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A Comparative Study on the Class of Algorithms Used in the Generation of Minimal Cutsets for S-T Network

[1]Vidyaathulasiraman and [2]S.P. Rajagopalan
[1]Department of HOD-MCA, Priyadarshini Engineering College, Vaniyambadi-625 751, India
[2]College of Development Council, University of Madras, Chennai-05, India

Abstract: This study provides a detailed review on the generation of the Minimal Cutsets and Minimal Pathsets between any single pair of terminals and for multi-terminal pairs for s-t network. A comparative study is made between the first technique, which deals with the generation of Minimal Cutsets by the node removal method over the given s-t network versus the second technique, which deals with the generation of Minimal Cutsets from Minimal Pathsets by the application of Inter-conversion over minimization technique for a given s-t network. The study concludes that the second technique is comparatively more advantageous, because in the calculation of network reliability, both the Minimal Pathsets and Minimal Cutsets play a vital role. The first technique generates only Minimal Cutsets and the generation of Minimal Pathsets should be done separately. But this is not so in the case of second technique, as it uses the Inter-conversion and minimization technique to convert Minimal Pathsets to Minimal cutsets for a given s-t network. Since this technique is applied over the generation of minimal pathsets for a given s-t networks based on the decomposition of the given network. Decomposition is performed, so that if 2 or more sub-networks are homogeneous, calculation of Minimal Pathsets, for each sub-network are not required. Thus the time required to generate Minimal Pathsets and from it Minimal Cutsets for the network is reduced. Thus the second technique is more efficient for large and complex networks.

Key words: Networks, network reliability, pathsets, cutsets, minimal pathsets, minimal cutsets, network decomposition

INTRODUCTION

The evaluation of network reliability is very important in engineering systems. Network reliability is an important factor in designing and in the operation of systems used in communication, transmission and distribution of electrical power, pipe lines, etc. Networks and systems are usually represented by Reliability Logic Diagrams (RLD) reliability graphs (Singh, 1998; Misra, 1993; Shooman, 1968; Amstadler, 1971; Green and Bourne, 1972; Singh and Billinron, 1977; Srinath, 1985). From reliability point of view RLD clearly depicts the functional logic of the system.

Enumeration of minimal paths or cutsets is one of the important step to evaluate the network reliability or unreliability of a complex system. Cutsets ares a set of edges which disconnects the connection between the source node and the terminal node. Cut is minimal, if it does not contain a subset as cutset. Pathsets are a set of edges which forms a connection between the source node and the terminal node. Pathsets are minimal, if it does not contain a subset as pathset.

The Role of Minimal Pathsets and Minimal Cutsets in the calculation of Network System Relibility (Rs) is:

If a system has i minimal pathsets denoted by P1,P2,…,Pi, then the system has a connection between input and output if atleast one pathset is intact. The system reliability is thus given by

$$Rs = P(P1+P2+\ldots+Pi)$$

The probability of system failure is given by the probability that atleast one minimal cutset fails. Let C1,C2,…,Cj represent the j minimal cutsets and Cj' the failure of the jth cutset, the system reliability is given by

$$Rs = 1- P(C1'+C2'+\ldots+Cj')$$

Several methods and techniques are available in the literature for enumerating cutsets and pathsets of complex systems. There are two categories of algorithms available for enumerating cutest and pathset of the RLD. They are:

- Algorithms that use the exhaustive search techniques (Misra, 1993; Pearson, 1977; Bansal *et al*., 1982; Deo, 1974).

Corresponding Author: Vidyaathulasiraman, Department of HOD-MCA, Priyadarshini Engineering College, Vaniyambadi-625751, India

- Algorithms that perform matrix operations with a connection matrix/adjacency matrix to generate paths of higher cardinalities (Samad, 1987; Biegel, 1977; Misra, 1979a, b; Misra and Misra, 1980).

Deo (1974) and Bansal *et al*. (1982) used node by node search tehniques for enumeration of paths for directed and non-directed RLDs. The limitations of these methods are their application to small networks. But the proposed method is highly efficient as the network becomes larger and larger.

Method proposed by Samad (1987) which generates many unwanted term, because of the 0 entries. As a result more memory and computation is involved here.Samad (1987) here also there is duplication of elements generated, where we have to discard them. Such drawbacks could be overcome in the proposed method. Misra (1979) here the algorithm contains more steps and is time consuming. Aziz *et al*. (1992) have proposed algorithm which uses the method of indexing non-zero terms. This method is more efficient than Samad (1987).

So far we had a review of several author's algorithm for generating minimal cutsets and pathsets for a s-t Network. Earlier researches have not used the decomposition concept for enumerating the minimal cutsets or minimal pathsets for s-t network, whereas Vidyaathulasiraman and Brijendra (1999, 2001) proposed algorithm deals with it. The inter-conversion method is applied to generate minimal cutsets from minimal pathsets, as inVidyaathulsiraman and Rajagopalan (2005).

An Overview on technique 1 and 2 is brought about. The algorithm, illustration and conclusion for technique 1 and 2 is brought about in Appendix A and B. We also notice that the assumptions in both the techniques are same. This helps us to conclude that both the algorithms can be applied in a similar situation. But technique 2 proves to be advantageous from the comparative study made.

ASSUMPTIONS

Nodes are perfectly reliable. The network/link have two states, viz., success/failure.

Note : Both technique 1 and 2 have the same assumption.

AN OVERVIEW ON TECHNIQUE 1

First we mark the source and terminal node for a given network. Secondly we start removing the nodes one by one which are available in the network, other than the source and terminal node in the given network. Thirdly we proceed with the previous step until the source and terminal node alone is left. This gives the Minimal Cutsets for the given network.

AN OVERVIEW ON TECHNIQUE 2

First divide the network. [Divide the Network in such a manner, so that the starting and the ending node are present in two separate sub-network and let us name them as Sub-network-start (S) and sub-network-destination (D). These Network can be divided into any number of Sub-networks apart from (S) and (D). Sub-network (S) has got only one source, but may have one or more destinations. In (D) there are many possible sources, but only one destination.] Second find the Minimal Pathsets for each sub-network. Third find the Minimal Cutsets using the Interconversion and minimization technique for each Sub-network separately. Fourth merge the nodes of each sub-network generated in step second. Fifth add the merged answers which we get at step 4. This gives the final Minimal Pathsets for the given network. Sixth step deals with the nodes generated in the third step. Here we add the nodes of each sub-network generated. Seventh step deals with merging of the results obtained in step 6. This gives the final Minimal Cutsets for the given network.

COMPARATIVE STUDY OF TECHNIQUE 1 AND 2

From the algorithm, illustration and conclusion specified for technique 1 and 2 in Appendix A and B, the following facts are noted.

- Technique 1 algorithm is simple, when compared to Technique 2. But both the techniques are implemented and executed using a computer. So there is no appreciable CPU time difference between the algorithms.
- The example reveal that the process of eliminating the repeated terms are performed in technique 1, which is absent in the case of technique 2.
- Technique 1 involves calculation of Minimal Cutsets only. For the generation of Minimal Pathsets an other algorithm should be executed. But this is not true in the case in Technique 2. Both Minimal Pathsets and Minimal Cutsets are getting generated.
- The conclusion of Technique 2 clearly states that the algorithm would best suite for larger networks. The Proposed method is more effecient for large and complex networks.This is because if 2 or more sub-networks are homogeneous, calculation of Minimal Pathsets, for each sub-network are not required. Thus the time required to generate Minimal Pathsets and Minimal Cutsets for the network is reduced, which is not true in the case of Technique 1.

CONCLUSION

From the comparative study made above, we understand that technique 2 is more advantageous than technique 1 in situations where both Minimal Pathsets and Minimal Cutsets calculations are required, in the process of Network Reliability calculation of a given S-T network.

APPENDIX A: TECHNIQUE 1

I. Algorithm:

a Start-Given a network as input is represented in the form of a matrix.

Here all the nodes present in the network are represented as rows and columns here (Always it starts with the source node, the last node represented in the matrix is the destination node. all the other nodes fall between them.) and we code the paths of each node to every other node in the network.

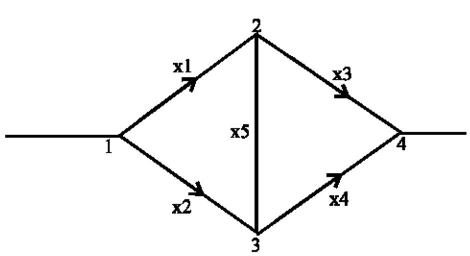
Note : For a node to itself, we code the path as 0.

b Remove each node one-by-one (except the 1st (source) and last (terminal) node) and then find the cuts for all the remaining nodes.

c Repeat tep b until the matrix contains the first (source) and Last (terminal) node alone.

d End- The cut that is available from the first (source) to last (terminal) node gives the minimal cutsets for the given network.

Illustration: Algorithm: Example 1: A given network



a)

	1	2	3	4
1	0	$\overline{x1}$	$\overline{x2}$	1
2	1	0	$\overline{x5}$	$\overline{x3}$
3	1	$\overline{x5}$	0	$\overline{x4}$
4	1	1	1	0

b By removing node 2 we get the resultant matrix as follows:

	1	3	4
1	0	$\overline{x1x2}+\overline{x2x5}$	$\overline{x1}+\overline{x3}$
3	1	0	$\overline{x3x4}+\overline{x4x5}$
4	1	1	0

Calculations: __

N11 = 0. (x1+1) = 0

$N13 = \overline{x2}.(\overline{x1}+\overline{x5}) = \overline{x1x2} + \overline{x2x5}$

$N14 = 1.(\overline{x1}+\overline{x3}) = \overline{x1}+\overline{x3}$

$N31 = 1.(1+\overline{x5}) = 1$

N33 = 0

$N34 = \overline{x4}.(\overline{x3}+\overline{x5}) = \overline{x3x4}+\overline{x4x5}$

N41 = 1

N43 = 1

N44 = 0

c By removing node 3 we get the resultant matrix as follows:

	1	4
1	0	$\overline{x1x2}+\overline{x2x3x5}+\overline{x3x4}+\overline{x1x4x5}$
4	1	0

Calculations:

N11 = 1

$N14 = (\overline{x1}+\overline{x3}).(\overline{x1x2}+\overline{x2x5}+\overline{x3x4}+\overline{x4x5})$

$= \overline{x1x2}+\overline{x1x2x5}+\overline{x1x3x4}+\overline{x1x4x5}+\overline{x1x2x3}+\overline{x2x3x5}+\overline{x3x4}+\overline{x3x4x5}$

$= \overline{x1x2}+\overline{x2x3x5}+\overline{x3x4}+\overline{x1x4x5}$ [By eliminating the repeated terms]

N41 = 1

N44 = 0

d The Minimal cutsets for the given network is

$\overline{x1x2}+\overline{x2x3x5}+\overline{x3x4}+\overline{x1x4x5}$

This method helps to evaluate the Minimal Cutsets for any complex and large network, provided it's source and destination is properly specified. It consumes less time. But of course there is generation of unwanted, repeated terms, which we have to eliminate by applying the basic purging rules.

APPENDIX B : TECHNIQUE 2

I. Algorithm:

a Start-take the given network.
b Divide the given network into any number of sub-networks.
c Start from (S) and proceed as follows:
i) From each source node consider all the possible destination nodes.
ii) Find Minimal Pathsets, for each of the source and destination combination by using the sub-algorithm.
iii) Generate the equivalent Minimal Cutsets form the Minimal Pathsets generated in step (ii) using the Interconversion and minimization technique.
iv) The destination nodes of the previous sub-network becomes the source nodes for the next sub-network.
v) Repeat steps (I), (ii), (iii) and (iv) until we find the Minimal Pathsets and Minimal Cutsets for (D) also.
d Now merge the Pathsets generated from each sub-network, so that the nodes which are considered to be the destination, are considered to be the source in next sub-network. Likewise proceed from (S) to (D) by combining the Minimal Pathsets separately to generate each possible cases (i.e., with the results obtained from step c (ii)).
e Stop-Add all the results obtained in step (d) which gives all possible Minimal Pathsets for the given network.
f Now add the cutsets generated from each sub-network, so that the nodes which are considered to be the destination, are considered to be the source in next sub-network. Likewise proceed from (S) to (D) by combining the Minimal Cutsets separately to generate each possible cases (i.e., with the results obtained from step c (iii)).
g Stop-Merge all the results obtained in step (f) which gives all possible Minimal Cutsets for the given network.

Sub-Algorithm:

a Start - Given a sub-network as input from the step c (ii) of the algorithm, is represented in the form of a matrix.

Here all the nodes present in the given network is represented as rows and columns (Always it starts with the source node, the last node represented in the matrix is the destination node and all the other nodes fall between them) and we code the paths of each node to every other node in the network.
Note : For a node to itself, we code the path as 1.

b Remove each node one-by-one (except the 1st and last node) and then find the paths for all the remaining nodes.
c Repeat step b until the matrix contains the first and Last node alone.
d End-The path that is available from the first to last node gives the Minimal Pathsets for the given Network.

The interconversion and minimization technique: This technique converts the minimal pathsets to minimal cutsets with the help of DeMorgan's theorems. The following are the DeMorgan's theorems:

$$\overline{XY} = \overline{X} + \overline{Y}$$

$$\overline{X+Y} = \overline{X} . \overline{Y}$$

Where X and Y are the Boolean variables.
In this method, the steps to be followed are:

Step 1: Input; Minimal path polynomial as the sum of the links in each minimal path.
Step 2: Express inverse of the path polynomial using De Morgan's theorem as the product of the inverses of terms.
Step 3: Initialize; Current polynomial = First term of the polynomial obtained from step 2. J = 2 (i.e. second term of polynomial)
Step 4: Multiply current polynomial by the Jth terms obtained from step 2.
Step 5: Simplify the expression obtained in step 4 with the help of following absorption rules:

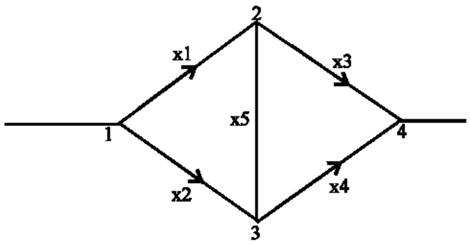
(i) $\overline{X} + \overline{X} = \overline{X}$

(ii) $\overline{X} + \overline{X.Y} = \overline{X}$

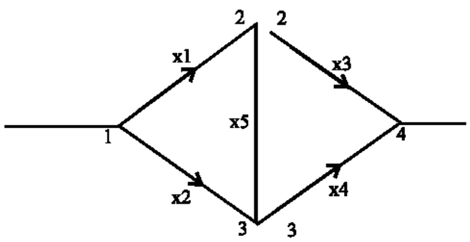
Step 6: $J \leftarrow (J + 1)$
Step 7: If J <= M, Where M is the total no. of terms in the path polynomial Then go to step 4. Else to step 8.
Step 8: Write the new current polynomial. Minimal cut polynomial-New current polynomial.
Step 9: List the terms as minimal cutsets of the minimal cut polynomial.
Step10: Stop.

Illustration : Algorithm: Example: A given network

a)



b) (i) (ii)



(S) (D)

Note : Here in this network we have only (S) and (D) sub-networks.

c) (i) First consider (S), Here 1-2,1-3 Here 2 and 3 are the possible destination nodes from node 1.

ii) We get the result
X2.X5 + X1 as the Minimal Pathsets for the path 1-2.
X2 + X1.X5 as the Minimal Pathsets for the path 1-3.
The results are obtained from the sub-algorithm.

iii) The interconversion and minimization technique (From Minimal Pathsets to Minimal Cutsets).

Step1: X2.X5 + X1 as the minimal pathsets for the path 1-2.

Step2: $(\overline{X2}+\overline{X5}).\overline{X1}$

Step3: Current polynomial = $(\overline{X2}+\overline{X5})$

J = $\overline{X1}$

Step4: $\overline{X1}.\overline{X2}+\overline{X1}.\overline{X5}$

Step5: $\overline{X1}.\overline{X2}+\overline{X1}.\overline{X5}$ ← New current polynomial

Step6: J = 3

Step7: Since J is > than M (where M = 2) the control goes to Step 8.

Step8: $\overline{X1}.\overline{X2}+\overline{X1}.\overline{X5}$ ← minimal cut polynomial

Step9: Minimal cutsets for the path 1-2 is:

$\overline{X1}.\overline{X2}$, $\overline{X1}.\overline{X5}$

Step 10: Stop

Likewise for the path 1-3, the minimal cutset is $\overline{X1}.\overline{X2}$,

$\overline{X2}.\overline{X5}$

iv) Now consider the 2nd sub-network (D). In the Previous case 2 and 3 were the destination nodes, so here 2 and 3 are the source nodes and the destination node is 4.

v) Once again repeat steps c i),ii),iii) and iv), we get : 2-4 and 3-4 as the possible paths.

$\overline{X3}$ is the Minimal pathset for 2-4.

$\overline{X4}$ is the Minimal pathset for 3-4.

$\overline{X3}$ is the Minimal cutset for 2-4.

$\overline{X4}$ is the Minimal Cutset for 3-4.

d Since in (S) we considered 1-2 path, Merge the pathsets of it with the pathsets generated for 2 as the source in (D). So we Merge 1-2 and 2-4 path's Minimal Pathsets and we get:

(X1.X3)+(X2.X3.X5) ----------(1)

Similarly for 3, it is 1-3 and 3-4 path's Minimal Pathsets are merged and we get :

(X2.X4)+(X1.X5.X4)----------(2)

e Adding the results (1) and (2), we get :

(X1.X3)+(X2.X3.X5)+ (X2.X4)+(X1.X4.X5)

This is the Minimal Pathset for the given Network.

f) Since in (S), we considered 1-2 path, Add the cutsets of it with the cutsets generated for 2 as the source in (D). So we Add 1-2 and 2-4 path's Minimal Cutsets and we get:

$(\overline{X1}.\overline{X2})+(\overline{X1}.\overline{X5})+(\overline{X3})$ ----------(3)

Similarly for 3, it is 1-3 and 3-4 path's minimal cutets are added and we get :

$(\overline{X1}.\overline{X2})+(\overline{X2}.\overline{X5})+(\overline{X4})$ ----------(4)

g Merge the results (3) and (4), we get:

$(\overline{X1}.\overline{X2})+(\overline{X2}.\overline{X3}.\overline{X5})+(\overline{X3}.\overline{X4})+(\overline{X1}.\overline{X4}.\overline{X5})$

This is the minimal cutset for the given network.

Sub-Algorithm: Here we receive 1-2 path. We consider all the nodes here.

Nodes	1	3	2
1	1	X2	X1
3	0	1	X5
2	0	X5	1

Removal of Node 3 from the above matrix,
Where Node 3 = k; the formula is we have to replace each element Nij as N'ij = Nij+(Nij.Nkj) where N'ij replaces the old Nij. Nij is the new matrix element of the ith row and jth column,

Nodes	1	2
1	1	X1+X2.X5
2	0	1

Therefore the minimal pathsets from 1-2 is X1+X2.X5
Likewise the minimal pathsets for :

1-3 path is X2+X1.X5
2-4 path is X3
3-4 path is X4

Once the minimal pathsets are generated, using inter-conversion method minimal cutsets could be evaluated. Separate evaluation of minimal cutsets could be avoided. Thus reducing computation time involved in the genaration of minimal pathsets/cutsets.

The proposed method is more effecient for large and complex networks.this is because if 2 or more sub-networks are homogeneous, calculation of minimal pathsets, for each sub-network are not required. Thus the time required to generate minimal pathsets and minimal cutsets for the network is reduced.

This theory could be further extended with the application of pattern recognition.

REFERENCES

Amstadler, B.L., 1971. Reliability Mathematics, McGraw-Hill, New York.

Aziz, M.A., M.A. Sobhan and M.A. Samad, 1992. Reduction of computations in enumeration of terminal and multiterminal pathsets by the method of indexing. Microelectronics Reliability, 32: 1067-1072.

Brijendra Singh, 1998. Quality Control and Reliability Analysis Published by Khanna Publishers, New Delhi.

Biegel, J.E., 1977. Determination of tie-sets and cutsets for a system without feedback. IEEE. Trans. Reliability, 26: 39-42.

Bansal, V.K., K.B. Misra and M.P. Jain, 1982. Minimal pathset and minimal cutset using search technique. Microelectronics Reliability, 22: 1067-1075.

Deo, N., 1974. Graph Theory with Applications to Engineering and Computer Science Prentice-Hall, Englewood Cliffs, NJ.

Green, A.E. and A.J. Bourne, 1972. Reliability Technology Wiley Interscience, London.

Misra, K.B., 1993. Reliability Analysis and Prediction, Eln Scden Publishers, Netherland.

Misra, R.B., 1979. Symbolic reliability evaluation of reducible networks. Microelectronics Reliability, 19: 253-257.

Misra, R.B.and K.B. Misra, 1980. An algorithm for reliability evaluation of complex systems. J. I. E. (India), 60: 77-80.

Misra, R.B., 1979. An algorithm for enumerating all simple paths in a communication network. Microelectronics Reliability, 19: 363-366.

Pearson, G.D.M., 1977. Computer Program for approximation of the reliability characteristics of acyclic directed graphs. IEEE. Trans. Reliability, 26: 32-38.

Shooman, M.L., 1968. Probablistic Reliability: An Engineering Approach, McGraw-Hill, New York.

Singh, C. and R. Billinton, 1977. System Reliability Modelling and Evaluation, Hutchinson, London.

Srinath, L.S., 1985. Concepts in Reliability Engineering, Affiliated East-West Press, New Delhi.

Samad, M.A., 1987. An efficient method for terminal and multiterminal pathset enumeration. Microelectronics Reliability, 27: 443-446.

Samad, M.A., 1987. An efficient algorithm for simultaneously deducing minimal paths as well as cuts of a communication network. Microelectronics Reliability, 27: 437-441.

Vidyaathulasiraman and Brijendra Singh-Generation of Minimal Pathsets for S-T Network, published in CSI'99 the 34th Annual National Conference held at Mumbai.

Vidyaathulasiraman and Brijendra Singh-Generation of Minimal Cutsets for S-T Network-published in QUEST, 2001 organized by Vikram Sarabai Space Centre (VSSC)-Tiruvananthapuram.

Vidyaathulasiraman and S.P.Rajagopalan-Generation, 2005. Minimal Cutsets from Minimal Pathsets for S-T Network- published and presented in National Conference organized by the Department of Mathematics and Computer Science, Sacred Heart College-Tirupattur.