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Simultaneous Design of T-S Fuzzy Controllers and Observers Using GA

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Abstract: Fuzzy control can be used to improve existing controller systems by adding an extra layer of intelligence to the current control method. But most of the cases it is very difficult to measure the state variables. Various design methods are available to design fuzzy observers which can be used to estimate the state variables. But simultaneous design of fuzzy controllers and observers using genetic algorithm is not reported so far. This study presents the Simultaneous Design of Takagi-Sugeno Fuzzy Controllers and Observers using Genetic Algorithm. Benchmarking control problem, namely the inverted pendulum system is simulated with the designed fuzzy controller and observer and the results are promising.

Key words: Fuzzy controllers, fuzzy observers, genetic algorithm

INTRODUCTION

Modeling techniques are believed to contain hidden intelligence that is able to solve a problem whenever the experimenter doesn't know what he is doing. A model is just a mapping from a given input to a given output. The essential element for the study of a nonlinear control problem is to get a tractable model of a dynamical system for use in control system design. The design model should be simple enough to work with, but must retain the essential features of the process. Literature says that Takagi-Sugeno (TS) fuzzy model (Takagi and Sugeno, 1985) along with Parallel Distributed Compensation (PDC) (Wang *et al*., 1995) scheme is best for control system design (Kaimal *et al*., 2003; Rajesh and Kaimal, 2007a-c; 2008, 2006, 2005a,b, 2003, 2002a,b, 2001, 2000.

In most of the cases, it is very difficult to measure the state outputs or there always exist noise in the measured output. Hence it is very necessary to have observers or state estimators to find out the values of the state variables. Various methods are available to design observers (Bergsten *et al*., 2002; Chang and Sun, 1999; Kim *et al*., 1999; Korba *et al*., 2003; Lee and Nam, 1990; Li *et al*., 2003; Park and Park, 2003; Teixeira *et al*., 2003).

But simultaneous design of fuzzy controllers and observers using genetic algorithm is not reported so far. This study presents the simultaneous design of Takagi-Sugeno fuzzy controllers and observers using genetic algorithms.

TAKAGI-SUGENO FUZZY CONTROLLERS AND OBSERVERS-A REVIEW

This study considers the continuous time Takagi-Sugeno fuzzy models, its controllers based on Parallel Distributed Compensation (PDC) and its observers.

The system dynamics of a nonlinear plant is captured by a set of fuzzy implications which characterize local relations in the state space and Takagi-Sugeno fuzzy model is constructed which expresses the local dynamics of each fuzzy implication (rule) by a linear system model. The overall fuzzy model of the system is achieved by fuzzy blending of the linear system models.

A generic non-autonomous continuous-time T-S model for a nonlinear plant rule can be written as follows:

$$\text{ith plant rule: IF } x_1(t) \text{ is } M_{i1} \text{ and } \ldots\ x_n(t) \text{ is } M_{in}$$
$$\text{Then } \dot{x} = A_i x + B_i u$$

where, $x \in R^{n\times 1}$ is the state vector; i = 1, … , r; r is the number of rules; M_{ij}s are input fuzzy sets; and $A_i, B_i \in R^{n\times n}$ are system matrices.

Using a singleton fuzzifier, product inference and weighted average defuzzifier, the aggregated fuzzy plant model can be written as:

$$\dot{x} = \sum_{i=1}^{r} \alpha_i(x)(A_i x + B_i u) \quad (1)$$

Where:

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$$\alpha_i(x) = \frac{w_i(x)}{\sum_{i=1}^{r} w_i(x)} \quad (2)$$

and w_i is defined as follows where $\mu_{ij}(x_j)$ is the grade of membership of $x_j(k)$ in M_{ij}.

$$w_i(x) = \prod_{j=1}^{n} \mu_{ij}(x_j) \quad (3)$$

Takagi-sugeno fuzzy controllers: The concept of Parallel Distributed Compensation (PDC) is utilized to design fuzzy controllers to stabilize fuzzy system (1). The main idea of the PDC is to design each local control rule so as to compensate each local rule of a fuzzy system. For instance, control rule i has the same structure as rule i of the fuzzy model. Linear control design techniques are used to design each local control rules. Thus, PDC controller consists of fuzzy If-Then rules where each rule is a local state-feedback controller and the overall controller is obtained by fuzzy blending of each individual linear controllers.
ith T-S controller rule can be written as:

i^{th} controller rule: IF $x_1(t)$ is M_{i1} and … $x_n(t)$ is M_{in} Then $u = - K_i x$

Hence, the overall fuzzy controller is

$$u(t) = -\sum_{i=1}^{r} \alpha_i(x) K_i x(t) \quad (4)$$

where, the α_i's are defined in (2). Note that the same fuzzy sets are used for both the controller rules and the plant rules.

Takagi-sugeno fuzzy observers: An implicit assumption in all previous sections for the design of fuzzy controllers was that the states are available for measurement. However, measuring the states can be physically difficult and costly. Moreover, sensors are often subject to noise and failure.

Classical control says that an observer can be used to estimate the states of an observable LTI system from output measurements. In fact, classical control theory also says how to estimate the states of a Linear Time-invariant (LTI) system in the presence of additive noise in the system and measurement noise in the output, using a Kalman filter (Kalman, 1960).

This study will describe the design of an observer in the continuous-time case (Rajesh and Kaimal, 2007) based on fuzzy implications, with fuzzy sets in the antecedents and an asymptotic observer in the consequents. Each fuzzy rule is responsible for observing the states of a locally linear subsystem.

Consider, the closed-loop fuzzy system described by r plant rules and r controller rules as follows:

$$\dot{x}(t) = \sum_{i=1}^{r} \alpha_i(y)(A_i x(t) + B_i u(t))$$

$$y(t) = \sum_{i=1}^{r} \alpha_i(y) C_i x(t) \quad (5)$$

A fuzzy observer is defined as a set of T-S If-Then rules which estimate the states of the system (5). A generic observer rule can be written as:

$$\begin{aligned} &\text{ith rule: if } y_1(t) \text{ is } M_{i1} \text{ and } \dots y_p(t) \text{ is } M_{ip} \\ &\text{Then } \dot{\hat{x}} = A_i \hat{x} + B_i u + L_i(y - \hat{y}) \end{aligned} \quad (6)$$

where, p is the number of measured outputs, $y_i = C_i x$ is the output of the ith T-S plant rule, $\hat{y}$ is the global output estimate and $L_i \in R^{n \times p}$ is the local observer gain matrix. The deffuzified global output estimate can be written as:

$$\hat{y}(t) = \sum_{j=1}^{r} \alpha_j(y) C_i \hat{x}(t) \quad (7)$$

where, the α_i's are the normalized membership functions. The aggregation of all fuzzy implications results in the following state equations:

$$\begin{aligned} \dot{\hat{x}} = &\sum_{i=1}^{r} \alpha_i(y)(A_i \hat{x} + B_i u) + \\ &\sum_{i=1}^{r} \sum_{j=1}^{r} \alpha_i(y) \alpha_j(y) L_i C_j (x - \hat{x}) \end{aligned} \quad (8)$$

Since,

$$\sum_{j=1}^{r} \alpha_j(y) = 1$$

Equation 8 can be written as

$$\dot{\hat{x}} = \sum_{i=1}^{r} \sum_{j=1}^{r} \alpha_i(y) \alpha_j(y) \left[\left(A_i - L_i C_j \right) \hat{x} + B_i u + L_i C_j x \right] \quad (9)$$

Note that the normalized membership function is a function of y instead of x since the antecedents are measured output variables, not states. The controller is also based on the estimate of the state rather than the state itself, i.e.,

$$u(t) = -\sum_{j=1}^{r} \alpha_j(x) K_j \hat{x}(t) \quad (10)$$

DESIGN OF FUZZY CONTROLLER AND OBSERVER FOR INVERTED PENDULUM SYSTEM

A famous benchmark problem namely the inverted pendulum control problem is chosen for study. The non-linear and non-stable behavior of the inverted pendulum problem renders the use of conventional method very difficult.

The inverted pendulum system consists of a pole hinged on a cart. The control objective is to balance the inverted pendulum for the approximate range of vertical angle, namely $x_1 \in (-\pi/2, \pi/2)$. The equations of motion of the pendulum (Wang *et al*., 1995) are:

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = \frac{g\sin(x_1) - amlx_2^2\sin(2x_2)/2 - a\cos(x_1)u}{4l/3 - aml\cos^2(x_1)}$$

where, x_1 denotes the angle (in radians) of the pendulum from the vertical and x_2 is the angular velocity, g is the gravity constant, m is the mass of the pendulum, M is the mass of the cart, 2l is the length of the pendulum, u is the force applied to the cart and a = 1/(m+M). In our simulation we used the values, g = 9.8 m/s^2, m = 2 Kg, M = 8 Kg, a = 0.1, 2l = 1m, β = cos 88°.

We approximate the system by the following two-rule Takagi Sugeno fuzzy model

$$R_1 : \text{If } x_1 \text{ is about } 0 \text{ then } \dot{x} = A_1x + B_1u$$

$$R_2 : \text{If } x_1 \text{ is about } \pm\pi/2 \text{ then } \dot{x} = A_2x + B_2u$$

where, $x = [x_1, x_2]^T$ and

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix}, A_1 = \begin{bmatrix} 0 & 1 \\ \frac{g}{4l/3 - aml} & 0 \end{bmatrix}, B_1 = \begin{bmatrix} 0 \\ \frac{-a}{4l/3 - aml} \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0 & 1 \\ \frac{2g}{\pi\left(4l/3 - aml\beta^2\right)} & 0 \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ \frac{-a\beta}{4l/3 - aml\beta^2} \end{bmatrix}$$

By utilizing the concept of Parallel Distributed Compensation (PDC), the following two rules are designed for the controller, where $\hat{x}$ is the state variables from the observer as explained in the previous section.

$$R_1 : \text{If } x_1 \text{ is about } 0 \text{ then } u_1 = K_1\hat{x}$$

$$R_2 : \text{If } x_1 \text{ is about } \pm\pi/2 \text{ then } u_2 = K_2\hat{x}$$

Genetic algorithm with population size 20 is used to find out the gain values. Each chromosome contains 8 real values. The structure of the chromosome is given below:

$$[K_{11}, K_{12}, K_{21}, K_{22}, L_{11}, L_{12}, L_{21}, L_{22}]$$

where, $K_1 = [K_{11}, K_{12}]$, $K_2 = [K_{21}, K_{22}]$, $L_1 = [L_{11}, L_{12}]$ and $L_2 = [L_{21}, L_{22}]$. The crossover rate is chosen as 0.7 and the mutation rate as 1/8. The fitness function is chosen as:

$$\frac{10 \times \text{max time}}{100x_1^2 + x_2^2}$$

where, maxtime is the maximum time the system is run (for this problem maxtime = 500 ms). After 100 generations the chromosome with maximum fitness is obtained as follows.

[0.9043, 0.1383, 1.2000, 0.1133, 0.0008, -0.0009, 0.0003, -0.0009]

Figure 1 shows the plot fitness values have converged after 80 generations. Figure 2 and 3 show the plot of θ and dθ/dt assumed to be obtained from the sensor [These values are obtained by adding a random noise of 0.05*sin(randn)]. Figure 4 and 5 show the plot of θ and dθ/dt given by the observer designed by the proposed method. Figure 6 and 7 show the plot of

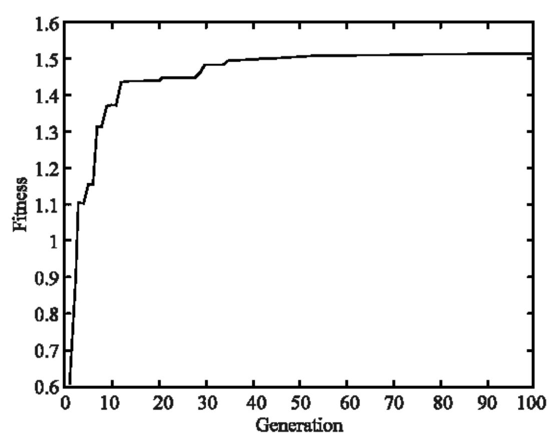

Fig. 1: Fitness vs generation

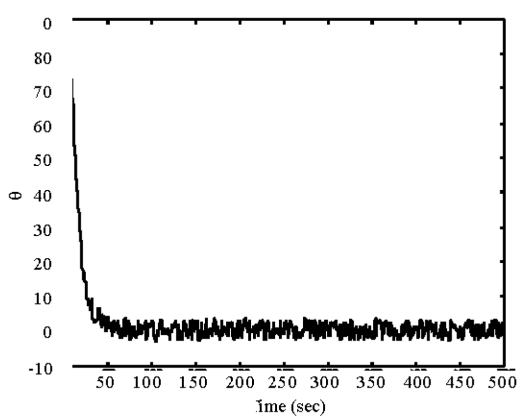

Fig. 2: θ from sensor

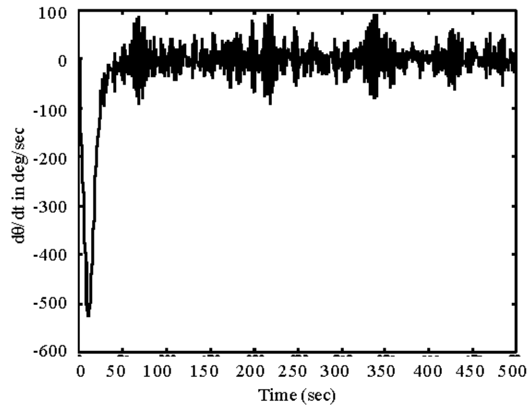

Fig. 5: dθ/dt from Observer

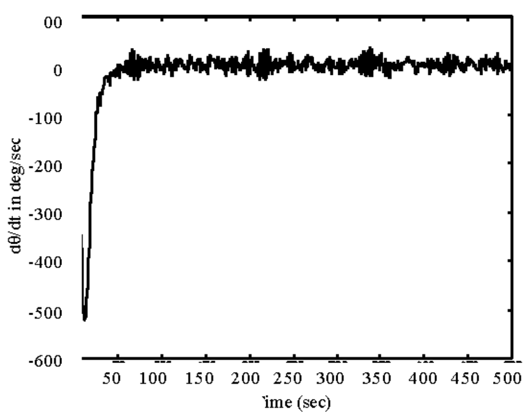

Fig. 3: dθ/dt from sensor

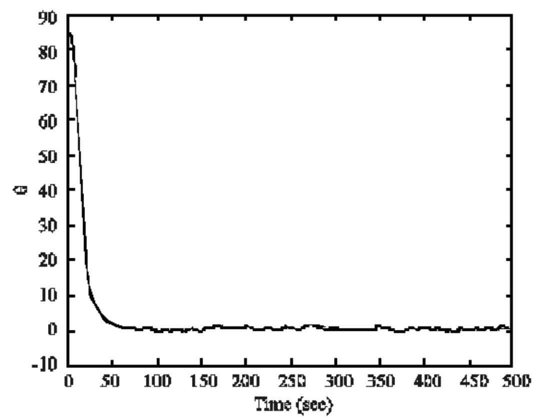

Fig. 6: Controlled value of θ

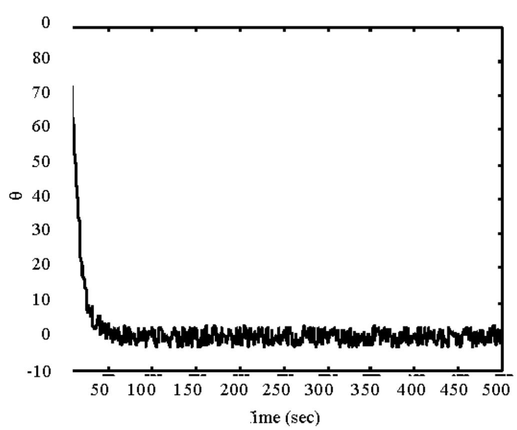

Fig. 4: θ from Observer

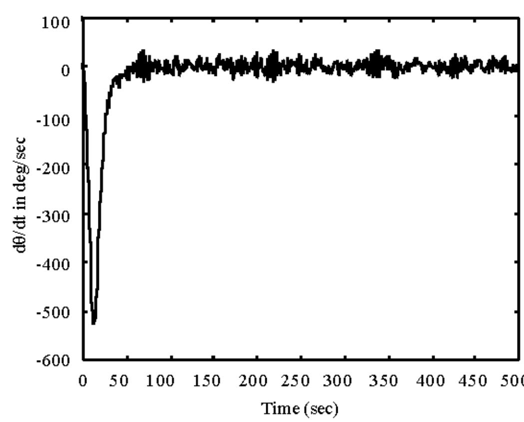

Fig. 7: Controlled value of dθ/dt

controlled θ and dθ/dt. Figure 8 shows the control force applied to the system. The simulations results shows the efficiency of the method of simultaneous design of T-S Fuzzy Controllers and Observers using GA.

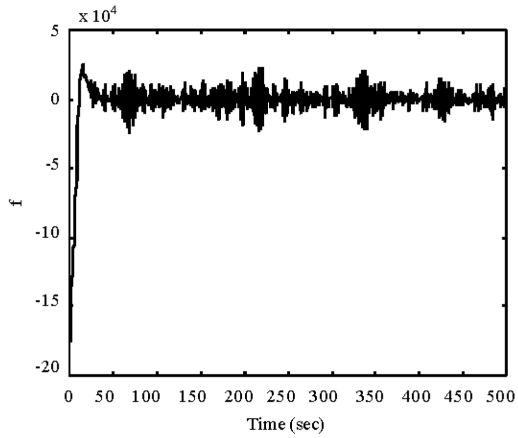

Fig. 8: Control force applied

CONCLUSION

Implementations using fuzzy logic controllers can work well without having to construct any mathematical model of the process or plant. Fuzzy logic control yields results superior to those using conventional control algorithms and their applications can also lead to reduced development cost. In many cases, fuzzy control can be used to improve existing controller systems by adding an extra layer of intelligence to the current control method. But most of the cases it is very difficult to measure the state variables. Fuzzy observers can be used to estimate the state variables.

This study has presented a method for the simultaneous design of fuzzy controllers and observers using genetic algorithm. Fuzzy controllers and observers for the benchmarking control problem of inverted pendulum is designed and the simulation results are promising.

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