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H∞ Robust Control for a Plasma CF4/O2 System

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Abstract: This study describes a multivariable H∞ robust control approach, for an ill conditioned, difficult to control system, given by a plasma chemical reactor CF4/O2. The control of such a system resulted, in the past, in a slow, oscillatory coupled response. The goals of applying H∞ robust control, is the achievement of a decent smooth response with minimum oscillations and overshoot. In the pursuit of this goal, we proceed to the quantification of uncertainties affecting the plasma process. Then, the robustness conditions in closed loop, to be achieved, are clearly stated. Only then, the H∞ algorithm is applied, after the construction of the augmented system resulting from the standard process form. The results obtained showed, clearly, the advantages of the H∞ robust control approach, within the robustness a-priori set conditions.

Key words: Robust control, H_∞ control, multi variable control, ill-conditioned problems

INTRODUCTION

Particular industrial applications, such as the aeronautics, chemical reactors as well as spatial structures are more than ever in the heart of the current control preoccupations. This revival is bound to the constant increase of quality requirements and performance of the controlled systems. Faced with these challenges, dynamic based control systems appears as an inescapable tool compared to graphically based control methods, not often suitable for control design of complex problems.

With the emergence of Computer-Aided design (C.A.O) tools, based on the convex optimization under constraints LMI (Boyd *et al*., 1994; Scorletti, 2005), classical frequency control methods witnessed considerable growth (Scorletti, 2005). Nowadays, a sufficient condition to efficiently control a system is to find an algorithm that minimizes a certain control criteria satisfying the control needs. One way of achieving the minimization of the mathematical control formulation, is firstly to express them on the H_∞ norm. The control approaches based on this norm, are then given the name of H_∞ control.

The control of linear systems using H_∞ control, is a solved issue with many applications in the 1990s (Doyle and Stein, 1981; Doyle *et al*., 1989; Font, 1995) it is, since then, gaining ground on nonlinear system control (Fromion, 1995; Fromion *et al*., 2001) with the main target remaining, difficult control problems. Keeping in mind the main goal; validating the synthesized H_∞ controller not only on the process model, but on the process itself. Ensuring the control of the physical process despite model/plant mismatch is the core idea of "Robust Control" (Safonov, 1980).

This study proposes H_∞ control for a multivariable chemical plasma reactor CF4/O2 difficult to control. The process was studied by McLaughlin *et al.* (1989). The multivariable process proved to be ill-conditioned and difficult to control. This reflected in a slow, oscillatory, coupled response with significant overshoot, in the time domain.

The objectives are then clearly stated, as suppose to, improve the response time and reduce oscillations using H_∞ robust control.

PROCESS DESCRIPTION OF THE PLASMA CF_4/O_2 REACTOR

The plasma process CF_4/O_2 is a square multivariable system, with four inputs and four outputs. The output variables to control are:

- [F] is the Fluor
- [CF2] is the Difluor of Carbon.
- [CO2] Dioxyde of Carbon and
- E/P: E is the Electric field and P is the Pressure.

While the process inputs are given by:

- O2: Is the oxygen
- Pr: Is the Pressure in the reactor chamber
- Pa: Is the Feeding power
- Db: Is the Flow rate

For more details refer to Mclaughline *et al.* (1989). The obtained process model is given by the following transfer function:

$$G(s)=\begin{bmatrix} 0.671/(1+0.105s) & 0.726/(1+0.105s) & 0.190/(1+0.105s) & 0 \\ 0 & 0.856/(1+0.053s). & 0.587/(1+0.011s) & 0 \\ 0.303/(1+0.105s) & 0.709/(1+0.105s) & 0.626/(1+0.105s) & 0 \\ 0 & -1.06/(1+0.053s) & 0.276/(1+0.053s) & 0.004/(1+0.053s) \end{bmatrix} \quad (1)$$

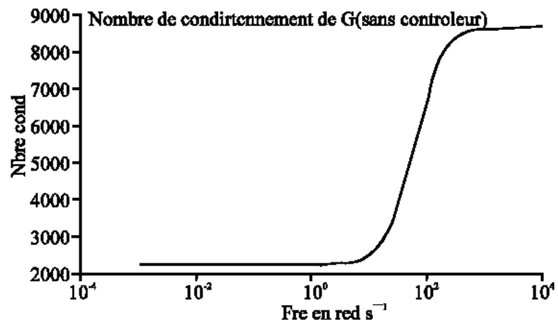

Fig. 1: Conditioning number

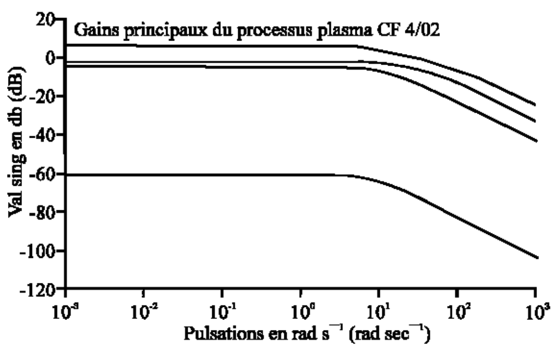

Fig. 2: Principal gains for the Plasma process CF4/O2

As stated in the previous section, the process is ill-conditioned, i.e., strongly coupled. This results in a high conditioning number, of around 2300, in low frequencies, as shown in Fig. 1. The ill-conditioning problem is emphasized by the fact that three main gains, are very close to each others and that the minimal principal gain, is relatively far away. The gains are shown in Fig. 2.

This process is thus very difficult to control, indeed the temporal response obtained by the authors, using Singular Value Decomposition (SVD) method and the Relative Gain Array (RGA) method, turn out to be slow (0.35mn) and an oscillatory coupled transitory response (Mclaughlin *et al*., 1989).

UNCERTAINTIES QUANTIFICATION

The model G_{mod} (s) obtained by identification, represents only the behavior of a given functioning input/output space, where high frequency dynamics are neglected along with some system nonlinearities.

We are thus in front of a dynamic uncertainty, which concerns the structure of the system. The initiative which we adopt to estimate the errors between the real system and its model consists in introducing a stable transfer $\Delta(s)$ representing a multiplicative output uncertainty, of the form:

$$\Delta(s)=(G_{real}(s) - G_{mod}(s))G^{-1}_{mod}(s) \quad (2)$$

$\Delta(s)$ is a non structured model uncertainty.

This relation (2) translates the fact that in low frequencies one cannot tolerate an error of 100% but that in high frequencies the error believes and exceeds the 100%.

The application of the small gain theorem allows guaranteeing the stability of the closed loop system. Therefore, the transfer matrix involved in the case of a multiplicative uncertainty, is the additional sensibility function. We shall thus have:

$$\|T(s)\Delta(s)_\infty\|_\infty < 1 \Rightarrow \|\Delta(s)\|_\infty < 1/\|T(s)\|_\infty \quad (3)$$

$$\Rightarrow \|\Delta(s)\|_\infty \le \|W_t(s)\|_\infty \quad (4)$$

The relation (4) implies:

$$\partial_{max}(\|\Delta(s)\|_\infty) \le \partial_{max}(\|W_t(s)\|_\infty) \quad (5)$$

Hence, the parametric disturbance is generally limited by a specification on the stability, like seen in relation (4) and which is illustrate in Fig. 3. That's because:

$$W_t(s) = \begin{vmatrix} 0.7(0.053s+1) & 0 & 0 & 0 \\ 0 & 0.7(0.053s+1) & 0 & 0 \\ 0 & 0 & 0.7(0.053s+1) & 0 \\ 0 & 0 & 0 & 0.7(0.053s+1) \end{vmatrix} \quad (6)$$

but we have:

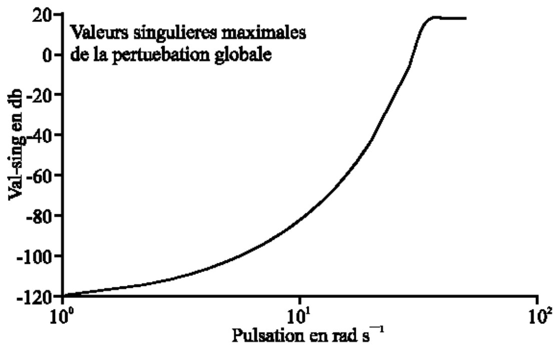

Fig. 3: Maximum singular values of the global disturbance

$$\|W_t(s)\|_\infty \leq 1/\|T(s)\|_\infty. \quad (7)$$

Then inequality (7) may then be rewritten as:

$$\partial_{max}(T(s)) \leq \partial_{max}(1/W_t(s)). \quad (8)$$

Where, the inequality (8) is the sufficient robustness condition on stability.

PREPARATION OF THE DATA

The H_∞ control approach is distinguished by two specific characteristics. The first one is the use of a system representation on its standard form. This form, allows obtaining a synthesized model configuration augmented by the weighted factor which translates the robustness objectives on performance and stability. The second concerns the use of the system on its frequency domain form, allowing a detailed description in closed loop.

The standard form which we are going to use within the control law synthesis framework of Plasma process CF4/O2 is known under the name of 'Mixed Sensitivity'. This form allows taking into account simultaneously the robustness objectives on performances and stability in the synthesis procedure considering only two transfer functions in closed loop: The sensibility and the complementary sensibility. This gave the name of ' 2 blocs' criteria ' to the formulated H∞ problem, Fig. 4.

Robustness condition: The robustness condition on stability is consequence of the application of the small gain theorem. The condition is given by:

$$\partial_{max}(T(s)) \leq \partial_{max}(1/W_t(s)). \quad (9)$$

The robustness condition on performance is given by:

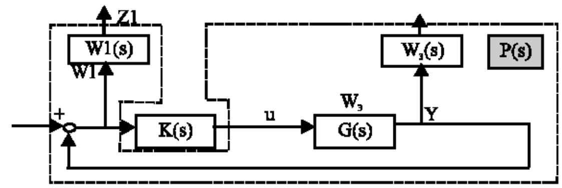

Fig. 4: 'Two blocs' criteria

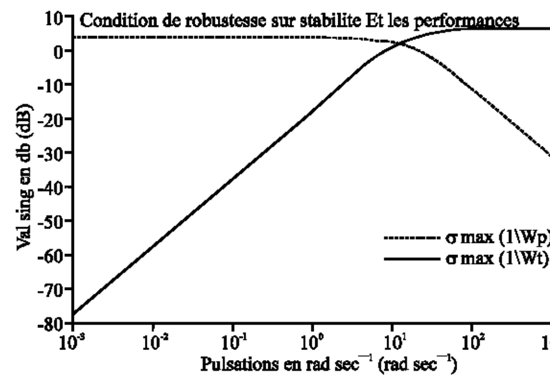

Fig. 5: Robustness conditions on stability and performances

$$\partial_{max}(S(s)) \leq \partial_{max}(1/W_p(s)). \quad (10)$$

The performance specifications are chosen so that the static error is null, response time in loop closed must be of the same order of the open loop one and finally the response of the system must be soft i.e. that there should not be important oscillations where:

$$W_p(s) = [(1+\alpha s)/\alpha s].\ I \quad (11)$$

$$W_p(s) = \begin{vmatrix} \frac{0.07s+1}{0.07s} & 0 & 0 & 0 \\ 0 & \frac{0.07s+1}{0.07} & 0 & 0 \\ 0 & 0 & \frac{0.07s+1}{0.07} & 0 \\ 0 & 0 & 0 & \frac{0.07s+1}{0.07s} \end{vmatrix} \quad (12)$$

We can remark that the robustness condition on performances appeals to the sensibility, which is no other than I-T(s). This completeness indicates the consideration necessity of the old dilemma stability/ performance. The importance to find a compromise between these two objectives is essential because any adaptation on stability engenders a performance adaptation, Fig. 5.

Construction of the augmented system: Starting from the augmented system configuration given by Fig. 4, the augmented system transfer matrix P(s), may be deducted from Fig. 4.
Where:

r : Set point
u : Control variables
w : Criteria inputs
y : Measured outputs
z : Controlled outputs

$$P(s) = \begin{bmatrix} W_1(s) & -W_1(s)G_{mod}(s) \\ 0 & W_3(s)G_{mod}(s) \\ I & -G_{mod}(s) \end{bmatrix} \quad (13)$$

Where,

$$W_1(s) = W_p(s) \text{ et } W_3(s) = W_t(s)$$

The transfer function in closed loop between and is the cost function and is given by:

$$\Rightarrow T_{ZW} = \begin{bmatrix} W_1 S \\ W_3 T \end{bmatrix} \quad (14)$$

Posing the H_∞ synthesis control problem, consist in minimizing the transfer according to the H_∞ norm, as follow:

$$\|T_{ZW}\|_\infty \le 1. \Rightarrow \|W_1 S\|_\infty < 1 \text{and } \|W_3 T\|_\infty < 1. \quad (15)$$

Taking: $W_1 = W_p$ and, $W_3 = W_t$ we get:

$$\|T_{ZW}\|_\infty \le 1. \Rightarrow \|W_p S\|_\infty < 1 \text{ and } \|W_t T\|_\infty < 1. \quad (16)$$

In order to realize the robust controller, the H∞ approach follows an algorithm solving two Ricatti equations obtained from the minimization of the control problem (Stein, 1987; Limebeer *et al.*, 1988; Chiang and Safonov, 1988).

RESULTS AND DISCUSSION

Applying the H∞ approach in order to synthesize a robust controller for the plasma CF4/O2 process, it was compulsory to go through the following steps:

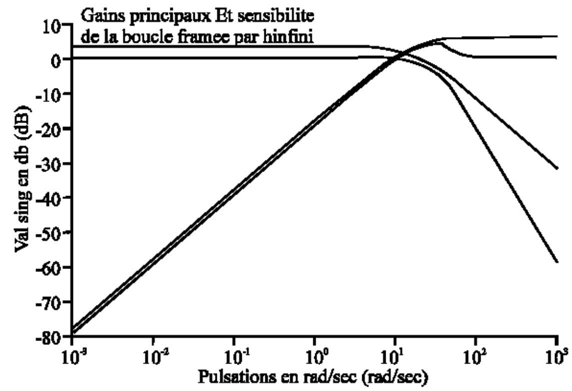

Fig. 6: Principal gains and sensitivity of the closed loop by application of H control σ max (T) (dashed line), σ max(S) (dot line)

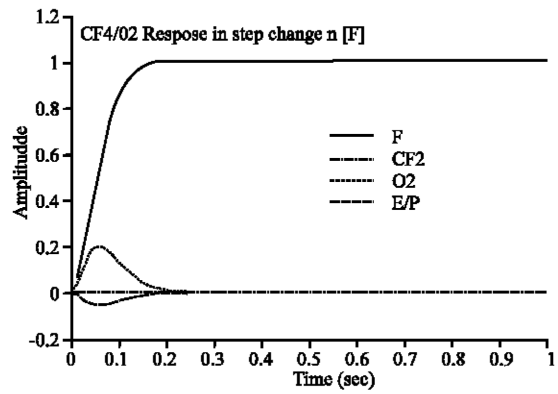

Fig. 7: Step response of CF4/O2: Step input on [F]

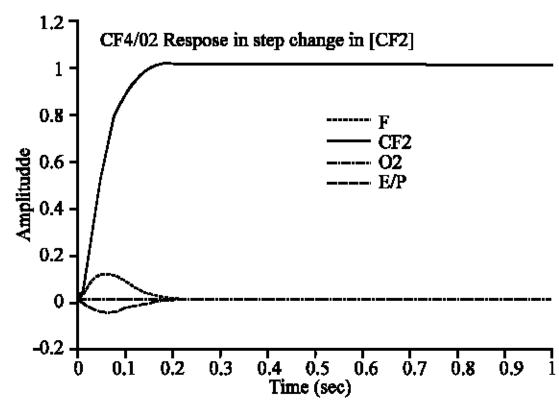

Fig. 8: Step response of CF4/O2: Step input on [CF2]

- Designing a process model P(s) augmented by the specifications on performance and stability.
- Application of the H∞ algorithm
- Representation of the results in the time and frequency domains.

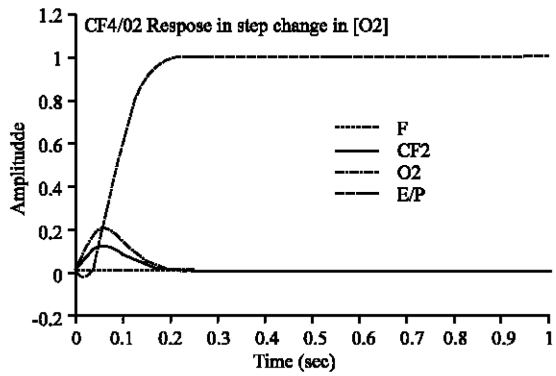

Fig. 9: Step response of CF4/O2: Step input on [O2]

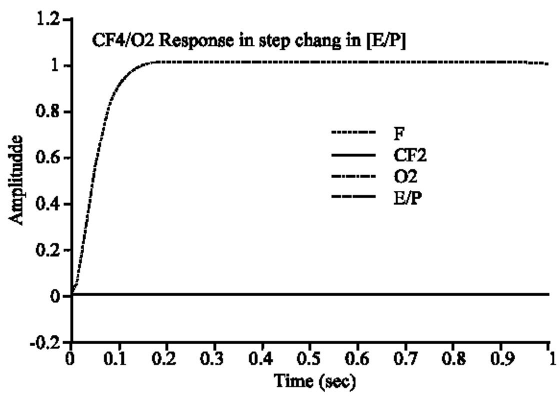

Fig. 10: Step response of CF4/O2: Step input on [E/P]

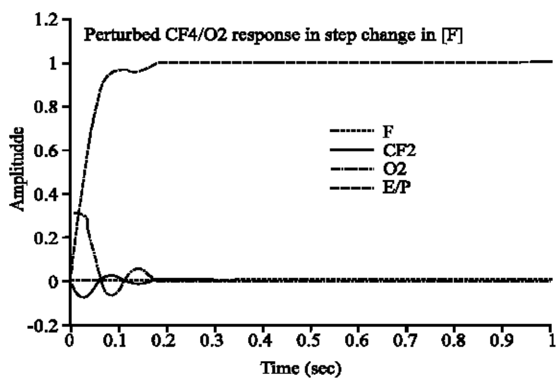

Fig.11: Step response of Perturbed CF4/O2: Step input on [F]

Performance in frequency domain: The H∞ robust controller, give the frequency domain results shown in Fig. 6, highlighting the following issues:

- A perfect principal gains superposition in closed loop, proving a total process decoupling.

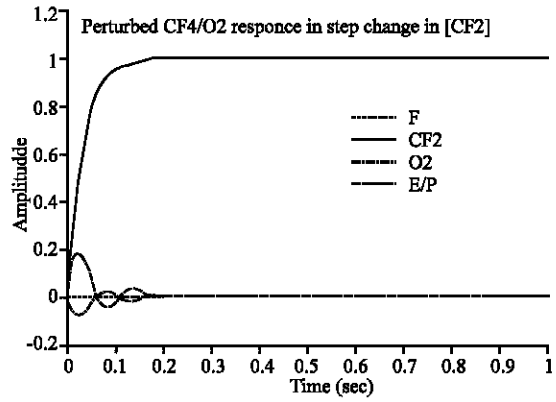

Fig. 12: Step response of Perturbed CF4/O2: Step input on [CF2]

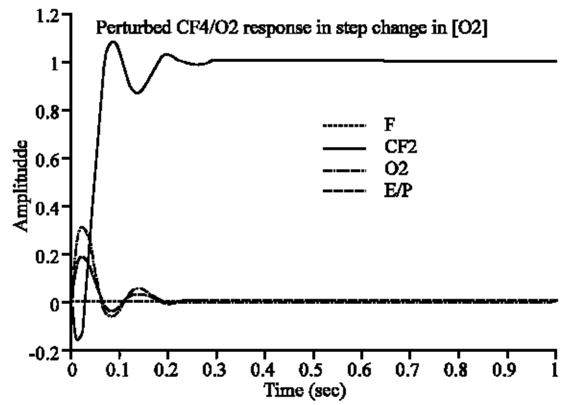

Fig. 13: Step response of Perturbed CF4/O2: Step input on [O2]

- Scrupulous respect of the robustness condition on stability. This is shown by the principal process gains are well under stability specification limit.
- Respect of the robustness condition on performance, highlighted by the maximum process stability situated under performance specification limit.

Performances in time domain: Step responses of the Plasma CF4/02 process; represent the time domain responses for input steps on:

[F] ($[1\ 0\ 0\ 0]^T$), [CF2] ($[0\ 1\ 0\ 0]^T$), [CO2] ($[0\ 0\ 1\ 0]^T$) and on [E/P] $[0\ 0\ 0\ 1]^T$

As can be seen, in Fig. 7-10, substantial improvements are observed when compared to the ones obtained by McLaughlin *et al.* (1989):

- A faster response time improving from 0.35-0.175mn.
- Pure elimination of overshoot.
- Sensible lowering of transient response interactions.

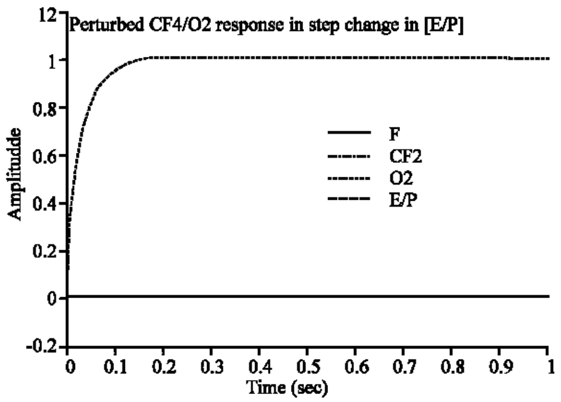

Fig. 14: Step response of Perturbed CF4/O2: Step input on [E/P]

The step responses of the disturbed model were also the subject of our interest and were considered through the application of the H_∞ control law on the perturbed process. The Fig. 11-14 illustrate these responses and show very satisfactory results.

CONCLUSION

It is clear from the results above that the application of H_∞ control to the Plasma CF4/O2 process, succeeded in improving frequency and time domain closed loop responses with respect to previously obtained results. Moreover, the H_∞ design successfully combined performance and stability objectives chosen, in order to implement a robust controller achieving good performances overcoming the dilemma stability/ performance.

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