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Robust PI Control Design: A Genetic Algorithm Approach

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Abstract: Control design goals can smartly be achieved using numerical optimisation methods such as Genetic Algorithms (GAs). Using GAs, an efficient numerical method to obtain robust PI tuning formulae for first order plus dead time processes is presented in this study. The design method is based on optimal load disturbance rejection. In order to obtain a robust controller, a constraint on the maximum sensitivity is used. In addition, the design method deals with setpoint response through setpoint weighting. The design procedure has 2 main steps. In the first step, PI controller parameters are determined such that the IAE criterion to a load disturbance step is minimized and the robustness constraint on maximum sensitivity is satisfied. In the second step, good setpoint regulation is achieved by using a two-degree of freedom control scheme. In order to show the performance and effectiveness of the proposed tuning formulae, they are applied to 2 simulation examples.

Key words: Robust, GAs, design, genetic, load, PI

INTRODUCTION

In spite of the continual advances in control theory, PID control is still, by far, the most commonly used algorithm in the process industry. According to a survey of more than 11000 industrial controllers, 97% of them were PID (Desbourough and Miller, 2002). The main reason is that a well designed and adequately tuned PID controller meets most control objectives (Fruehauf *et al*., 1994). Therefore, good methods of tuning PID controllers are highly desirable due to their widespread.

Generally, good load disturbance rejection is the primary objective. Also, the closed-loop system should be robust against model errors. Optimizing load disturbance rejection with sensitivity constraints was suggested by Shinskey (1990). He used a constraint in terms of a rectangle around the critical point. The idea to use a constraint on the maximum sensitivity was proposed in Persson and Astrom (1992). The use of both maximum sensitivity and maximum complementary sensitivity as design parameters was suggested in Schei (1983-1989).

In order to consider both performance requirements and robustness issues, the design method is aimed at optimizing load disturbance rejection with a constraint on the maximum sensitivity. In addition, good setpoint regulation is obtained using setpoint weighting. This has no influence on the load disturbance response but plays a significant role in improving the setpoint response.

PI control is sufficient for a large number of control problems, particularly when process dominant dynamics are of the first order and the design requirements are not too rigorous (Astrom and Hagglund, 1995). It is a common and well accepted practice to approximate high order processes by low order plus dead time models. A large number of industrial plants can be approximately modelled by a First Order Plus Dead Time (FOPDT) transfer function, as shown in Eq. 1.

$$G_p(s) = \frac{K_p e^{-\tau_d s}}{Ts+1} \tag{1}$$

Although a FOPDT model does not capture all the features of a high order process, it often reasonably describes the process gain, dominant time constant and effective dead time of such a process (Dougherty and Cooper, 2003). Considering the importance of this category of industrial plants, optimal PI tuning formulae for FOPDT processes are proposed in this study.

CONTROL REQUIREMENTS

Load disturbance rejection: The most common disturbances in process control are load disturbances. These low frequency signals are added to the control

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signal at the process input and drive the system away from its desired operating point (Astrom and Hagglund, 1995). Good rejection of such signals is the first design goal.

Robustness against model uncertainties: Typically, the controller parameters are obtained from the model parameters. Due to model uncertainties, the controller parameters should be chosen in such a way that the closed-loop system is not too sensitive to variations in process dynamics. Sensitivity to modelling errors can be expressed as the largest value of the sensitivity function, as shown in Eq. 2.

$$M_s = \max_{\omega} \left| \frac{1}{1 + G_p(j\omega)G_c(j\omega)} \right| \tag{2}$$

M_s is the inverse of the shortest distance from the Nyquist curve of the loop transfer function to the critical point. Smaller values of M_s show little or no overshoot while larger ones result in faster responses. A constraint on the maximum sensitivity is employed to obtain a robust controller.

Setpoint regulation: The primary design goal is to reject the load disturbance signals. However, it is also important to have good setpoint responses. Because responses to load disturbance and setpoint signals are usually conflicting, the first design goal may result in bad setpoint responses. As the secondary design goal, good setpoint responses are obtained using setpoint weighting.

DESIGN PROCEDURE

Consider a two-degree of freedom structure, as shown in Fig. 1.

Where r, d and y refer to the setpoint, load disturbance and output signals, respectively. $G_p(s)$ refers to the process model whereas $G_c(s)$ and $G_{ff}(s)$ are PI and feed forward controllers, shown in Eq. 3 and 4.

$$G_c(s) = K_c\left(1 + \frac{1}{T_i s}\right) \tag{3}$$

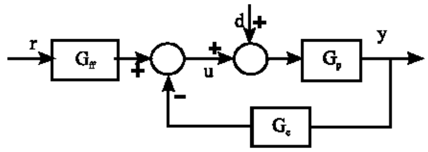

Fig. 1: Block diagram of two-degree of freedom control system

$$G_{ff}(s) = K_c\left(b + \frac{1}{T_i s}\right) \tag{4}$$

Equation 5 describes the closed-loop control system.

$$y = \frac{G_p(s)G_{ff}(s)}{1 + G_p(s)G_c(s)} r + \frac{G_p(s)}{1 + G_p(s)G_c(s)} d \tag{5}$$

The input output relationship for the controller is described by Eq. 6.

$$u(t) = K_c\left(br(t) - y(t) + \frac{1}{T_i}\int_0^t (r(\tau) - y(\tau))d\tau\right) \tag{6}$$

The design objective is to determine $G_c(s)$ and G_s to obtain good load disturbance and setpoint responses. A constraint on maximum sensitivity is used to guarantee robustness to model uncertainties. $M_s = 2$ is considered as the robustness constraint, in this study.

Load disturbance response: The objective function is to minimize the IAE criterion, shown in Eq. 7, subject to a constraint on maximum sensitivity.

$$IAE = \int_0^{\infty} \left| r(t) - y(t) \right| dt \tag{7}$$

The design procedure has 2 main steps. In the first step, the setpoint signal is considered to be zero and $G_c(s)$ is determined so that load disturbances are attenuated and the robustness constraint is satisfied. For the $G_c(s)$ determined in the first step and in absence of load disturbances, $G_{ff}(s)$ is then tuned to achieve good setpoint responses, in the second step.

In order to obtain the optimal PI tuning formulae for the FOPDT model in (1), the PI parameters can be defined based on the model parameters, as shown in Eq. 8 and 9.

$$K_c = f_1(K_p, \tau_d, T) \tag{8}$$

$$T_i = f_2(K_p, \tau_d, T) \tag{9}$$

Functions f_1 and f_2 should be determined such that the load disturbance response is minimized and the robustness constraint is satisfied. However, it is very difficult to determine these functions because each parameter of the controller is a function of three parameters of the model. In order to overcome this difficulty, the procedure for determining f_1 and f_2 is simplified using dimensional analysis, see appendix.

Considering the process model in Eq. 1, the unit of both dead time (τ_d) and Time constant (T) is the second. The unit of process gain (K_p) depends on the ature of the system. Because process gain along with either dead time or time constant cover all the units in Eq. 8 and 9, m is equal to 2. Therefore, there is only one dimensionless number in the model, namely $\tau d/T$, which is named dimensionless dead time. Considering the PI controller in Eq. 4, the unit of integral time (T_i) is the second. The unit of controller gain is the inverse of the unit of process gain. As a result, other dimensionless numbers for the combined model and controller are dimensionless gain (K_p K_c) and dimensionless integral time T_i/T or T_i/τ_d. Based on Buckingham's pi theorem (Zlokarnik, 1991) these dimensionless numbers are functions of the dimensionless number in the plant model. Therefore, the P I parameters can be obtained through determining K_p K_c and T_i/τ_d (or T_i/T) from, τ_d/Tas shown in Eq. 10 and 11.

$$K_pK_c = g_1(\frac{\tau_d}{T}) \tag{10}$$

$$\frac{T_i}{\tau_d} = g_2(\frac{\tau_d}{T}) \tag{12}$$

In order to determine g_1 and g_2 and generate PI tuning formulae, the following procedure is proposed.

Step 1: The values of τ_d/T are selected

Step 2: For each value of τ_d/T, the optimal values of K_c and T_i that minimise the desired objective function are determined using GAs (Goldberg, 1989; Fleming and Purshouse, 2002).

Step 3: The optimal values of K_p K_c and T_i/τ_dversus τ_d/T are drawn.

Step 4: g_1 and g_2 are determined using curve fitting techniques.

In order to take FOPDT processes with small, medium and fairly long dead time into account, the values of dimensionless dead time are considered from 0.1 to 2. The optimal values of K_p K_c and T_i/τ_d, resulting from step 2 are shown in Table 1.

Figure 2 and 3 show the optimal values of the dimensionless gain and the dimensionless integral time across the selected values of the dimensionless dead time,

Table 1:Optimal PI parameters for a FOPDT model

τ_d/T	K_p K_c	T_i/τ_d	b
0.1	6.1710	4.0917	0.5950
0.2	3.1724	3.1610	0.6170
0.3	2.1808	2.5977	0.6459
0.4	1.6665	2.1721	0.6672
0.5	1.3820	1.9119	0.7072
0.6	1.1976	1.7387	0.7223
0.7	1.0629	1.5905	0.7509
0.8	0.9666	1.4785	0.7793
0.9	0.8971	1.4119	0.7964
1.0	0.8339	1.3184	0.8188
1.1	0.7832	1.2476	0.8469
1.2	0.7494	1.2082	0.8697
1.3	0.7039	1.1264	0.8930
1.4	0.6764	1.0909	0.9176
1.5	0.6547	1.0527	0.9263
1.6	0.6298	1.0077	0.9425
1.7	0.6062	0.9767	0.9616
1.8	0.5894	0.9428	0.9798
1.9	0.5844	0.9426	0.9837
2.0	0.5689	0.9238	1.0303

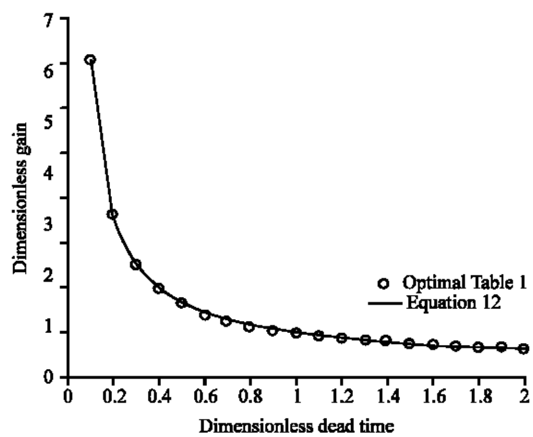

Fig. 2: Optimal values of the dimensionless gain,K_p K_c and values of K_p K_c given by Eq. 12 versus the dimensionless dead time, τ_d/T

respectively. It can be seen from Fig. 2 that the dimensionless gain is a function of the dimensional dead time as shown in Eq. 12. Similarly, the values of T_i/τ_d are determined from the values of τ_d/T, using Eq. 13.

$$K_pK_c = A_1 + \frac{B_1}{\frac{\tau_d}{T}} \tag{12}$$

$$\frac{T_i}{\tau_d} = \frac{A_2\frac{\tau_d}{T} + B_2}{\frac{\tau_d}{T} + C_2} \tag{13}$$

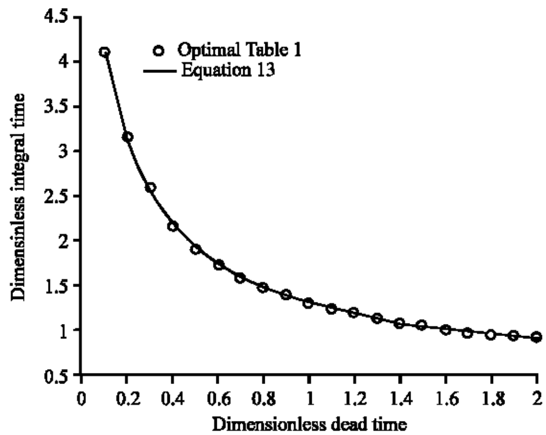

Fig. 3: Optimal values of the dimensionless integral time, and T_i/τ_d values of T_i/τ_d given by Eq. 13 versus the dimensionless dead time, τ_d/T.

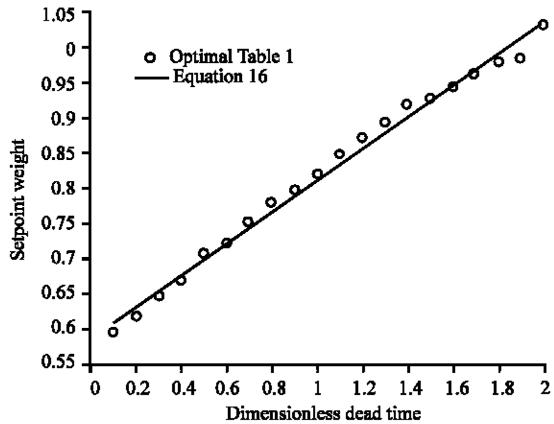

Fig. 4: Optimal values of setpoint weight, b and the values of *b* given by Eq. 16 versus the dimensionless dead time, τ_d/T.

Using the least squares method, A_1, B_1, A_2, B_2 and C_2 are determined for the best match with Table 1. The optimal values of A_1, B_1, A_2, B_2 and C_2 are, 3/11, 4/7, 9/20, 13/12 and 3/17, respectively.

Setpoint response: In this step, the load disturbance signal is considered to be zero and G_{ff} (s) is determined to obtain a good setpoint response. First, for each value of τ_d/T the optimal values of K_c and T_i are determined using Eq. 12 and 13. Next, the setpoint response is improved using setpoint weight, b, which is a function of the process parameters, as shown in Eq. 14.

$$b = f_3(K_p, \tau_d, T) \tag{14}$$

This equation can be simplified to Eq. 15, using dimensional analysis.

$$b = g_3\left(\frac{\tau_d}{T}\right) \tag{15}$$

Using a numerical optimization technique, the optimal value of b is determined so that the objective function in Eq. 7 is minimized. Table 1 and Fig. 4 show the optimal values of b versus τ_d/T.

Using the least squares method, optimal value of b can be calculated from Eq. 16.

$$b = \frac{9}{40}\frac{\tau_d}{T} + \frac{7}{12} \tag{16}$$

There is no need to employ setpoint weighting if the setpoint response is good. The setpoint signal is not weighted if the value of b is chosen equal to one. Hence, the setpoint weight will not be far from one if the setpoint signal is fairly good. However, for small values of, τ_d/T the dimensionless gain, K_p K_c, given by Eq. 12 is large to reject load disturbance signals well. Therefore, the setpoint response is expected to be too oscillatory leading to a value of b which is far from one.

INTEGRATING PROCESSES

If the time constant, T, becomes very large, a FOPDT process is converted to an integrating process with dead time, as shown in Eq. 17.

$$G_p(s) = \lim_{T\to\infty} \frac{K_p e^{-\tau_d s}}{Ts+1} = \frac{K_p^{'} e^{-\tau_d s}}{s} \tag{17}$$

Where K^1_p is given by Eq. 18.

$$K_p^{'} = \frac{K_p}{T} \tag{18}$$

Therefore, PI tuning formulae for the integrating process in Eq. 17 are obtained by using Eq. 12, 13 and 16 for a very large time constant, as shown in Eq. 19-21.

$$K_p^{'} K_c = \frac{\frac{4}{7}}{\tau_d} \tag{19}$$

$$T_i = \frac{\frac{13}{12}}{\frac{3}{17}}\tau_d \approx \frac{43}{7}\tau_d \tag{20}$$

$$b = \frac{7}{12} \quad (21)$$

SIMULATION RESULTS

In this study, performance of the proposed method is compared with that of the method presented in (Astrom *et al*., 1998), which is one of the most prevalent techniques in PI tuning. For simplicity, the latter method is abbreviated as APH. Both methods aim to reject load disturbance signals and improve setpoint responses through setpoint weighting whilst having a constraint on maximum sensitivity of $M_s = 2$

Example 1:

$$G_1(s) = \frac{1}{(s+1)^3}$$

G_1 (s) is a third order model although the proposed PI tuning formulae are optimal for FOPDT processes. In order to obtain PI parameters suggested by the proposed method, the transfer function should be approximated by a FOPDT model. A simple method based on analysis of the open loop step response is given in Toscano (2005). Parameters of the FOPDT model are obtained using Eq. 22-24.

$$K_p = y_\infty \quad (22)$$

$$\tau_d = 2.8t_1 - 1.8t_2 \quad (23)$$

$$T = 5.5(t_2 - t_1) \quad (24)$$

where y_∞ is the final value of the step response of the process and t_1 (t_2) is the time when the output attains 28% (40%) of its final value. Applying this model reduction method to G_1 (s), its FOPDT approximation is given by

$$\hat{G}_1(s) = \frac{e^{-1.039s}}{2.448s + 1}$$

The closed-loop step responses given by the proposed and APH methods are shown in Fig. 5. The comparison results are shown in Table 2, where M_t is the maximum complementary sensitivity. P_d refers to the peak of the step disturbance response. T_d is the time required for the disturbance response to settle to within a tolerance of ±0.02.

Tuning is a trade off between conflicting design objectives. Fast speed of response and good load disturbance rejection are design goals in conflict with good robustness (Skogestad, 2003). As shown in Table 2, the proposed controller results in a faster response and a better load disturbance rejection but at the cost of having a larger maximum sensitivity.

An advantage of the proposed method is that the controller parameters are directly given by Eq. 12, 13 and 16 for FOPDT processes. For a higher order process, the

Table 2: Comparison of the performance of the proposed and aph methods to control G_1 (s)

Method	Proposed	APH
K_c	1.619	1.220
T_i	2.203	1.780
b	0.679	0.500
M_s	2.151	2.000
Mt	1.000	1.000
PO	12.77	11.94
T_s	12.23	11.07
Pd	0.418	0.458
Td	11.93	13.14

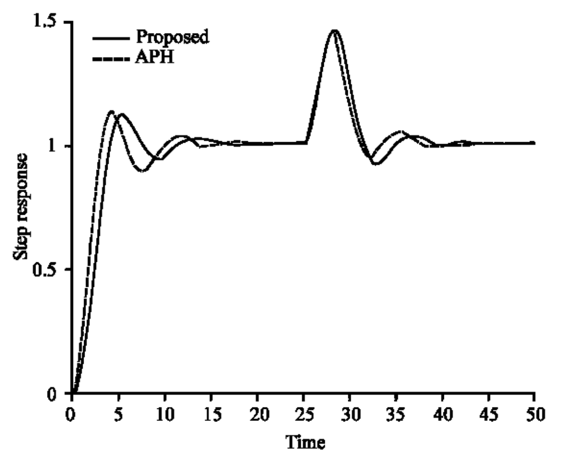

Fig. 5: Closed-loop step responses resulting from applying the proposed and APH methods to $G_1(s)$

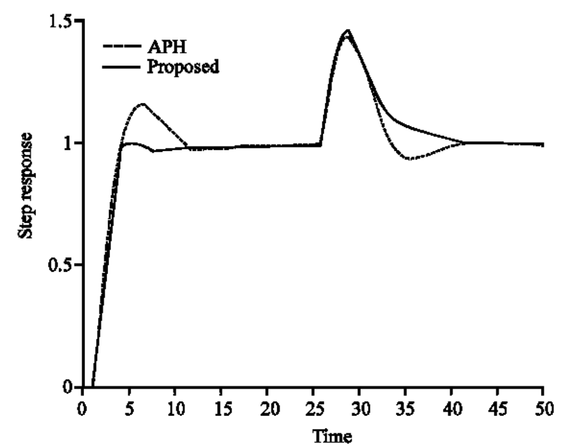

Fig. 6: Closed-loop step responses resulting from applying the proposed and APH methods to $G_2(s)$

Table 3: Comparison of the performance of the proposed and aph methods to control $G_1(s)$

Method	Proposed	APH
K_c	0.571	0.488
T_i	6.140	3.725
b	0.583	0.460
M_s	1.913	2.003
M_t	1.444	1.821
PO	0.550	17.21
T_s	10.20	10.66
Pd	0.455	0.477
Td	16.21	13.80

tuning formulae can be used after an appropriate model reduction. However, parameters of the APH controller are not explicitly given by a set of tuning formulae. They should be computed through a procedure.

Example 2:

$$G_2(s) = \frac{e^{-s}}{s}$$

$G_2(s)$ is an integrating process with dead time. The proposed and APH methods result in closed-loop step responses shown in Fig. 6.

The proposed method performs better in setpoint regulation and gives a smaller M_s, however, a faster load disturbance rejection is given by the APH controller. The comparison results are shown in Table 3.

CONCLUSION

Using GAs, a new set of robust PI tuning formulae for FOPDT processes is presented in this study. As an integrating process with dead time is a special case of a FOPDT process, tuning formulae for such processes are also given. The design problem considers three essential requirements of control problems. These requirements are load disturbance rejection, setpoint regulation and robustness of the closed-loop system against model uncertainties. The primary design goal is to optimize load disturbance rejection. Robustness is guaranteed by requiring that the maximum sensitivity is less than or equal to a specified value. In the first step, PI controller parameters are determined such that the IAE criterion to a load disturbance step is minimized and the robustness constraint on maximum sensitivity is satisfied. In the second step, good setpoint regulation is achieved by using a structure with two degrees of freedom, which introduces an extra parameter, the setpoint weight. The main advantage of the proposed method is its simplicity. As soon as the equivalent FOPDT model is determined, the PI parameters are explicitly given by a set of tuning formulae. Simulation studies for two examples show that the proposed PI controller can effectively deal with conflicting design requirements.

As the FOPTD processes are not representative for all processes encountered in process control, new research will attempt to consider a larger test batch to develop the tuning formulae.

APPENDIX

Dimensional analysis is often used to simplify a problem by reducing the number of its variables to the smallest number of essential ones (Zlokarnik, 1991). Using this technique, relations between variables in a physical system can be defined as relations between dimensionless numbers in the system with no change in the system behavior. A dimensionless number is a pure number without any physical unit. Such a number is typically defined as a product or ratio of quantities that do have units, in such a way that all units can be cancelled.

Assume that a system is expressed by Eq. 25.

$$x_1 = f(x_2, x_3, \ldots, x_n) \quad (25)$$

Where x_1, x_2,......., x_n are non-zero variables. Based on Buckingham's pi theorem (Zlokarnik, 1991), Eq. 25 can be replaced by Eq. 26.

$$\pi_1 = g(\pi_2, \pi_3, \ldots, \pi_{n-m}) \quad (26)$$

Where π_2,...., π_{n-m} are independent dimensionless numbers and m is the minimum number of x_2, x_3,......., x_n, which includes all the units in x_1, x_2,......., x_n.

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