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Multi-Item Inventory Control with Backlogging

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Abstract: This study presents a cost optimization model for multi-item inventory system having deterministic demansd. We assume that backlogging is allowed. The maximum capacity of the inventory is assumed to be constant Q_i. The total annual variable cost is derived in terms of Q_i and it is a non-linear function of Q_i. First the deterministic inventory problem is solved using constrained optimization technique. In addition to the above, we assume that the total expenditure and floor space are not precisely measured function. A non-linear programming problem is formulated and solved by Lagrange multiplier method. Numerical examples are provided for deterministic inventory problem to emphasize the results in crisp.

Key words: Multi-item inventory, non-linear programming, optimization

INTRODUCTION

Traditional inventory control problem deals with single item maintained in a location in which demands are fully backordered or backorder are not allowed or lost sales due to inability to meet the demand. Dealing the model with multi-item is tedious rather than the maintenance of single item. Various kinds of interactions between the items stocked. Some of them are floor space, capital budget investment and number of orders per year.

In the literature (Silver, 1981; Nadoor, 1966; Hamdy, 2005) a substantial portion is devoted to models of a single item in isolation from all other items. Multiple item problems (with intedependencies) can take variety of forms including, overall constraint as budget or space used by a group of items, coordinated control to save as replenishment costs, substitutable items: When a particular item is not in stock, the customer may be willing to accept a substitute product and complementary demand-certain products tend to be demanded together, in fact, the customer may not accept one without the other. Yet another kind of independently namely multiple stocking points (supply chain). All these kinds of problems have been studied by many researchers since 1950. Multi-item inventory with stochastic demand, with space or budget constraint was studied by Brown and Harray. Exchange curve principle and Lagrange multipliers are used as tools by researchers to analyze the problems to make the optimal decisions.

In this study we shall consider the problem of determining optimal reorder quantity for multi-item inventory problem under crisp. In the literature (Cinler, 1958; Silver and Peterson, 1985) the three types of constraints are considered separatively and solved them in crisp cost and other parameters. They considered constraints of three types separately and solved them in crisp cost and other parameters.

MODEL FORMULATION

Let us begin with a review of the model to be used in its crisp situation. An extensive discussion of this model is given in Hadely and Whitin (1963) and we shall summarize here that the material will be of use to us. A group of n items are placed in stock for a long period. There can be many sorts of interactions between items. Here we shall consider cases where there are constraints on the floor space and/or on the maximum number of orders per year which may be placed and/or on the maximum dollar investment in inventory. We assume that the yearly demand rate for the ith item be λ_i and ordering quantity be Q_i. Assume that backorders are allowed in the system and backorders are filled immediately after the next procurement arrives. In this case, for simplicity we consider only the constraint on floor space. We employ the following notations for the description of the system.

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Assumptions: Here we assume that the following cost structure proposed in the inventory control systems for each item i (i = 1, 2,..., n).

- Multi item inventory.
- Fuzzy model.
- Under one restrictions.
- Total floor space. (Not more than f sq. feet).
- Back orders are allowed.
- Lead time is allowed.

The inventory carrying cost per cycle for the ith item is

$$IC\int_0^{T_1}(Q_i - s_i - \lambda_i t)dt = I_iC_i\left[(Q_i - s_i)t - \frac{\lambda_i t^2}{2}\right]_0^{T_1}$$

$$= \frac{I_iC_i}{2\lambda_i}(Q_i - s_i)^2$$

Also it is area of the triangle 1 in the Fig. 1. There are $\frac{\lambda}{Q}$ cycles per year on average. So that the average yearly cost of carrying inventory is

$$\frac{IC(Q-s)^2}{2Q}$$

The back order cost per cycle for the ith item is

$$\pi_i s_i + \hat{\pi}\int_0^{T_2}\lambda t\,dt = \pi_i s_i + \frac{1}{2}\hat{\pi}\lambda_i\left(\frac{s_i}{\lambda_i}\right)^2$$

$$= \pi_i s_i + \frac{\hat{\pi} s_i^2}{2\lambda_i}$$

Hence the average annual cost of backorders is

$$\frac{\pi s_i + \frac{\hat{\pi} s_i^2}{2\lambda}}{T} = \frac{1}{Q_i}\left[\pi_i \lambda_i s_i + \frac{\hat{\pi} s_i^2}{2}\right]$$

Therefore, the average annual variable cost K includes the cost of ordering, holding inventory and back orders for all the items

$$K = \sum_{i=1}^{n}\left[\frac{\lambda_i}{Q_i}A + \frac{I_iC_i}{2Q_i}(Q_i - s_i)^2 + \frac{1}{Q_i}\left(\pi_i\lambda_i s_i + \frac{\hat{\pi} s_i^2}{2}\right)\right]$$

$$K = \sum_{i=1}^{n}\left[Q_i^{-1}\lambda_i A + 2h_i Q_i^{-1}(Q_i - s_i)^2 + Q_i^{1}\left(\pi_i\lambda_i s_i + \frac{\hat{\pi} s_i^2}{2}\right)\right]$$

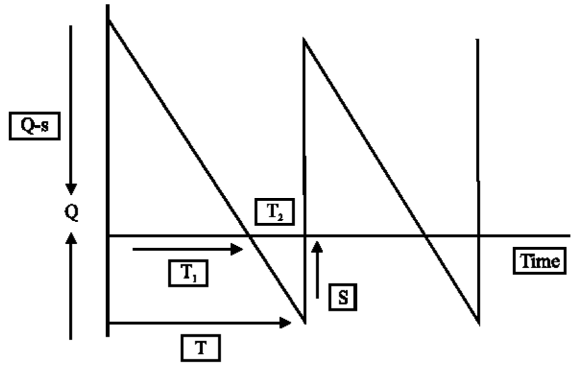

Fig. 1: Inventory system with backlogging

Here K is the function of Q_i and s_i. Here we wish to find the absolute minimum of K in the region $0<Q_i<\infty$, $0 = s_i$. We impose constraint with respect to the warehouse floor space. Assume there is an upper limit f to the sq. ft. of warehouse floor space. Suppose that n items are being stocked and that one unit of item i takes up f_i sq. ft. of floor space. Here Q_i is the order quantity for the item i, then.

$$\sum_{i=1}^{n} f_i(Q_i - s_i) \le f$$

CRISP OPTIMIZATION PROBLEM

The cost involved in the system is inventory carrying cost and ordering costs for each procurement. Then the average annual variable cost for all the items is,

$$K = \sum_{i=1}^{n}\frac{\lambda_i A}{Q_i} + \frac{I_iC_i}{2Q_i}(Q_i - s_i)^2 + \frac{1}{Q_i}\left(\pi_i\lambda_i s_i + \frac{\hat{\pi} s_i^2}{2}\right)$$

We desire to find the absolute minimum of K in the region $0<Q_j<\infty$, $s_i \ge 0$. The cons traints involved in the problem are the floor space for which the items in stock complete. Each item in stock has its share in floor space f_i sq. feet and the constraint is expressed as

$$\sum_{i=1}^{n} f_i(Q_i - s_i) \le f$$

Where f is the maximum floor space available. Now, the proposed problem becomes a Non Linear Programming Problem.

$$\text{Minimize}\, K = \sum_{i=1}^{n}\frac{\lambda_i A}{Q_i} + \frac{I_iC_i}{2Q_i}(Q_i - s_i)^2 + \frac{1}{Q_i}\left(\pi_i\lambda_i s_i + \frac{\hat{\pi} s_i^2}{2}\right) \quad (1)$$

Subject to

$$\sum_{i=1}^{n} f_i(Q_i - s_i) \le f \qquad (2)$$

where $Q_i \ge 0$ $s_i \ge 0$. The optimal value of Q and s are obtained from (1) by just ignoring the constraint (2) using classical differentiation technique. The optimal Q, i.e., Q* must indeed satisfy 0< Q* <∞, if the optimal s, i.e, s* satisfies 0<s* <∞, the differential calculus requires that since K is differential everywhere in the region of interest Q* and s* must satisfy the equation

$$\frac{\partial K}{\partial Q} = 0 \qquad \text{and} \qquad \frac{\partial K}{\partial s} = 0$$

$$\frac{\partial K}{\partial Q_i} = 0 \Rightarrow \frac{1}{2} Q_i^{\,2} = \frac{1}{I_i C_i}\left[\lambda_i A_i + \pi_i \lambda_i s_i + \frac{1}{2}\hat{\pi} s_i^{\,2}\right] + \frac{1}{2} s_i^{\,2}$$

$$\frac{\partial K}{\partial Q_i} = 0 \Rightarrow \frac{1}{2} Q_i = \frac{\pi_i \lambda_i}{I_i C_i} + s_i\left(1 + \frac{\hat{\pi}}{I_i C_i}\right), \quad i = 1, 2, \ldots, n$$

Solving the above equations for Q_i and s_i we obtain,

$$s_i^{\,*} = (\hat{\pi} + I_i C_i)^{-1}\left\{-\pi_i \lambda_i + \left[\begin{array}{l} 2\lambda_i A_i I_i C_i - \left(1 + \frac{I_i C_i}{\hat{\pi}}\right) \\ - \frac{I_i C_i}{\hat{\pi}}(\pi_i \lambda_i)^2 \end{array}\right]^{\frac{1}{2}}\right.$$

and

$$Q_i^{\,*} = \left[\frac{\hat{\pi} + I_i C_i}{\hat{\pi}}\right]^{\frac{1}{2}}\left[\frac{2\lambda_i A_i}{I_i C_i} - \frac{(\pi_i \lambda_i)^2}{I_i C_i(\hat{\pi} + I_i C_i)}\right]^{\frac{1}{2}}$$

Suppose $\hat{\pi}$ and π are nonzero constants.

Suppose $\hat{\pi}_i = 0$, but $\pi_i \ne 0$ for the item i, then obviously $s_i = 0$ or $s_i = \infty$ is optimal. When $s^*_i = 0$, the corresponding optimal value of Q is Q_w given as $Q_{iw} = \sqrt{\frac{2\lambda_i A_i}{h_i}}$. and the minimum root is $K_w = \sqrt{2\lambda_i A h_i}$. When s* is infinity we get an unrealistic inventory system.

$$\text{Let } \delta_i = \sqrt{\frac{2Ah_i}{\lambda_i}} \text{ for } I = 1, 2, \ldots, n$$

If $\pi_i > \delta_i$ then the optimal solution for s is s* = 0 and $Q^*_{iw} = \sqrt{\frac{2\lambda_i A_i}{h_i}}$. If $\pi_i < \delta_i$ then the inventory system is ceased to exist. If $\pi_i = \delta_i$, any value of s in the interval [0, ∞] is optimal (the optimal Q* depends on chosen value of s).

Next we consider the case, when $\pi_i = 0$. In this case the optimal values s_i^* and Q^*_i are given by,

$$s_i^{\,*} = \left[\frac{2\lambda_i A_i I_i C_i}{\hat{\pi}(\hat{\pi} + I_i C_i)}\right]^{1/2}$$

and

$$Q_i^{\,*} = \left[\frac{\hat{\pi} + h_i}{\hat{\pi}}\right]^{1/2}\sqrt{\frac{2\lambda_i A_i}{h_i}}, \quad i = 1, 2, \ldots, n$$

Here we observe that when $\pi_i = 0$, then $s_i^* > 0$ unless $\hat{\pi}_i > \infty$. Thus under optimal operating conditions, some backorders will always be incurred. Our next concern is the feasibility of constraint which restricts the floor space available for inventory in business. If the optimal s_i^* and Q_i^* are obtained the above satisfy the inequality (2). Then these Q_i's and s_i's are optimal solutions to our NLPP. This means that sufficient floor space is available, so that the average annual variable cost cannot be reduced by increasing the floor space used for stock.

On the other hand, if s_i^* and Q_i^* are computed the above does not satisfy the constraint relation (2), the constraint (floor space) is active and the Q_i's obtained are not optimal. The optimal s_i^* and Q_i^* can be obtained by applying Lagrange multiplier method. Consider the Lagrange multiplier function for the given cost structure is

$$L(s_i, Q_i, \theta) = \sum_{i=1}^{n}\left[\begin{array}{l} \frac{\lambda_i}{Q_i} A + \frac{h_i}{2Q_i}(Q_i - s_i)^2 \\ + \frac{1}{Q_i}(\pi_i \lambda_i s_i + \frac{\hat{\pi}\, s_i^2}{2}) \\ + \theta\left(\sum_{i=1}^{n} f_i(Q_i - s_i) - f\right) \end{array}\right]$$

where, θ is the Lagrange multiplier. The set of values Q_i, i = 1, 2, …, n which yields the absolute minimum of K subject to constraint (2) are solution to the set of equations

$$\frac{\partial L}{\partial Q_i} = 0 = -\lambda_i A Q_i^{-2} + \frac{h_i}{Q_i}(Q_i - s_i) - \frac{1}{2}(Q_i - s_i)^2 h Q_i^{-2} - \frac{1}{Q_i^2}\left[\lambda_i s_i + \hat{p}\frac{s_i^{\,2}}{2}\right] + \theta \sum_{i=1}^{n} f_i$$

Solving the above equations we get the optimal values of and which gives the absolute minimum of *K* subject to the given constraint (2) as

$$Q_i^* = \sqrt{\frac{2\lambda_i A(h_i + \hat{p}) - (p_i \lambda_i)^2}{\hat{p}(h_i + 2?\, f_i) - (\theta\, f_i)^2}} \quad (3)$$

and

$$s_i^* = \frac{1}{\hat{\pi} + h_i}\left[(h_i + \theta f_i)\sqrt{\frac{2?_i A(h_i + \hat{\pi}) - (\pi_i \lambda_i)^2}{\hat{\pi}\,(h_i + 2?\, f_i) - (?\, f_i)^2}} - \pi_i \lambda_i\right] \quad (4)$$

where θ is the solution of the Equation,

$$\sum_{i=1}^{n} f_i \left[\begin{matrix}\sqrt{\frac{2\lambda_i A(h_i + \hat{p}) - (p_i ?_i)^2}{\hat{p}(h_i + 2\theta f_i) - (\theta\, f_i)^2}} - \frac{1}{\hat{\pi} + h_i} \\ \left[(h_i + \theta f_i)\sqrt{\frac{2\lambda_i A(h_i + \hat{p}) - (p_i \lambda_i)^2}{\hat{p}(h_i + 2\theta f_i) - (\theta\, f_i)^2}} - \pi_i \lambda_i\right]\end{matrix}\right] - f = 0$$

Substitute the value of è in (3) and (4) we get the optimal values of s_i and Q_i. Then finally we determine the value of total annual variable cost.

NUMERICAL EXAMPLE FOR CRISP MODEL

To illustrate the above method we consider the following the parametric values of 3 items (n=3). Here we find the optimum values for the above two cases. That is for constraint and unconstraint cases. In the constraint case, we find the optimal solution is obtained by using Lagrange's multiplier method. Then we compute optimal values of's.

For example The collected data from the system controls (Hadley and Whitin, 1963) depicts as follows (Table 1).

The management derives, never more than 2500 square feet are floor area and backorder cost varying with time $\hat{\pi} = \$10$ and fixed order cost A=10. In this problem, we have, $h_i = I_i C_i$ ($h_1 = 7.5$, $h_2 = 2.5$, $h_3 = 5$).

Table 1: Collected data from the system control

ITEM	1	2	3
Demand rate λ_i	1000	3000	2000
Variable cost C_i	30	10	20
Carrying cost I_i	0.25	0.25	0.25
Floor space f_i	5	10	8
Backorder cost π_i	0.271	0.091	0.158

Case (i): If 0 and $\hat{\pi} = \$10$ for all i = 1, 2, 3, The optimal values of s_i and Q_i given by

$s_1^* = 10.291$ $Q_1^* = 60.55$

$s_2^* = 11.034$ $Q_2^* = 164.369$

$s_3^* = 12.271$ $Q_3^* = 100.034$

The corresponding annual variable cost is K = 1198.968

Case (ii): If $\pi_1 \neq = 0$ and $\hat{\pi} \neq 0$ The optimal values of s_i and Q_i given by

$s_1^* = 29.277$ $Q_1^* = 68.319$

$s_2^* = 34.641$ $Q_2^* = 173.2$

$s_3^* = 36.514$ $Q_3^* = 109.57$

The corresponding annual variable cost is K = 1004.33

Case (iii): If $\hat{\pi} = 0$

When $\pi_i\, \delta_i >= \sqrt{\frac{2Ah_i}{\lambda_i}}$ and $s^* = 0$

$Q_1 = 51.64$, $Q_2 = 154.919$ and $Q_3 = 89.443$.

The corresponding annual variable cost is K = 1221.816

When $\pi_i \delta_i <$ then we have no solution (inventory cases).

When $\pi_i = \delta_i$ $0 \leq s \leq \infty$

Choose $s_1 = 12$ $Q_1 = 63.6$

$s_2 = 14$ $Q_2 = 168.8$

$s_3 = 16$ $Q_3 = 103.6$

The corresponding annual variable cost is K = 1221.933

In the case of constraint optimization, Lagrange multiplier method is used to compute the optimal solution for $\pi = 0$, $\hat{\pi} = 0$

Case (i): If $\pi = 0$ and $\hat{\pi} \neq 0$ ($10)

$s_1 = 29.309$ $Q_1 = 65.213$

$s_2 = 35.561$ $Q_2 = 137.621$

$s_3 = 36.716$ $Q_3 = 98.629$

The corresponding annual variable cost is K = 1020.161

Case (ii): If $\pi \neq 0$ and, $\hat{\pi} \neq 0$

Here we obtain θ = -0.00169

$s_1 = 10.539$ $Q_1 = 60.7929$

$s_2 = 11.035$ $Q_2 = 165.49$

$s_3 = 12.271$ $Q_3 = 100.28$

The corresponding annual variable cost is K = 1198.462

Analysis: A structure of the annual average cost expression is considered for analysis.

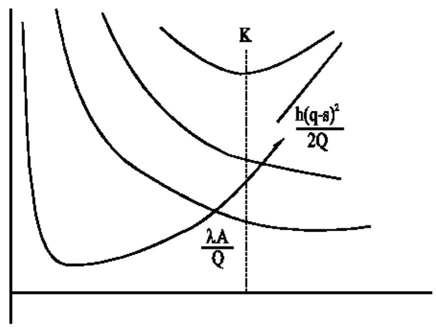

Fig. 2: Cost curve for K

$$\Sigma\left[\frac{\lambda_i}{Q_i}A + \frac{2\lambda_i}{Q_i}(Q_i - s_i)^2 + \frac{1}{Q_i}(\pi_i\lambda_i s_i + \frac{\pi^{\Lambda} s_i^{2}}{2})\right]$$

This contains three different components, namely ordering cost, inventory carrying costs and backordering costs. Plotting the cost curves alone and its total K as a function of Q for fixed s. [When $\hat{\pi} = 0$ and = $\pi_i\sqrt{\frac{2Ah_i}{\lambda_i}}$ all possible s>0 are optimal]. For convenience we first take the case $\hat{\pi} = 0$ and π_i 0 when $\pi_i\sqrt{\frac{2Ah_i}{\lambda_i}}$ =. The plotted curves are Cost curve for K. The Fig. 2 shows that the optimal Q* is unique and given by the relation $\frac{d^2k}{d\,Q^2} \succ 0$ for all Q>0 and fixed s>0

The average annual cost of procurement and holding inventory is given by

$$K = \sum_{i=1}^{n}\left[\frac{\lambda_i}{Q_i}A + \frac{2\lambda_i}{Q_i}(Q_i - s_i)^2 + \frac{1}{Q_i}(\pi_i\lambda_i s_i + \frac{\pi^{\Lambda} s_i^{2}}{2})\right]$$

CONCLUSION

In this study, multi-item inventory models with backlogging have been proposed with floor space constraint. In literature, most of the multi-item problems with backlogging have been considered with single constraint or no constraint. Models with a single constraint in deterministic have been solved. Multi-item inventory systems with lost sales cases can be considered for further research.

Notations

- n = Number of items
- D = Inventory capital i.e., upper limit to the dollar investment in inventory.
- λ_i = Yearly demand rate for the ith item.
- Q_i = Ordering quantity for the item i.
- I_i = Carrying charge for the item i.
- C_i = Unit cost for the item i.
- h_i = Holding cost for the item i. where $h_i = I_i C_i$
- f = Maximum floor space.
- θ = Lagrange multiplier
- A = Fixed ordering cost for all items.
- π = Cost of back orders.(fixed)
- $\hat{\pi}$ t = The value is proportional to the length of the time for which theback orders exists.
- s_i = Number of backorders when a procurement of Q_i units arrives (>0) for ith item.
- Q_i - s_i = On hand inventory after satisfying the backorders.
- T_1 = The time required for Q-s units to be demanded. i.e., $\frac{Q_i - s_i}{\lambda_i}$
- T_2 = The length of the time over which backorders will be incurred.(T- T_1) where T is the length of each cycle.

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