International Journal of Soft Computing 1(3): 199-206, 2006
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A Deterministic Inventory Model for Deteriorating Items with Partially Backlogged and Stock and Time Dependent Demand Under Trade Credit

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Abstract: This study discusses a deterministic EOQ model under conditions of permissible delay in payments. In many inventory systems suppliers offer some fixed credit periods to retailers for settling the accounts for the goods. This study explores a deterministic inventory model for deteriorating items with stock and time dependent demand for which the supplier permits a fixed trade credit period. In this study we assume that excess demand is partially backlogged. Finally numerical examples are given to elucidate the model.

Key words: Inventory, stock and time dependent demand, deteriorating items, trade credit period

INTRODUCTION

Keeping an inventory for future sale or use is very common in business. Retail firms, wholesalers, manufacturing companies and even blood banks generally have a stock of goods on hand. Usually the demand rate is decided by the amount of stock level. The motivational effect on the people may be caused by the presence of stock at times. Large quantities of goods displayed in markets according to seasons lure the customers to buy more. If the stock is insufficient the customers may prefer some other brands, as a result the shortage will fetch loss to the producer. There are two trade options in markets. 1. The supplier gets his amount immediately after delivering the goods. 2. The supplier he himself gives some credit period to the retailer either to market his products or to boost the trade. Some goods are deteriorating in due course of time.

More researchers are attracted by stock-dependent demand rate patterns. Silver and Peterson[1] noted that sales at the retail level tend to be proportional to the stock level. Gupta and Vrat[2] assumed that the demand rate was a function of initial stock level. Mandal and Phaujdar[3] developed a production inventory model for deteriorating items with uniform rate of production and linearly stock dependent demand. Backer and Urban[4], Datta and Pal[5] and Goh[6] concentrated on the situation that defined the demand rate depends on the instantaneous stock level. Many researchers in this area may refer to Padmanabhan and Vrat[7] Ray and Chaudhuri[8], Sarker *et al*.[9], Giri and Chaudhuri[10] and Mandal and Maiti[11].

In real life, suppliers offer some credit periods to the buyers to stimulate the demand. Goyal[12] developed an EOQ model under conditions of permissible delay in payments. Chung[13] presented the DCF (Discounted Cash Flow) approach for the analysis of the optimal policy in the presence of trade credit. Shinn *et al*.[14] extended Goyal's[12] model and considered quantity discounts for freight cost. Aggarwal and Jaggi[15] and Hwang and Shinn[16] extended Goyal's[12] model to consider the deterministic inventory model with a constant deterioration rate. Shah and Shah[17] developed a probabilistic inventory model when delay in payment is permissible. They developed an EOQ model for deteriorating items in which time and deterioration of units are treated as continuous variables and demand is a random variable. Excess demand also decide suppliers success. Jamal *et al*.[18] extended Aggarwal and Jaggi's[15] model to allow for shortages to make it more suitable in real life.

Many researchers consider the situation in which shortages are either completely backlogged or completely lost. Wee[19] and Yan and Cheng[20] assume that a part of the demand will be lost and the remaining portion is backlogged. Padmanabhan and Vrat[7] developed an EOQ model for perishable items with stock dependent demand in which they considered that the stock out period demand linearly depends on the stock. The acceptance of the backlogged demand depends on the waiting time. Hence the waiting time for the next replenishment plays a vital role in this acceptance. Chang and Dye[21] developed a model for deteriorating items with time varying demand and shortages in which the backlogging rate is assumed to be inversely proportional to the waiting time for the next replenishment.

Chung-Yuan Dye[22] developed an inventory model for deteriorating items with stock dependent demand and partial backlogging under conditions of permissible delay

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in payments. In this study we extend Chung's[22] model with stock and time dependent demand under conditions of permissible delay in payments. The shortages are partially backlogged and the backlogged rate is inversely proportional to the waiting time for the next replenishment.

NOTATIONS AND ASSUMPTIONS

To construct the proposed model we use the following notations and assumptions.

Notations:

- K = Ordering cost of inventory, \$ per order
- $I(t)$ = The inventory level at time t
- θ = Deterioration rate, a fraction of the on-hand inventory
- P = Purchase cost, \$ per unit
- h = Holding cost excluding interest charges, \$ per unit per year
- s = Shortage cost, \$ per unit per year
- π = Opportunity cost due to lost sales, \$ per unit
- I_e = Interest which can be earned per unit
- I_r = Interest charges which invested in inventory, \$ per year, $I_r \geq I_e$.
- M = Permissible delay in payments and $0 < M < 1$
- T = The length of replenishment cycle
- T_1 = Time at which shortage starts $0 \leq T_1 \leq T$
- $TVC(T_1, T)$ = The average total inventory cost per unit time
- $TVC_1(T_1, T)$ = The average total inventory cost per unit time for $T_1 \geq M$ in study 1
- $TVC_2(T_1, T)$ = The average total inventory cost per unit time for $T_1 < M$ in study 2

Assumptions:

- The inventory system involves only one item.
- The replenishment occurs instantaneously at an infinite rate.
- There is no repair or replacement of deteriorated units.
- The demand rate function R(t) is deterministic and is a known function of instantaneous stock level I(t) and time t: The functional R(t) is given by

$$R(t) = \{\alpha + \beta I(t) + ct\ ;\ 0 \leq t < T_1\ \ \alpha + ct\ ;\ T_1 \leq t < T$$

where $\alpha > o$ & $o < \beta < 1$, $o < c < 1$.

- Shortages are allowed and the backlogged rate is defined to be $1/[1+\delta(T-t)]$ when inventory is negative. The backlogged parameter δ is a positive constant and $T_1 \leq t < T$.

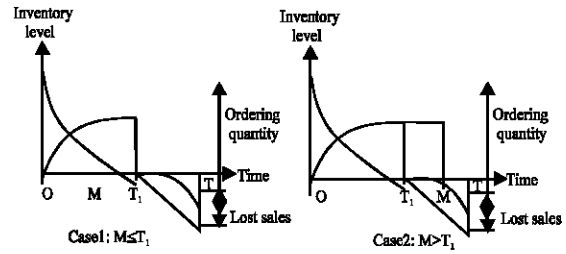

Fig. 1: Graphical representation of inventory system

Model formulation: Due to the combined effects of the demand and deterioration in the interval $[0, T_1)$ the inventory is exhausted. The excess demand is partially backlogged in the interval $[T_1, T)$ as shown in Fig. 1.

The differential equation of I(t) with respect to time can be given as

$$\frac{dI(t)}{dt} = \begin{cases} -\alpha - \beta I(t) - ct - \theta I(t), 0 \leq t \prec T_1 \\ -ct - \dfrac{\alpha}{1+\delta(T-t)}\ , T_1 \leq t \prec T \end{cases} \quad (1)$$

with boundary condition $I(T_1) = 0$

$$I(t) = \begin{cases} \left[\dfrac{\alpha(\beta+\theta)-c}{(\beta+\theta)^2}\right]\left[e^{(\beta+\theta)(T_1-t)} - 1\right] + \left[\dfrac{c}{(\beta+\theta)}\right]\left[T_1 e^{(\beta+\theta)(T_1-t)} - t\right] & \text{when } 0 \leq t \prec T \\ \dfrac{c}{2}(T_1^2 - t^2) - \dfrac{\alpha}{\delta}\{\log[1+\delta(T-T_1)] - \log[1+\delta(T-t)]\} & \text{when } T_1 \leq t \prec T \end{cases} \quad (2)$$

The stock holding cost in the interval $[0, T_1)$ denoted by HC can be written as $HC = h\int_0^{T_1} I(t)\,dt$.

$$HC = h/2(\beta+\theta)^3\ \{2[c - \alpha(\beta+\theta)]\ [1+T_1(\beta+\theta) - e^{(\beta+\theta)T_1}] - [cT_1(\beta+\theta)]\ [2+T_1(\beta+\theta) - 2e^{(\beta+\theta)T_1}]\} \quad (3)$$

Deterioration cost in $(0, T_1)$ denoted by DC is given by

$$DC = P\theta \int_0^{T_1} I(t)\,dt$$

$$DC = P\theta/2(\beta+\theta)^3\ \{2[c - \alpha(\beta+\theta)]\ [1+T_1(\beta+\theta) - e^{(\beta+\theta)T_1}] - [cT_1(\beta+\theta)]\ [2+T_1(\beta+\theta) - 2e^{(\beta+\theta)T_1}]\} \quad (4)$$

We have to consider two costs in the shortage period. The first is to derive the shortage cost for the backlogged items and the second is finding the opportunity cost due to lost sales.

The shortage cost in the interval $[T_1, T)$ denoted by SC is given by

$$SC = s\int_{T_1}^{T} I(t)\,dt$$

$$\text{Shortage cost} = \frac{sc}{2}\left[-T_1^{\,2}T + \frac{T_3}{3} + \frac{2T_1^{\,3}}{3}\right] + \frac{s\alpha}{\delta^2} \tag{5}$$
$$\left\{\delta(T - T_1) - \log[1 + \delta(T - T_1]\right\}$$

The opportunity cost due to lost sales denoted by OC is given by

$$OC = \pi\int_{T_1}^{T}\left[\alpha + ct - ct - \frac{\alpha}{1 + \delta(T - t)}\right]dt \tag{6}$$
$$OC = \frac{\pi\alpha}{\delta}\left\{\delta(T - T_1) - \log[1 + \delta(T - T_1)]\right\}$$

Now consider the supplier's credit period M in settling the accounts. There are two studys. Study 1:$M \le T_1$ or Study 2:$M > T_1$ we shall discuss these two studys one by one.

Study 1:$M < T_1$: Since the length of the period with positive inventory stock of the items is greater than the credit period, the buyer can use the sale revenue with an annual rate I_e in $[0, T_1)$.

The interest earned denoted by IE_1 is

$$IE_1 = PI_e\int_0^{T_1}(T_1 - t)\,R(t)\,dt$$

$$IE_1 = PI_e\left\{\begin{array}{l}[e^{(\beta+\theta)T_1} - 1]\left[\dfrac{c\beta - \alpha\beta(\beta+\theta)}{(\beta+\theta)^4}\right] + \dfrac{\alpha T_1^{\,2}}{2}\left[1 - \dfrac{\beta}{(\beta+\theta)}\right] \\ + \dfrac{ct_1^{\,3}}{6}\left[2 - \dfrac{\beta}{(\beta+\theta)}\right] + \dfrac{\alpha T_1\beta e^{(\beta+\theta)T_1}}{(\beta+\theta)^2} + \dfrac{cT_1^{\,2}\beta}{2(\beta+\theta)^2}[1 + 2e^{(\beta+\theta)T_1}] \\ + \dfrac{cT_1\beta}{(\beta+\theta)^3}\,[1 - 2e^{(\beta+\theta)T_1}]\end{array}\right\} \tag{7}$$

After the credit period the buyer has to pay the interest for the goods still in stock with annual rate I_r. We can find the interest payable denoted by IP as follows.

$$IP = PI_r\int_M^{T_1} I(t)\,dt$$

$$IP = PI_r\left\{\begin{array}{l}\left[\dfrac{\alpha(\beta+\theta) - c}{(\beta+\theta)^3}\right][e^{(\beta+\theta)(T_1 - M)} - 1 - (\beta+\theta)(T_1 - M)] \\ + \left[\dfrac{c}{(\beta+\theta)^2}\right]\left[T_1\,[e^{(\beta+\theta)(T_1 - M)} - 1] - \dfrac{(\beta+\theta)}{2}(T_1^{\,2} - M^2)\right]\end{array}\right\} \tag{8}$$

Therefore the total average cost in study 1 in given by

Our objective is to minimize the total average cost. For this, we have to find the optimal solutions of T_1 and T (say T_1^* and T^*). They can be found by solving the

$$TVC_1 = \frac{K + HC + DC + SC + OC + IP + IE_1}{T}$$

$$TVC_1 = \frac{1}{T}\left\{\begin{array}{l}K + \left[\dfrac{h + P\theta}{2(\beta+\theta)^3}\right]\left[2[c - \alpha(\beta+\theta)][1 + T_1(\beta+\theta) - e^{(\beta+\theta)T_1}]\right] \\ -cT_1(\beta+\theta)[2 + T_1(\beta+\theta) - 2e^{(\beta+\theta)T_1}]\end{array}\right\} + \frac{sc}{2}\left[-T_1^{\,2}T + \frac{T_3}{3} + \frac{2T_1^{\,3}}{3}\right]$$
$$+ \frac{\alpha(s + \pi\delta)}{\delta^2}\delta(T - T_1)\left\{-\log[1 + \delta(T - T_1)]\right\} + PI_r\left\{\begin{array}{l}\left[\dfrac{\alpha(\beta+\theta) - c}{(\beta+\theta)^3}\right][e^{(\beta+\theta)(T_1 - M)} - 1 - (\beta+\theta)(T_1 - M) \\ + \left[\dfrac{c}{(\beta+\theta)^2}\right]\left[T_1\,[e^{(\beta+\theta)(T_1 - M)} - 1] - \dfrac{(\beta+\theta)}{2}(T_1^{\,2} - M^2)\right]\end{array}\right\} \tag{9}$$
$$-PI_e\left\{\left[\frac{c\beta - \alpha\beta(\beta+\theta)}{(\beta+\theta)^4}\right][e^{(\beta+\theta)T_1} - 1] + 1 - \frac{\alpha T_1^{\,2}}{2}\left[\frac{\beta}{(\beta+\theta)}\right] + \frac{cT_1^{\,3}}{6}\right\}\left\{\begin{array}{l}\left[2 - \dfrac{\beta}{(\beta+\theta)}\right] + \dfrac{\alpha T_1\beta e^{(\beta+\theta)T_1}}{(\beta+\theta)^2} + \dfrac{cT_1^{\,2}\beta}{2(\beta+\theta)^2}[1 + 2e^{(\beta+\theta)T_1}] \\ + \dfrac{cT_1\beta}{(\beta+\theta)^3}[1 - 2e^{(\beta+\theta)T_1}]\end{array}\right\}$$

$\partial TVC_1(T_1, T)/\partial T_1 = 0$ and $\partial TVC_1(T_1, T)/\partial T = 0$ (10)
following equations simultaneously. provided they satisfy the sufficient conditions

$$\left.\frac{\partial\, TVC_1(T_1,T)}{\partial T_1^{\,2}}\right|_{(T_1^*,T^*)} > 0, \quad \left.\frac{\partial\, TVC_1(T_1,T)}{\partial T_1^{\,2}}\right|_{(T_1^*,T^*)} > 0 \text{ and}$$

$$\left[\frac{\partial^2 TVC_1(T_1,T)}{\partial T_1^{\,2}}\right]\left[\frac{\partial^2 TVC_1(T_1,T)}{\partial T^2}\right]-\left[\frac{\partial^2 TVC_1(T_1,T)}{\partial T_1\,\delta T}\right] > 0$$

$$(T_1^*,T^*)\ \frac{\partial\, TVC_1(T_1,T)}{\partial T} = 0 \Rightarrow$$

$$\frac{1}{T}\left\{\left[\frac{h+P\theta}{2(\beta+\theta)^2}\right]\left[\begin{array}{l}2[c-\alpha((\beta+\theta)][1-e^{(\beta+\theta)T_1}]-c[2+\\ T_1(\beta+\theta)-2e^{(\beta+\theta)T_1}]-cT_1(\beta+\theta)[1-2e^{(\beta+\theta)T_1}]\\ +sc(TT_1+T_1^{\,2})-\dfrac{\alpha(s+\pi\delta)(T-T_1)}{1+\delta(T-T_1)}\end{array}\right]\right\}+PI_r\left\{\begin{array}{l}\left[\dfrac{\alpha(\beta+\theta)-c}{(\beta+\theta)^2}\right][e^{(\beta+\theta)(T_1-M)}-1]+\left[\dfrac{c}{(\beta+\theta)^2}\right]\\ \left[[e^{(\beta+\theta)(T_1-M)}-1][1+T_1(\beta+\theta)]\right]\end{array}\right\}$$

$$+\frac{\alpha\beta e^{(\beta+\theta)T_1}}{(\beta+\theta)^2}+\frac{\alpha\beta T_1 e^{(\beta+\theta)T_1}}{(\beta+\theta)}+\frac{cT_1\beta}{(\beta+\theta)^2}+\frac{\alpha\beta T_1^2 e^{(\beta+\theta)T_1}}{(\beta+\theta)}$$
$$-\frac{2c\beta e^{(\beta+\theta)T_1}}{(\beta+\theta)^3}+\frac{c\beta}{(\beta+\theta)^3}\Big\}\Big\}=0 \qquad (11)$$

$$\frac{\partial^2 TVC_1(T_1,T)}{\partial T_1^{\,2}} = 0 \Rightarrow$$

$$\frac{1}{T}\left\{\frac{scT_1^{\,2}}{2}+\frac{scT^2}{2}+\frac{\alpha(s+\pi\delta)(T-T_1)}{1+\delta(T-T_1)}\right\}K+\frac{1}{T^2}\left\{\left[\frac{h+P\theta}{2(\beta+\theta)^3}\right]\right.$$

$$\left\{2[c-\alpha(\beta+\theta)[1+T_1(\beta+\theta)-e^{(\beta+\theta)T_1}]-cT_1(\beta+\theta)\left[2+T_1(\beta+\theta)-2e^{(\beta+\theta)T_1}\right]\right\}$$

$$+\frac{sc}{2}\left[-T_1^{\,2}T+\frac{T_3}{3}+\frac{2T_1^{\,3}}{3}\right]+\frac{\alpha(s+\pi\delta)}{\delta^2}\{\delta(T-T_1)-\log[1+\delta(T-T_1)]\}$$

$$+PI_r\left\{\left[\frac{\alpha(\beta+\theta)-c}{(\beta+\theta)^3}\right]\left[e^{(\beta+\theta)(T_1-M)}-1-(\beta+\theta)(T_1-M)\right]+\left[\frac{c}{(\beta+\theta)^2}\right]\left\{\begin{array}{l}T_1[e^{(\beta+\theta)(T_1-M)}\\ -1-\dfrac{(\beta+\theta)}{2}(T_1^{\,2}-M^2)\end{array}\right\}\right.$$

$$-PI_e\left\{\begin{array}{l}[e^{(\beta+\theta)T_1}-1]\left[\dfrac{c\beta-\alpha\beta(\beta+\theta)}{(\beta+\theta)^4}\right]+\dfrac{\alpha T_1^{\,2}}{2}\left[1-\dfrac{\beta}{(\beta+\theta)}\right]\\ +\dfrac{cT_1^{\,3}}{6}\left[2-\dfrac{\beta}{(\beta+\theta)}\right]-\dfrac{\alpha T_1\beta e^{(\beta+\theta)T_1}}{(\beta+\theta)^2}+\dfrac{cT_1^{\,2}\beta}{2(\beta+\theta)^2}[1+2e(\beta+\theta)T_1]+\dfrac{cT_1\beta}{(\beta+\theta)^3}[1-2e(\beta+\theta)T_1]\end{array}\right\}=0 \qquad (12)$$

To obtain the optimal values of T_1 & T, we use the following algorithm.

Algorithm-1

Step 1:

- Start with $T_{1,(1)} = M$
- Substituting $T_{1,(1)}$ in equation (11) to find $T_{(1)}$
- Using $T_{(1)}$ in equation (12) we can find $T_{1,(2)}$
- . Repeat (ii) and (iii) until no change occurs in the values of T_1 and T.

Step 2: compare T_1 and M

- If $M < T_1$, T_1 is feasible. Then go to step (3).
- If $M > T_1$, T_1 is not feasible. Set $T_1 = M$ in the equation (12) to calculate T and then go to step 3.

Step 3: Calculate the corresponding TVC_1 (T_1*,T*).

Study 2: $T_1 < M$: In this study, the buyer need not pay interest and he earns the interest with annual interest rate I_e. The interest earned denoted by IE_2 is given by

$$IE_2 = PI_e\left\{\int_0^{T_1}(T_1 - t)(\alpha+\beta I(t)+ct)dt + (M-T_1)\int_0^{T_1}(\alpha+\beta I(t)+ct)dt\right\}$$

$$IE_2 = PI_e\left\{\begin{matrix}[e^{(\beta+\theta)T_1}-1]\left[\frac{c\beta-\alpha\beta(\beta+\theta)}{(\beta+\theta)^4}\right]\left[\frac{\alpha T_1^{\ 2}}{2}\right]\left[1-\frac{\beta}{(\beta+\theta)}\right] \\ +\frac{cT_1^{\ 3}}{6}\left[2-\frac{\beta}{(\beta+\theta)}\right]+\left[\frac{\alpha\beta T_1 e(\beta+\theta)T_1}{(\beta+\theta)^2}\right]\left[\frac{cT_1^{\ 2}\beta}{2(\beta+\theta)^2}\right][1+2e^{(\beta+\theta)T_1}]\end{matrix}\right\} + PI_e(M-T_1)\left[\alpha T_1 + \frac{cT_1^{\ 2}}{2}\right]\frac{PI_e(M-T_1)\beta}{2(\beta+\theta)^3} \quad (13)$$

$$\left\{2[c-\alpha((\beta+\theta)]\left[1+T_1(\beta+\theta)-e^{(\beta+\theta)T_1}\right]-cT_1(\beta+\theta)\left[2+T_1(\beta+\theta)-2e(\beta+\theta)T_1\right]\right\}$$

We have the total average cost in this study as

$$TVC_2 = \frac{K+HC+DC+SC+OC+IP+IE_2}{T}$$

$$TVC_2 = \frac{1}{T}\left\{\begin{matrix}K+\left[\frac{h+P\theta}{2(\beta+\theta)^3}\right]\left[2[c-\alpha(\beta+\theta)][1+T_1(\beta+\theta)-e^{(\beta+\theta)T_1}]\right] \\ -cT_1(\beta+\theta)[2+T_1(\beta+\theta)-2e^{(\beta+\theta)T_1}]\end{matrix}\right\}+\frac{sc}{2}\left[-T_1^{\ 2}T+\frac{T_3}{3}+\frac{2T_1^{\ 3}}{3}\right]$$

$$+\frac{\alpha(s+\pi\delta)}{\delta^2}\delta(T-T_1)\{-\log[1+\delta(T-T_1)]\}-PI_r\left\{\begin{matrix}\left[\frac{\alpha(\beta+\theta)-c}{(\beta+\theta)^3}\right][e^{(\beta+\theta)(T_1-M)}-1-(\beta+\theta)(T_1 \\ +\left[\frac{c}{(\beta+\theta)^2}\right]\left[T_1[e^{(\beta+\theta)(T_1-M)}-1]-\frac{(\beta+\theta)}{2}(T_1^{\ 2}-M^2)\right]\end{matrix}\right\} \quad (14)$$

$$-PI_e\left\{\left[\frac{c\beta-\alpha\beta(\beta+\theta)}{(\beta+\theta)^4}\right][e^{(\beta+\theta)T_1}-1]+1-\frac{\alpha T_1^{\ 2}}{2}\left[\frac{\beta}{(\beta+\theta)}\right]+\frac{cT_1^{\ 3}}{6}\right\}$$

Our purpose is to minimize total average inventory cost. To achieve this, we have to find the optimal values of T_1 and T which are the solutions of the following equations.

$\partial TVC_2(T_1, T)/\partial T_1 = 0$ and $\partial TVC_2(T_1, T)/\partial T = 0$ (15)

provided they satisfy the sufficient conditions

Table 1: Calculated optimal values of δ

M		δ 1	2	3	4	5
5	TVC	2757.19	3066.27	3326.02	3551.36	3751.08
	T_1*	0.0274	0.0689	0.0688	0.0687	0.0559
	T*	0.0491	0.1219	0.1103	0.1052	0.0803
	T_1*/ T*	0.5580	0.5652	0.6238	0.6530	0.6961

$$\left|\begin{matrix} >0, & \dfrac{\partial^2 TVC_1(T_1,T)}{\partial T_1^{\,2}} \\ (T_1*,T*) & (T_1*,T*) \end{matrix}\right| > 0 \text{ and } \left[\frac{\partial^2 TVC_1(T_1,T)}{\partial T_1^{\,2}}\right]\left[\frac{\partial^2 TVC_1(T_1,T)}{\partial T_1^{\,2}}\right] - \left[\frac{\partial^2 TVC_1(T_1,T)}{\partial T_1\,\delta T}\right] > 0\,(T_1*,T*)\frac{\partial^2 TVC_1(T_1,T)}{\partial T_1^{\,2}} = 0 \Rightarrow$$

$$\frac{1}{T}\left\{\left[\frac{h+P\theta}{2(\beta+\theta)^2}\right]\left[\begin{matrix} 2[c-\alpha((\beta+\theta)][1-e^{(\beta+\theta)T_1}]-c[2+ \\ T_1(\beta+\theta)-2e^{(\beta+\theta)T_1}]-cT_1(\beta+\theta)[1-2e^{(\beta+\theta)T_1}] \\ +sc(TT_1+T_1^{\,2})-\dfrac{\alpha(s+\pi\delta)(T-T_1)}{1+\delta(T-T_1)} \end{matrix}\right]\right\} + PI_r\left\{\begin{matrix}\left[\dfrac{\alpha(\beta+\theta)-c}{(\beta+\theta)^2}\right][e^{(\beta+\theta)(T_1-M)}-1]+\left[\dfrac{c}{(\beta+\theta)^2}\right] \\ \left[[e^{(\beta+\theta)(T_1-M)}-1][1+T_1(\beta+\theta)]\right]\end{matrix}\right\}$$

$$-PI_e\left\{\begin{matrix}\left[\dfrac{c\beta-\alpha\beta(\beta+\theta)}{(\beta+\theta)^3}\right]e^{(\beta+\theta)T_1}+\alpha T_1 1\left[-\dfrac{\beta}{(\beta+\theta)}\right]+\dfrac{cT_1^{\,2}}{2}2\left[-\dfrac{\beta}{(\beta+\theta)}\right] \\ +\dfrac{\alpha\beta e^{(\beta+\theta)T_1}}{(\beta+\theta)^2}+\dfrac{\alpha\beta T_1 e^{(\beta+\theta)T_1}}{(\beta+\theta)}+\dfrac{cT_1\beta}{(\beta+\theta)^2}+\dfrac{\alpha\beta T_1^{\,2}e^{(\beta+\theta)T_1}}{(\beta+\theta)} \\ -\dfrac{2c\beta e^{(\beta+\theta)T_1}}{(\beta+\theta)^3}+\dfrac{c\beta}{(\beta+\theta)^3}\end{matrix}\right\}\frac{\partial^2 TVC_1(T_1,T)}{\partial T_1^{\,2}} = 0 \Rightarrow \frac{1}{T}\left\{\begin{matrix}\dfrac{scT_1^{\,2}}{2}+\dfrac{scT^2}{2}+ \\ \dfrac{\alpha(s+\pi\delta)(T-T_1)}{1+\delta(T-T_1)}\end{matrix}\right\} \quad (16)$$

Table 2: Results of 4 different values of M

M		δ=1
5	TVC	2722.53
	T_1*	0.2449
	T*	0.2698
	T_1*/ T*	0.9077
10	TVC	2474.08
	T_1*	0.0872
	T*	0.0947
	T_1*/ T*	0.9208
15	TVC	2256.50
	T_1*	0.0879
	T*	0.0946
	T_1*/ T*	0.9291
40	TVC	1535.37
	T_1*	0.0190
	T*	0.2033
	T_1*/ T*	0.0934

$$+PI_r\left\{\left[\frac{\alpha(\beta+\theta)-c}{(\beta+\theta)^3}\right]\left[e^{(\beta+\theta)(T_1-M)}-1-(\beta+\theta)(T_1-M)\right]+\left[\frac{c}{(\beta+\theta)^2}\right]\right\}\left\{\begin{matrix}T_1[e^{(\beta+\theta)(T_1-M)} \\ -1-\dfrac{(\beta+\theta)}{2}(T_1^{\,2}-M^2)\end{matrix}\right\}$$

$$-PI_e\left\{\begin{matrix}[e^{(\beta+\theta)T_1}-1]\left[\dfrac{c\beta-\alpha\beta(\beta+\theta)}{(\beta+\theta)^4}\right]+\dfrac{\alpha T_1^{\,2}}{2}\left[1-\dfrac{\beta}{(\beta+\theta)}\right] \\ +\dfrac{cT_1^{\,3}}{6}\left[2-\dfrac{\beta}{(\beta+\theta)}\right]-\dfrac{\alpha T_1\beta e^{(\beta+\theta)T_1}}{(\beta+\theta)^2}+\dfrac{cT_1^{\,2}\beta}{2(\beta+\theta)^2}[1+2e(\beta+\theta)T_1] \\ +\dfrac{cT_1\beta}{(\beta+\theta)^3}[1-2e(\beta+\theta)T_1]\end{matrix}\right\}=0 \quad (17)$$

To find the optimal values T_1* and T*, we use the following algorithm.

Algorithm 2
Step 1: Perform (i)-(iv)

- Start with $T_{1,(1)} = M$
- Substituting $T_{1,(1)}$ in equation (16) to find $T_{(1)}$
- Using $T_{(1)}$ in equation (17) we can find $T_{1,(2)}$
- Repeat (ii) and (iii) until no change occurs in the values of T_1 and T

Step 2: compare T_1 and M

- If $T_1 < M$, T_1 is feasible. Then go to step (3).
- If $T_1 \geq M$, T_1 is not feasible. Set $T_1 = M$ in the equation (17) to calculate T and then go to step 3.

Step 3: Calculate the corresponding $TVC_2\ (T_1^*,T^*)$

Our main aim is to find the optimal values of T_1 and T which minimize TVC (T_1,T), we find that

$TVC\ (T_1^*, T^*) = \text{Min}\ \ TVC_1\ (T_1^*, T^*), TVC_2\ (T_1^*, T^*)$.

NUMERICAL EXAMPLES

To elucidate the preceding theory the following examples are given.

Example 1: Let K=200, α=1000, β=0.3, C=0.4, P=200, h=12, S=30, π=15, I_e=0.13, I_r=0.15, θ=0.08, δ=2, M=10/365.

The computational result shows the following optimal values.

T_1^*=0.2881, T^*=0.6468, TVC=(T_1^*,T^*)=2846.87.

Example 2: Let K=200, α=1000, β=0.3, C=0.4, P=20, h=1.2, S=30, π=15, I_e=0.13, I_r=0.15, θ=0.08, δ=2, M=35/365.
We get the optimal values as
T_1^*=0.2883, T^*=0.03066, TVC=(T_1^*,T^*)=1724.92

Example 3: Let K=200, α=1000, β=0.3, C=0.4, P=200, h=1.2, S=30, π=15, I_e=0.13, I_r=0.15, θ=0.08, M=5/365, (δ=1,2,3,4,5). Five different values of δ are taken. δ=1,2,3,4,5. For each value of δ the calculated optimal values are given in Table 1.

Example 4: Let K=200, α=1000, β=0.3, C=0.4, P=20, h=1.2, S=30, π=15, I_e=0.13, I_r=0.15, θ=0.08, δ=1. Four different values of M are considered computed results are given in Table 2.

CONCLUSION

In this study, we developed an inventory model with stock and time dependent demand with shortages under the condition of permissible delay in payments. The backlogging rate is considered to be a decreasing function of the waiting time for the next replenishment.

This assumption is more realistic. For the condition C=0 the model reduces to the model by Chung-Yuan Dye[22]. For the conditions with β=0, C=0 and δ=0 the model reduces to the model by Aggarwal S.P. and Jaggi C.K.[15] and moreover they did not allow shortages.

Furthermore, the results of the sensitivity analysis are also consistent with the economic incentives. For fixed M, the proportion of customers who would like to accept backlogging at time t decreases as δ increases. For fixed δ, increasing the value of M will result in a significant decrease in the optimal average inventory cost (From Table 2).

This study may be extended to multi-items. Another possible extension of this study may consider the assumption of variable deterioration rate.

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