International Journal of Soft Computing 1(2) : 118-120, 2006

The Classes of Left and Right Sigmoidal Signals for Feed Forward Neural Networks

[1]M. Ramakrishnan, [1]C.M. Velu, [2]K. Ekambavanan, [3]P. Thangavelu and [2]P. Vivekanandan
[1]Department of Computer Science and Engineering, Vel. Tech. Engineering College, Avadi, Chennai-600062, India
[2]Department of Mathematics, Anna University, Chennai-600025, India
[3]Department of Mathematics, Aditanar College, Tiruchendur-628216, India

Abstract: The authors recently introduced left sigmoidal signals and right sigmoidal signals to prove certain function approximation theorems for Feed Forward Neural Networks. In this study we identify a class LS (resp. RS.) of left sigmoidal (resp.Right sigmoidal) signals having the property that the envelope of the derivatives of members of a class LS (resp. RS) is a left sigmoidal (resp.Right sigmoidal) signal.

Key words: Feed forward networks, activation functions, sigmoidal signals, approximation

INTRODUCTION

Sigmoidal signals are widely used in function approximations and universal function approximations for feed forward neural networks with one or more hidden layers. Yogesh Sing and Pravin Chandra[1] gave a class of sigmoidal functions used to prove the universal function approximation theorems, such that the envelope of the derivatives of the members of the class is again a sigmoidal function. In this study we introduce the two classes of sigmoidal functions that are used to prove certain function approximation theorems in neural networks, having the property the envelope of the derivatives of the members of the class is again a corresponding sigmoidal function.

PRELIMINARIES

Definition 1: A function σ: R → R is called a generalized sigmoidal function[2], if

$$\lim_{x\to-\infty} \sigma(x) = 0 \quad \text{and} \quad \lim_{x\to+\infty} \sigma(x) = 1.$$

Definition 2: A function σ: R → R is said to be a generalized left sigmoidal[3], if

$$\lim_{x\to-\infty} \sigma(x) = 0.$$

Definition 3: A function σ: R→R is said to be a generalized right sigmoidal[3], if

$$\lim_{x\to+\infty} \sigma(x) = 1.$$

SIGMOIDAL CLASS

Let LS denote a class of generalized left sigmoidal functions $l_m(x)$ that are defined as

$$l_m(x) = \begin{cases} \left(1+e^{e^{-x}}\right)^{-m}, & x < 0 \\ 3^{-m}, & x \geq 0 \end{cases}$$

where $m \in (0,\infty)$.

And let RS denote a class of generalized right sigmoidal functions $r_m(x)$ that are defined as

$$r_m(x) = \begin{cases} \left(1-e^{e^{-x}}\right)^{-m}, & x \geq 0 \\ 2^{-m}, & x < 0 \end{cases}$$

where $m \in (0,\infty)$.

Proposition 1: Every member of the class LS is monotonically increasing.

Proof:

$$(*)\ \frac{dl_m(x)}{dx} = \begin{cases} m\left(1+e^{e^{-x}}\right)^{-m-1} e^{e^{-x}} e^{-x}, & \text{for } x < 0 \\ 0, & x \geq 0 \end{cases}$$

where $m \in (0,\infty)$.

Corresponding Author: M. Ramakrishnan, Department of Computer Science and Engineering, Vel Tech Engineering College, Avadi, Chennai- 600062, India

This proves that

$$\frac{dl_m(x)}{dx} \geq 0$$

for all real x and hence the theorem.

Proposition 2: Every member of the class RS is monotonically decreasing.

Proof:

$$(**)\ \frac{dr_m(x)}{dx} = \begin{cases} -m(1-e^{-e^x})^{-m-1}e - e^x, & x \geq 0 \\ 0 & x < 0 \end{cases}$$

where m ∈ (0,∞).

This proves that $r'_m(x) \leq 0$ for all real x and hence $r_m(x)$ is decreasing.

Proposition 3: Every member $l_m(x)$ of the class LS is bounded below by 0.

Proof: $l_m(x)>0$ for all real x and therefore $l_m(x)$ is bounded below by 0.

Proposition 4: Every member $r_m(x)$ of the class RS is bounded above by 1.

Proof: $r_m(x)<1$ for all real x and therefore $r_m(x)$ is bounded above by 1.

Theorem 5: The envelope of the derivative curves of the members of LS is a left sigmoidal signal.

Proof:

Let L(x, m) = $\frac{dl_m(x)}{dx}$. Therefore by (*)

$$L(x,m) = \begin{cases} m\left(1+e^{e^{-x}}\right)^{-m-1} e^{e^{-x}}, e^{-x}, & \text{for } x < 0 \\ 0 & , \quad x \geq 0 \end{cases}$$

where m ∈ (0,∞).

$$\frac{dL}{dm} = \left(1+e^{e^{-x}}\right)^{-m-1} e^{e^{-x}} e^{-x} - m\left(1+e^{e^{-x}}\right)^{-m-1} e^{e^{-x}} e^{-x} \log(1+e^{e^{-x}}),$$

for x<0 and = 0 for x≥0.

If $\frac{dL}{dm}$ = 0 then

$$m = \left(\log\left(1+e^{e^{-x}}\right)\right)^{-1}$$

Using the value of m in L(x, m) we take

$$y(x) = [e\log(1+e^{e^{-x}})]^{-1} \frac{e^{-x}}{1+e^{-e^{-x}}}.$$

It is easy to prove that

$$\lim_{x\to-\infty} y(x) = e^{-1}$$

Then h(x) = y(x)-e^{-1} is a generalized left sigmoidal signal. Thus the envelope of the derivative curves of members of LS is a generalized left sigmoidal signal that can be used for the function approximation by feed forward neural networks.

$e^{-e^x}(1-e^{-e^x})^{-m-1}$

Theorem 6: The envelope of the derivative curves of the members of RS is a right sigmoidal signal.

Proof:

$$\text{Let } R(x,m) = \frac{dr_m(x)}{dx}$$

by using (**)

R(x, m) = -m e^x for x≥0 and = 0 for x<0.

$$\frac{dR}{dm} = -e^x e^{-e^x}(1-e^{-e^x})^{-m-1} + me^x\ e^{-e^x} \left[\log(1-e^{-e^x})(1-e^{-e^x})^{-m-1}\right]$$

If $\frac{dR}{dm} = 0$ then $m = \frac{1}{\log(1-e^{-e^x})}$

Using the value of m in R(x, m) we take

$$y(x) = -[e\log(1-e^{-e^x})]^{-1}\left(\frac{e^x\ e^{-e^x}}{1-e^{-e^x}}\right)$$

It is easy prove that

$$\lim_{x\to\infty} y(x) = -\frac{1}{e}$$

Let

h(x) = -e y(x)

Thus the envelope of the derivative curves of members of RS is a right sigmoidal signal that can be used for the function approximation by feed forward neural networks.

CONCLUSION

Thus we have introduced two classes of generalized left and right sigmoidal signals that have been the function approximators[3]. The important property that has been proved here is that the envelope of the derivatives of the members of the class is a sigmoidal signal that will be used as a activation function for feed forward neural networks with one hidden layer.

REFERENCES

1. Sing, Y. and C. Pravin, 2003. A class+1 sigmoidal activation functions for FFANNs, J. Econom. Dyna. Con., 28: 183-187.
2. Chen T., H. Chen and R.W. Liu, 1995. Approximation capability in $C(\bar{R}^n)$ by multiplayer feed forward networks and related problems. IEEE Transactions on Neural Networks, 6: 25-35.
3. Ramakrishnan, M., C.M. Velu, K. Ekambavanan, N. Prabakaran, P. Thangavelu and P. Vivekanandan, 2006. Function Approximation Using Feed Forward Neural Networks with sigmoidal signals, Intl. J. Soft Comput., 1: 76-82, 2006.