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A Modified Particle Swarm Optimization for Economic Dispatch Problems with Non-Smooth Cost Functions

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Abstract: This study proposes a new versatile optimization algorithm called modified particle swarm optimization algorithm (MPSO) for solving economic dispatch problems (ED) with non smooth objective functions. In this algorithm, particles not only studies from itself and the best one but also from other individuals. By this enhanced study behavior, the opportunity to find the global optimum is increased and the influence of the initial position of the particles is decreased. To show its efficiency and effectiveness, the MPSO is applied to sample ED problems with smooth cost function as well as non-smooth cost functions. The results of the MPSO are compared with those of the conventional numerical method, evolutionary programming approach and the classical PSO approach.

Key words: Economic dispatch, particle swarm optimization, modified particle swarm optimization

INTRODUCTION

To supply reliable and economic electric energy to consumers, electric utilities face many economic and technical problems in the operation, planning and control of power systems. Most of power system optimization problems including economic dispatch (ED) have complex and nonlinear characteristics with heavy equality and inequality constraints (Lee and Sharkawi, 2002).

The primary objective of the ED problem is to determine the optimal combination of power outputs of all generating units so as to meet the required load demand at minimum operating cost while satisfying system equality and inequality constraints. In the traditional ED problem, the cost function for each generator has been approximately represented by a single quadratic function and is solved using mathematical programming based on the optimization techniques such as lambda-iteration method (Wood and Wollenberg, 1984), gradient method (Wood and Wollenberg, 1984) and dynamic programming method (Liang and Glover, 1992), etc. These methods require incremental fuel cost curves which should be monotonically increasing to find global optimum solution. Unfortunately, the input-output characteristics of generating units are inherently highly nonlinear because of valve point loadings, multiple effects, etc. Thus, the practical ED problem with valve point and multi-fuel effects is represented as a non-smooth optimization problem with equality and inequality constraints and this makes the problem of finding the global optimum difficult. Over the past few years, in order to solve this problem, many salient methods have been developed such as hierarchical numerical (Lin and Viviani, 1984), genetic algorithm (Walters and Sheble, 1993), evolutionary programming (Yang *et al*., 1996; Sinha *et al*., 2003; Park *et al*., 1998), Tabu search (Lin *et al*., 2002), neural network approaches (Lee *et al*., 1998) and particle swarm optimization (Park *et al*., 2005).

Recently, Eberhart and Kennedy suggested a particle swarm optimization (PSO) based an analogy of swarm of bird and school of fish (Victoire and Jeyakumar, 2004). The PSO mimics the behaviors of individuals in a swarm to maximize the survival of the species. In PSO, each individual decides his decision using his own experience as well as best individual experience. It can be used to solve many complex optimization problems, which are nonlinear, non-differentiable and multi-modal. The most prominent merit of PSO is its fast convergence speed. In addition, PSO algorithm can be realized simply for less parameters need adjusting. Now it was applied successfully in various fields of power system optimization such as power system stabilizer design, reactive power and voltage control.

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Like other optimization algorithms, PSO also has the disadvantage of premature convergence. For single-modal problem, PSO can find out the global optimal solution accurately and rapidly. But for complex multi-modal optimization problems, PSO is easy to be trapped by local optimum. Besides, the initial value of the particles, in some degree, decides whether it could find the global optimal solution. Moreover, in the basic PSO, particle adjusts its velocity only according to its best experiences and that of the best one in the population, without considering other particle's information. Based on the analysis above, this study proposes a modified particle swarm optimization (MPSO) algorithm (Kennedy and Eberhart, 2001). In MPSO, the particle not only studies from itself and the best one but also simulate other individuals.

In this study, we propose a novel approach for solving the ED problems with non-smooth cost functions using an modified PSO (MPSO). The feasibility of the MPSO for ELD problems with quadratic and piecewise quadratic cost functions is demonstrated and compared with existing approaches.

FORMULATION OF ECONOMIC DISPATCH

Basic economic dispatch formulation: Economic dispatch is one of the most important problems to be solved in the operation and planning of a power system. The objective of the economic dispatch problem is to minimize the total fuel cost of thermal power plants subjected to the operating constraints of a power system. In general, it can be formulated mathematically with an objective function and two constraints (Wood and Wollenberg, 1984).

$$F_T = \sum_{i=1}^{N} F_i(P_i) \qquad (1)$$

$$F_i(P_i) = a_i + b_i P_i + c_i P_i^2 \qquad (2)$$

where,

F_T : Total generation cost.
F_i : Cost function of generator i.
a_i, b_i, c_i : Cost coefficients of generator i.
P_i : Power of generator i.
N : Number of generators.

Active power balance equation: For power balance, an equality constraint should be satisfied. The total generated power should be the same as total demand plus the total line loss:

$$\sum_{i=1}^{N} P_i = P_D + P_{Loss} \qquad (3)$$

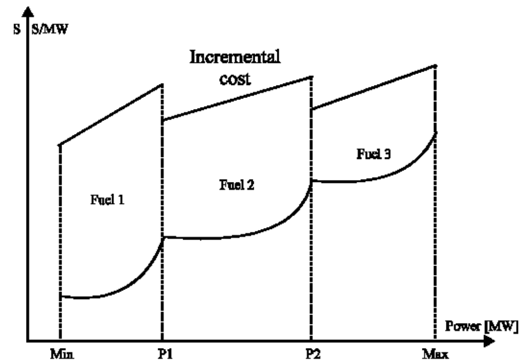

Fig. 1: Piecewise quadratic and Incremental cost function of a generator

where, P_D is the total system demand and P_{Loss} is the total line loss. However, the transmission loss is not considered in this paper for simplicity (i.e., $P_{Loss} = 0$).

Minimum and maximum power limits: Generation output of each generator should be laid between maximum and minimum limits. The corresponding inequality constraints for each generator are

$$P_{i,min} \le P_i \le P_{i,max} \qquad (4)$$

where, $P_{i,min}$ and $P_{i,max}$ are the minimum and maximum output of generator i, respectively.

Non-smooth cost functions with multi-fuels: Since, the dispatching units are practically supplied with multi-fuel sources, each unit should be represented with several piecewise quadratic functions reflecting the effects of fuel type changes. In general, a piecewise quadratic function is used to represent the input-output curve of a generator with multiple fuels (Lin and Viviani, 1984) and described as (5).

$$F_i(P_i) = \begin{cases} a_{i1} + b_{i1}P_i + c_{i1}P_i^2 & \text{if } P_i^{min} \le P_i \le P \\ a_{i2} + b_{i2}P_i + c_{i2}P_i^2 & \text{if } P_{i1} \le P_i \le P_{i2} \\ \vdots & \vdots \\ a_{im} + b_{im}P_i + c_{im}P_i^2 & \text{if } P_{im-1} \le P_i \le P_i^{max} \end{cases} \qquad (5)$$

where, a_{ij}, b_{ij}, c_{ij}: Cost coefficients of generator i for the j-th power level (Fig. 1).

OPTIMIZATION METHODOLOGIES FOR ED PROBLEMS

Overview of the PSO: Kennedy and Eberhart (2004) developed a particle swarm optimization (PSO) algorithm based on the behavior of individuals (i.e., particles or agents) of a swarm. Its roots are in zoologist's modeling of the movement of individuals (i.e., fishes, birds and insects) within a group. It has been noticed that members of the group seem to share information among them, a fact that leads to increased efficiency of the group. The PSO algorithm searches in parallel using a group of individuals similar to other AI-based heuristic optimization techniques. Each individual corresponds to a candidate solution to the problem. Individuals in a swarm approach to the optimum through its present velocity, previous experience and the experience of its neighbors.

In a physical n-dimensional search space, the position and velocity of individual i are represented as the vectors $X_i = (x_{il}, ..., x_{in})$ and $V_i = (v_{il}, ..., v_{in})$ in the PSO algorithm. Let $Pbest_{ii} = (x_{il}^{Pbest}, ..., x_{in}^{Pbest})$ and $Gbest_{ii} = (x_{il}^{Gbest}, ..., x_{in}^{Gbest})$ be the best position of individual i and its neighbors' best position so far, respectively. Using the information, the updated velocity of individual i is modified under the following equation in the PSO algorithm:

$$\begin{aligned} V_i^{k+1} = \omega V_i^k + c_1 rand_1 \times \left(Pbest_i^k - X_i^k\right) \\ + c_2 rand_2 \times \left(Gbest^k - X_i^k\right) \end{aligned} \quad (6)$$

where,

V_i^k	:	Velocity of individual i at iteration k.
ω	:	Weight parameter.
c_1, c_2	:	Weight factors.
$rand_1$, $rand_2$	:	Random numbers between 0 and 1.
X_i^k	:	Position of individual i at iteration k.
$Pbest_i^k$	:	Best position of individual i until iteration k.
$Gbest_i^k$	:	Best position of the group until iteration k.

In this velocity updating process, the values of parameters such as ω, c_1 and c_2 should be determined in advance. In general, the weight ω is set according to the following Eq. 1 and 11:

$$\omega = \omega_{max} - \frac{\omega_{max} - \omega_{min}}{Iter_{max}} \times Iter \quad (7)$$

where,

ω_{max}, ω_{min},	:	Initial, final weights,
$Iter_{max}$	:	Maximum iteration number,
Iter	:	Current iteration number.

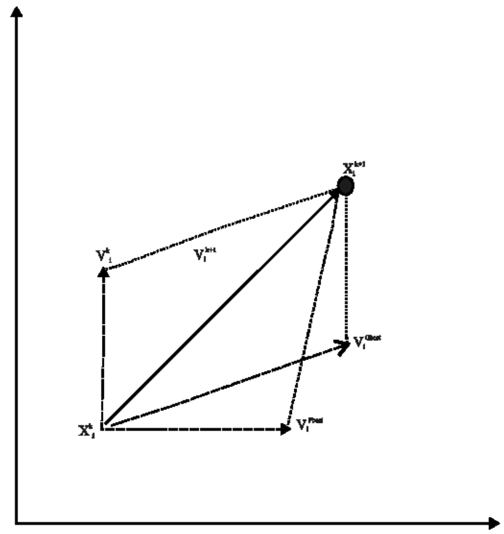

Fig. 2: The searching mechanism of the particle swarm optimization

Each individual moves from the current position to the next one by the modified velocity in (6) using the following equation:

$$X_i^{k+1} = X_i^k + V_i^{k+1} \quad (8)$$

Figure 2 shows the concept of the searching mechanism of PSO using the modified velocity and position of individual i based on (6) and (8) if the values of ω, c_1, c_2, $rand_1$, $rand_2$ are 1.

Modified particle swarm optimization: In basic PSO, particle updates its flying velocity and position only according to its own best position and the best of the groups. During the searching process, most particles contract quickly to a certain specific position. If it is a local optimum, then it is not easy for the particles to escape from it. Research shows that in the initial stage the particles convergence very quickly, however, with the iterations goes on, particles become very similar and almost have no ability to explore new area. The reasons for this phenomenon are that in the basic PSO, the particle does not consider the information of other particles except its own and the best one. The algorithm only can search the area nearby the best one in detail, but cannot explore other areas adequately. In addition, the performance of basic PSO is greatly affected by the initial position of the particles, if the initial population is far away from the real optimal solution and then the algorithm is very hard to success. In the reality, the individual not only studies from itself and the best one but also simulate other individual's behavior frequently. The proverb that three heads are better than one is just this point. Especially in the initial stage, this simulation behavior should dominate the study behavior of particle Based on the above recognition, the MPSO algorithm is proposed.

In the MPSO, the particle also adjusts its velocity according to two extremes. One is the best position of its own and the other is not always the best one of the group, but selected randomly from the group. In each generation, the particle studies randomly from the group at the beginning with bigger probability and tends to the best one of the populations in the later. A selection policy is given as follows:

In the i-th generation, r is a randomly number between 0 and 1 evenly. R_t is calculated according to the formula (9), if $r > R_t$ choose a particle randomly except itself and the best one, use its position, record as X_{rnd}, $X_{rnd} = (X_{r1}, X_{r2}, \ldots, X_{rn})$ instead of Gbest in the equation and update particle's velocity, that is to say, Eq. (6) is changed to (10), shown as follows, or else update as Eq. 9.

$$R_t = t / \text{MaxGen} \quad (9)$$

$$V_i^{k+1} = \omega V_i^k + c_1 \text{rand}_1 \times \left(\text{Pbest}_i^k - X_i^k\right) + c_2 \text{rand}_2 \times \left(X_{rnd}{}^k - X_i^k\right) \quad (10)$$

Where, t is the number of current generation, MaxGen is the maximum number of generation.

Implementation of MPSO for ELD problems: In this study, we will describe the approach to implement the MPSO algorithm in solving the ELD problems. Its implementation consists of following steps.

Step 1: The particles are randomly generated between the maximum and minimum operating limits of the generators. It is very important to create a group of individuals satisfying the equality constraint (3) and inequality constraints (4) . That is, summation of all elements of individual

$$i \ (\text{i.e.,} \sum_{j=1}^{n} P_{ij})$$

should be equal to the total system demand P_D and the created element j of individual i at random (i.e., P_{ij}) should be located within its boundary.

Step 2: The particle velocities are randomly generated.

Step 3: Objective function values of the particles are evaluated. Penalties are given for violation of inequality constraints (4). These values are set as the Pbest value of the particles.

Step 4: The best value among all the Pbest values (Gbest) is identified.

Step 5: New velocities are calculated using Eq. (10).

Step 6: The positions of each particles are updated using Eq. 8. If any element of an individual violates its inequality constraint due to over/under speed, then the position of the individual is fixed to its maximum/minimum operating point.

Step 7: New objective function values are calculated for the new positions of the particles. If the new value is better than previous Pbest, the new value is set to Pbest. The best value among new Pbest (Gbest) is identified.

Step 8: The proposed MPSO is terminated if the iteration approaches to the predefined maximum iteration.

CASE STUDIES

To access the efficiency and effectiveness of the proposed MPSO, it has been applied to ELD problems where the cost functions used are the quadratic and piecewise quadratic cost functions. The results obtained for the test systems are compared with those of the numerical lambda-iteration method (Wood and Wollenberg, 1984), the hierarchical numerical method (HM) (Lin and Viviani, 1984), the improved evolutionary programming (IEP) (Park *et al*., 1998) and the classical PSO approach (Park *et al*., 2005).

The proposed MPSO is applied to the ELD problem with 3 generators where the cost functions used are the quadratic cost functions. Table 1 shows the cost functions and the related minimum/maximum operating points of 3 generators. Here, the system demand is 850 MW. Table 2 shows the comparison of results from MPSO, NM (the lambda-iteration method), IEP and PSO.

Figure 3 illustrates the convergence characteristics of the proposed MPSO where faster convergence is achieved. As seen in Table 2, the proposed MSPO has also provided the global solution, while satisfying the equality and inequality constraints.

The proposed MPSO has been applied to the ELD problem with piecewise quadratic functions and 10 generators. The piecewise cost coefficients and the related constraints of generators are given in Lee *et al.*

(1998). In this case, the load system demand is 2400 MW. The results from proposed MPSO are compared with those of hierarchical method (HM) (Lin and Viviani, 1984),

Table 1: Cost coefficients of test system with 3 generators

Unit	a_i	b_i	c_i	$P_{i,min}$	$P_{i,max}$
1	561.0	7.92	0.001562	150.0	600.0
2	310.0	7.85	0.00194	100.0	400.0
3	78.0	7.97	0.00482	50.0	200.0

Table 2: Comparison of results of each method

Unit	NM	IEP	PSO	MPSO
1	393.170	393.17009	393.16983	392.3614
2	334.604	334.60337	334.60375	334.9850
3	122.26	122.22654	122.22642	122.6537
TP	850.000	850.000	850.0000	850.0000
TC	8194.35612	8194.35614	8194.35612	8194.4000

*TP: total power [MW], TC : total generation cost [$]

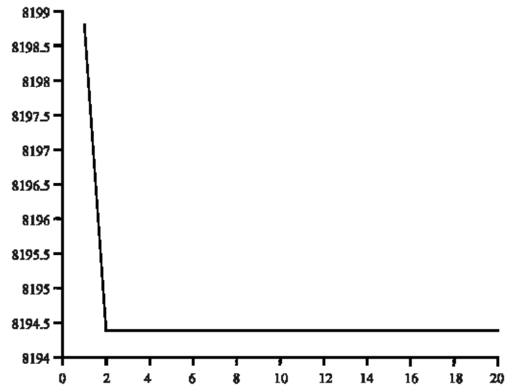

Fig. 3: Convergence characteristics of MPSO for smooth cost functions

Table 3: Comparison of optimization methods (demand = 2400[MW])

	HM		IEP		PSO		MPSO	
U	F	GEN	F	GEN	F	GEN	F	GEN
1	1	193.2	1	190.9	1	189.7	1	188.6838
2	1	204.1	1	202.3	1	202.3	1	208.2526
3	1	259.1	1	253.9	1	253.0	1	270.0613
4	3	234.3	3	233.9	3	233.0	3	234.8858
5	1	249.0	1	243.8	1	241.8	1	229.1797
6	1	195.5	3	235.0	3	233.0	3	242.9074
7	1	260.1	1	253.2	1	253.3	1	241.9124
8	3	234.3	3	232.8	3	233.0	3	230.4737
9	1	325.3	1	317.2	1	320.4	1	332.6084
10	1	246.3	1	237.0	1	239.4	1	221.0348
TP		2401.2		2400.0		2400.0		2400.000
TC		488.500		481.700		481.723		480.8062

Table 4: Comparison of optimization methods (demand = 2500[MW])

	HM		IEP		PSO		MPSO	
U	F	GEN	F	GEN	F	GEN	F	GEN
1	2	206.6	2	203.1	2	206.5	2	204.4
2	1	206.5	1	207.2	1	206.5	1	211.8
3	1	265.9	1	266.9	1	265.7	1	253.9
4	3	236.0	3	234.6	3	236.0	3	229.3
5	1	258.2	1	259.9	1	258.0	1	253.9
6	3	236.0	3	236.8	3	236.0	3	227.4
7	1	269.0	1	270.8	1	268.9	1	282.1
8	3	236.0	3	234.4	3	235.9	3	241.0
9	1	331.6	1	331.4	1	331.5	1	355.7
10	1	255.2	1	254.9	1	255.1	1	240.0
TP		2501.1		2500.0		2400.0		2500.000
TC		526.700		526.304		526.304		525.7859

Table 5: Comparison of optimization methods (demand = 2600[MW])

	HM		IEP		PSO		MPSO	
U	F	GEN	F	GEN	F	GEN	F	GEN
1	2	216.4	2	213.0	2	216.5	2	210.9
2	1	210.9	1	211.3	1	210.9	1	215.7
3	1	278.5	1	283.1	1	278.5	1	273.0
4	3	239.1	3	239.2	3	239.1	3	238.9
5	1	275.4	1	279.3	1	275.5	1	267.4
6	3	239.1	3	239.5	3	239.1	3	243.1
7	1	285.6	1	283.1	1	285.7	1	279.1
8	3	239.1	3	239.2	3	239.1	3	253.8
9	1	343.3	1	340.5	1	343.5	1	339.7
10	1	271.9	1	271.9	1	272.0	1	277.8
TP		2600.0		2600.0		2600.0		2600.000
TC		574.030		574.473		574.381		573.15

Table 6: Comparison of optimization methods (demand = 2700[MW])

	HM		IEP		PSO		MPSO	
U	F	GEN	F	GEN	F	GEN	F	GEN
1	2	218.4	2	219.5	2	218.3	2	216.7
2	1	211.8	1	211.4	1	211.7	1	214.0
3	1	281.0	1	279.7	1	280.7	1	274.6
4	3	239.7	3	240.3	3	239.6	3	248.2
5	1	279.0	1	276.5	1	238.5	1	287.1
6	3	239.7	3	239.9	3	239.6	3	240.5
7	1	289.0	1	289.0	1	288.6	1	260.0
8	3	239.7	3	241.3	3	239.6	3	238.0
9	3	429.2	3	425.1	3	428.5	3	435.5
10	1	275.2	1	277.2	1	274.9	1	284.9
TP		2700.0		2700.0		2700.0		2700.000
TC		625.180		623.851		623.809		622.476

IEP (Park *et al*., 1998) and PSO (Park *et al*., 2005) in Table 3. Unlike in the case of smooth cost functions, it is impossible to find the global solution with the numerical approach for the ELD problems with non-smooth cost functions.

As shown in Table 3-6, the MPSO has provided better solutions HM, IEP and PSO. Moreover, it has provided solutions' satisfying the equality and inequality constraints, while HM does not satisfy the equality constraints. When compared with PSO, it gives better solution. However, the generation configurations are not similar between PSO and MPSO.

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CONCLUSION

This study presents a new approach for solving non-smooth ED problems with valve-point and multi-fuel effects based on the modified PSO (MPSO) algorithm. The suggested method includes new velocity equation, equality and inequality constraints treatment methods and creation of initial position. The application of new velocity calculation in a PSO is a powerful strategy to improve the global searching ability and escape from local minima. Also, the equality and inequality constraints treatment methods have always provided the solutions satisfying the constraints without disturbing the optimum process of the PSO. The proposed MPSO outperforms other state-of-the-art algorithms in solving economic dispatch problems with valve-point and multi-fuel effects.

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