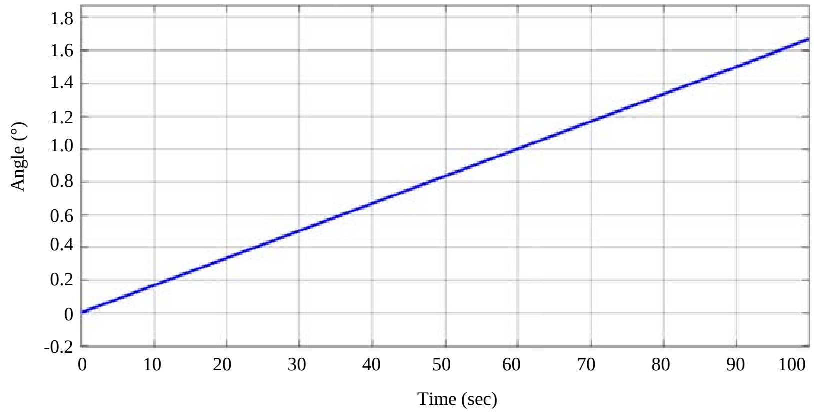

Fig. 3: Open loop response; Open loop pendulum angular position to suspended mass force of 0.1 N

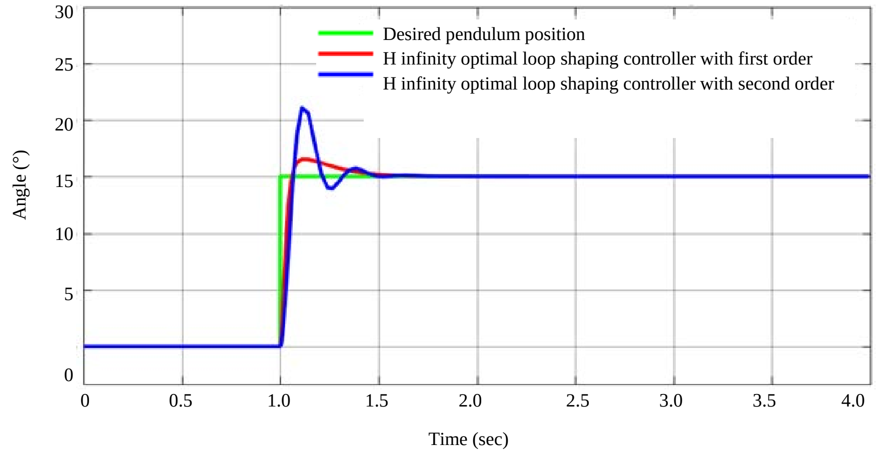

Fig. 4: Step response; Actual pendulum angular position response to desired pendulum angular position input

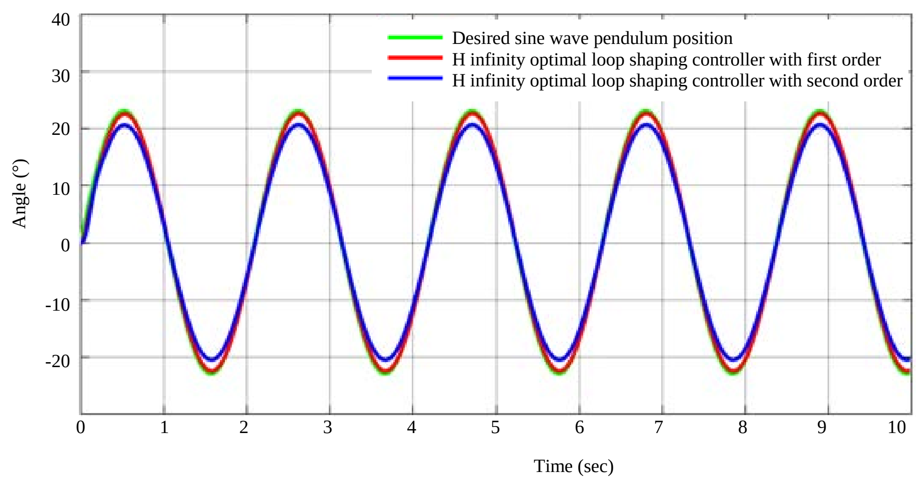

Fig. 5: Sine wave response; Actual pendulum angular position response to desired pendulum angular position input

As Table 2 shows that the pendulum with suspended mass using H∞ optimal loop shaping controller with first order desired loop shaping function controller improves the performance of the system by minimizing the rise time, percentage overshoot and settling time (Fig. 5).

Table 2: Step response datas

Performance data	First order	Second order
Rise time	1.05 sec	1.12 sec
Per. overshoot	13.3 (%)	40 (%)
Settling time	1.38 sec	1.45 sec
Peak value	17°	21°

Comparison of the sine wave response of pendulum with suspended mass using h∞ optimal loop shaping controller with first and second order desired loop shaping function controllers: The simulation result of the sine wave response of pendulum with suspended mass using H∞ optimal loop shaping controller with first and second order desired loop shaping function is shown in Fig. 5.

As Fig. 5 shows that the pendulum with suspended mass using H∞ optimal loop shaping controller with first order desired loop shaping function controller improves the performance of tracking the set point input to the system[5].

CONCLUSION

In this study, the design and simulation of a horizontally moving suspended mass pendulum base is done using H∞ optimal loop shaping with first and second order desired loop shaping function controllers. Comparison of the proposed system with H∞ optimal loop shaping with first and second order desired loop shaping function controllers have been done to track the desired angular position of the pendulum using step and sine wave input signals. The step input signal response shows that the pendulum with suspended mass using H∞ optimal loop shaping controller with first order desired loop shaping function controller improves the performance of the system by minimizing the rise time, percentage overshoot and settling time while the sine wave input signal response shows that the pendulum with suspended mass using H∞ optimal loop shaping controller with first order desired loop shaping function controller improves the performance of tracking the set point input to the system. Finally, the simulation comparison results prove that the system with H∞ optimal loop shaping controller with first order desired loop shaping function controller improved the system performance better.

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