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Wavelet Based Image Denoising Using Adaptive Subband Thresholding

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Abstract: This study proposes an adaptive, data-driven threshold for image denoising via wavelet soft-thresholding based on the Generalized Gaussian Distribution (GGD) widely used in image processing applications. The proposed threshold is simple and it is adaptive to each sub band because it depends on data-driven estimates of the parameters. In this proposed method, the choice of the threshold estimation is carried out by analyzing the statistical parameters of the wavelet sub band coefficients like standard deviation, variance. Our method describes a new method for suppression of noise in image by fusing the wavelet denoising technique with optimized thresholding function improving the denoised results significantly. Simulated noise images are used to evaluate the denoising performance of proposed algorithm along with another wavelet-based denoising algorithm. Experimental results show that the proposed denoising method outperforms standard wavelet denoising techniques in terms of the PSNR and the prevented edge information in most cases. We have compared this with various denoising methods like wiener filter, VisuShrink and BayesShrink.

Key words: Gaussian noise, image denoising, wavelet transform, thresholding

INTRODUCTION

Many scientific data sets are contaminated with noise either because of data acquisition process or because of naturally occurring phenomenon. For example during the image acquisition, the performance of imaging sensors is affected by a variety of factors, such as environmental conditions and by the quality of the sensing elements themselves. For instance, in acquiring images with a CCD camera, light levels and sensor temperature are major factors affecting the amount of noise in the resulting image. Images are also corrupted during transmission, due to interference in the channel used for transmission. Image denoising techniques are necessary to remove such random additive noises while retaining as much as possible the important signal features. The main objective of these types of random noise removal is to suppress the noise while preserving the original image details. Especially for the case of additive white Gaussian noise a number of techniques using wavelet-based thresholding have been proposed. Donoho and Johnstone (1995) proposed hard and soft thresholding methods for Denoising . This exterminates many wavelet coefficients that might contain useful image information. However, the major problem with both methods and most of its variants is the choice of a suitable threshold value.

The definition of coefficient independent threshold given by Donoho (1994) depends on the noise power and the size of the image. In practice, however, one deals with images of finite size, where the applicability of such a theoretical result is rather questionable. In addition, most signals show a spatially non-uniform energy distribution, which motivates the choice of a non-uniform threshold. Besides wavelet-thresholding, many other approaches have been suggested as well. For example, wavelet-based denoising using Hidden Markov Trees, which was initially proposed by Crouse *et al*. (1998) and Romberg *et al.* (1999) has been quite successful and it gave rise to a number of other HMT-based schemes. They tried to model the dependencies among adjacent wavelet coefficients using the HMT and used the Minimum Mean-Squared Error (MMSE)-like estimators for suppressing the noise. Since wavelet provides an appropriate basis for separating noisy signal from image signal there has been a fair amount of research on wavelet thresholding and threshold selection for signal and image denoising. In this study we present an efficient thresholding technique for image denoising by analyzing the statistical parameters of the wavelet coefficients based on Maarten (2001), Vattereli and Kovacevic (1995), Chang *et al.* (2000 a, b, c) and Javier (2002). The experimental

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results show that this algorithm can outperform the traditional ones, improving the quality of the denoised images significantly.

MATERIALS AND METHODS

Wavelet thresholding: Wavelet thresholding is a simple non-linear technique, which operates on one wavelet coefficient at a time. In its most basic form, each coefficient is threshold by comparing against threshold, if the coefficient is smaller than threshold, set to zero; otherwise it is kept or modified. Replacing the small noisy coefficients by zero and inverse wavelet transform on the result may lead to reconstruction with the essential signal characteristics and with less noise. Wavelet thresholding involves threes steps A linear forward wavelet transform, nonlinear thresholding step and a linear inverse wavelet transform.

Let us consider a signal $\{x_{ij}, i, j = 1, 2 \ldots N\}$ denote the N.N matrix of the original image to be recovered and N is some integer power of 2. During transmission the signal is corrupted by noise

$$y_{ij} = x_{ij} + \varepsilon_{ij}, I, j = 1, 2 \ldots\ldots\ldots\ldots N$$

Where, ε_{ij} independent and identically distributed (i.i.d) zero mean, white Gaussian Noise with standard deviation σ i.e. N $(0, \sigma^2)$. From this noisy signal yi, we want to find an approximation $\hat{x}_i$. The goal is to estimate the signal x_{ij} from noisy observations y_{ij} such that Mean Squared Error (MSE) is minimum. i.e.,

$$\text{MSE}(\hat{x}) = \frac{1}{N^2}\sum_{i,j=1}^{N}\left(\hat{x}_{ij} - x_{ij}\right)^2$$

Let $y = \{y_{ij}\}_{i,j}$, $x = \{x_{ij}\}_{i,j}$, $\varepsilon = \{\varepsilon_{ij}\}_{i,j}$ will denote the matrix representation of the signals under consideration. Let $D = W_y$, $C = W_x$, $\varepsilon = W_z$ denote the matrix of wavelet coefficients of y, x, z, respectively. Where, W is the two-dimensional dyadic orthogonal wavelet transform operator. It is convenient to label the sub bands of the transform as in Fig. 1. The sub bands, HH_k. HL_k. LH_k are called the details, where k = 1,2…J is the scale, with J being the largest (or coarsest) scale in the decomposition and a sub band at scale k has size $N/2^k$ X $N/2^k$. The sub band LL_j is the low resolution residual and is typically chosen large enough such that $N/2^j \geq N$, $N/2^j \geq 1$. The wavelet-thresholding denoising method filters each coefficient Y_{ij} from the detail sub bands with a threshold

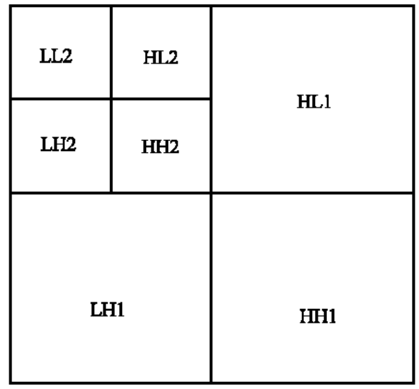

Fig. 1: Two-Level Image decomposition by using DWT

function to obtain $\hat{x}_{ij}$. The denoised estimate is then $\hat{y} = W^{-1}\hat{x}$, where, W^{-1} is the inverse wavelet transform.

Wavelet transform of noisy signal should be taken first and then thresholding function is applied on it. Finally the output should be undergone inverse wavelet transformation to obtain the estimate $\hat{x}$. There are two thresholding functions frequently used, i.e. a hard threshold, a soft threshold. The hard-thresholding function keeps the input if it is larger than the threshold; otherwise, it is set to zero. It is described as:

$$\eta_1(w) = wI(|w| > T) \qquad (1)$$

Where, w is a wavelet coefficient, T is the threshold and I(x) is a function the result is one when x is true and zero vice versa. The soft-thresholding function (also called the shrinkage function) takes the argument and shrinks it toward zero by the threshold. It is described as:

$$\eta_2(w) = (w - \text{sgn}(w)T)\, I(|w| > T) \qquad (2)$$

Where sgn(x) is the sign of x. The soft-thresholding rule is chosen over hard-thresholding, for the soft-thresholding method yields more visually pleasant images over hardthresholding.

While the idea of thresholding is simple and effective, finding a good threshold is not an easy task. For one-dimensional (1-D) deterministic signal of length , Donoho and Johnstone (1995) proposed for VisuShrink the universal threshold, which results in an estimate asymptotically optimal in the minimax sense (minimizing the maximum error over all possible -sample signals). One other notable threshold is the SURE threshold , derived from minimizing Stein's unbiased risk estimate when soft-thresholding is used. The SureShrink method is a hybrid of the universal and the SURE threshold, with the choice being dependent on the energy of the particular sub band.

Estimation of parameters for threshold value: Finding an optimum value for thresholding is not an easy task. A small threshold value will pass all the noisy coefficients and hence the resultant denoised signal may still be noisy. A large threshold value on the other hand, makes more number of coefficients as zero which leads to smooth signal and destroys details and in image processing may cause blur and artifacts. So, optimum threshold value should be found out, which is adaptive to different sub band characteristics. Here, we describe an efficient method for fixing the threshold value for denoising by analyzing the statistical parameters of the wavelet coefficients.

$$\text{Threshold T is given as } T=\beta\frac{\sigma^2}{\sigma_X} \quad (1)$$

This study focuses on the estimation of the GGD parameters, σ_x and β which in turn yields a data-driven estimate of $T(\sigma_x)$, that is adaptive to different sub band characteristics. The noise variance σ^2 needs to be estimated first. It may be possible to measure σ^2 based on information other than the corrupted image. If such is not the case, it is estimated from the sub band HH_1 by the robust median estimator,

$$\sigma^2 = \left[\frac{\text{median}\left|Y_{ij}\right|}{0.6745}\right]^2 \quad (2)$$

Where, σ^2s the noise variance, σ_x the signal standard deviation. The parameter α in the expression, σ_x the signal standard deviation needs to be estimated.

From the observation model $Y = X + \varepsilon$, with X and ε independent o each other we have

$$\sigma_Y^2 = \sigma_X^2 + \sigma^2$$

Where $\sigma^2{}_Y$ is the variance of Y. It can be found by

$$\sigma_Y^2 = \frac{1}{n^2}\sum_{i,j=1}^{n} Y_{ij}^2$$

From this σ_x can be derived as

$$\sigma_X = \sqrt{\max\left(\sigma_Y^2 - \sigma^2, 0\right)}. \quad (3)$$

Then the parameter α can be found as

$$\beta=\log\left(\frac{L}{k}\right) \quad (4)$$

Where, L is the number of wavelet decomposition level, k is the level at which the subband is available (for HL_2, k=2)

Image denoising algorithm: This study describes the image-denoising algorithm, which achieves near optimal soft thresholding in the wavelet domain for recovering original signal from the noisy one. The algorithm is very simple to implement and computationally more efficient. It has the following steps:

1. Perform the DWT of the noisy image up to 2 levels (L=2) to obtain seven sub bands, which are named as HH_1, LH_1, HL_1, HH_2, LH_2, HL_2 and LL_2.
2. Compute the threshold value T for each sub band, except the LL_2 band using Eq. 1.
3. Obtain the noise variance using the Eq. 2
4. Calculate the signal standard deviation $\sigma_x|$ by the Eq. 3.
5. Find out the parameter α from Eq. 4
6. Threshold the all sub band coefficients using Soft Thresholding given in Eq. 1 by substituting the threshold value obtained from the step 2.
7. Perform the inverse DWT to reconstruct the denoised image.

RESULTS AND DISCUSSION

The 512*512 grayscale images "Lena," and "gold hill," are used as test images with different noise levels σ = 10, 20, 30. The wavelet transform employs Daubechies' least asymmetric compactly supported wavelet with eight vanishing moments with four scales of orthogonal decomposition. To evaluate the performance of the proposed method, it is compared with BayesShrink, Normal shrink, oracle shrink using Peak Signal to Noise Ratio (PSNR), which is defined as

$$PSNR = 10\log_{10}\frac{255}{MSE}$$

Where, MSE denotes the Mean Square Error between the original and denoised images and is given as

$$MSE = \frac{1}{MN}\sum_{i=1}^{M}\sum_{j=1}^{N}\left(X(i,j) - Y(i,j)\right)^2$$

Where, M, N- Width, Height of image,Y - Noisy Image X - Original Image

We have also made comparisons with the Wiener filter, the best linear filtering possible. The version used is the adaptive filter, wiener 2, in the MATLAB image

Table1: Comparison of PSNR of different wavelet filters for different images corrupted by Gaussian noise

Image (512*512)	Peak Signal to Noise Ratio in dB (PSNR)					
	σ	Weiner filter	Oracle shrink	Bayes shrink	NormalShrink	Proposed method
Lena	10	33.5793	33.6114	33.4106	33.5390	33.6241
Gold hill		31.0043	31.2734	31.6704	31.5108	31.6346
Lena	20	28.9868	30.3813	30.2258	30.3530	30.3724
Goldhill		28.2604	28.7682	28.6570	28.6590	28.7524
Lena	30	25.6915	28.6009	28.4901	28.5330	28.6121
Goldhill		25.3490	27.1687	27.2133	27.0963	27.1246

Fig. 2: Comparing the performance of (a) Noisy Lena at σ = 30 with (b) Wiener filter (c) NormalShrink and (d) proposed Shrink

processing toolbox, The PSNR results are shown in Table 1 and they are considerably worse than the nonlinear thresholding methods, especially when σ is large. The image quality is also not as good as those of the thresholding methods. It is clear from Table 1 that the proposed thresholding technique outperforms the NormalShrink and the filters like wiener. The proposed method removes noise significantly and remains within 3% of the NormalShrink. Moreover, the computational time is 4% for BayesShrink. Figure 2 shows the noisy image and resulting images of wiener filter, Normal Shrink and proposed method of Lena image.

CONCLUSION

Since, the proposed threshold estimation method is based on the analysis of statistical parameters like standard deviation, variance of the sub band coefficients, it is more sub band adaptive. The image-denoising algorithm uses soft thresholding to provide smoothness and better edge preservation at the same time. Experiments are conducted on different natural images corrupted by various noise levels to validate the performance of proposed thresholding method in comparison with NormalShrink, BayesShrink and filters like wiener. Since the denoising of images technique has possessed better PSNR, this method applicable to images those are corrupted during transmission, which is normally random in nature.

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