

A New Interleaver Design for Iteratively Decoded Bit-Interleaved Coded Modulation

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Abstract: Bit-interleaved coded modulation with iterative decoding (BICM-ID) is a bandwidth efficient transmission scheme, which is a low complexity alternative to turbo code. The high performance of BICM-ID depends on the interleaver design. A new interleaver based on the chaos theory and golden section is proposed in the study, which outperforms the random interleaver usually used in a BICM-ID system. In comparison to the golden interleaver, the new interleaver has a low correlation and exhibits a significant performance improvement with an acceptable complexity addition. Simulation results in Both Additive White Gaussian Noise (AWGN) channel and Rayleigh fading channel demonstrate the effectiveness of the proposed approach.

Key words: Interleaver, chaos, golden section, iterative decoding, bit-interleaved coded modulation (BICM)

INTRODUCTION

The demands for high data rates and spectrally efficient modulation techniques are increasing due to the evolution of multimedia service in wireless communication systems. To satisfy the increasing subscriber number and diverse service requirements, the development of wireless communication systems is towards the next-generation mobile communication systems which provide large system capacity and multimedia service capability with higher data transmission efficiency. However, due to the limits of the wireless channel bandwidth and complicated multipath propagation conditions, it is a hard task to provide high data rate transmission over wireless channels. Bit-interleaved coded modulation (BICM) (Zehavi, 1992; Caire *et al.*, 1998) is a bandwidth efficient transmission scheme, which is the concatenation of an encoder, a bit-wise interleaver and a symbol mapper. The advantages of BICM are its simple and flexible implementation and the good performance in fading channels. The performance of BICM can be greatly improved through iterative information exchange between the inner demapper and the outer decoder at the receiver. This system, introduced in Li *et al.* (2002), is usually

referred to as BICM with iterative decoding (BICM-ID). BICM-ID shows excellent performance both in AWGN and Rayleigh fading channels. Compared with turbo codes (Berrou *et al.*, 1993), the main advantage of BICM-ID is that it only requires one soft-input soft-output (SISO) decoder instead of 2 as usually used in turbo decoding. Besides its lower complexity than turbo codes, BICM-ID has flexible implementation and is well combined with multiple-input multiple-output (MIMO) detection (El-Azizy *et al.*, 2006), OFDM (Trung and Nguyen, 2006), etc.

With the emergence of chaos theory onto the scientific field with a big bang, many research institutions are studying the behavior of the chaotic systems. Chaos exists in many nonlinear dynamical systems and is widely used in different fields such as physics, biology, engineering and communications. Chaotic maps have sensitive dependence on initial conditions and system parameters and produce pseudorandom sequences of good auto-correlations. In recent years, there has been growing interest in applying chaos theory to signal processing and communications, such as secure communications (Papadimitriou *et al.*, 1997), design of pseudorandom sequences for spread spectrum systems (Heideri-Bateni and McGillem, 1994), detection in radar

signal processing (Leung, 1995) and pattern recognition (Andreyev *et al.*, 1997). In fact chaotic techniques have been introduced in almost all fields of the digital communication systems, including information encoding/decoding, information encryption/decryption, spectrum spreading/dispersing and modulation/demodulation. As is well known, channel coding plays a key role in reducing the bit error rate (BER) of digital communication systems. However, there are very few reports on application of chaos in channel coding.

The high performance of BICM-ID depends on the interleaver design and a random interleaver is generally used in BICM-ID (Li *et al.*, 2002). In this study, we focus on the interleaver design for BICM-ID to further improve the system performance. The classical use of interleaver is to randomize the locations of errors introduced in transmission, allowing for the use of random error correcting codes at the receiver. Such a situation occurs in burst error channels or concatenated coding systems. Recently, interleavers have become an integral part of the coding and decoding strategy, as in the case of turbo codes and the BICM-ID system. Therefore, the problem of finding optimal interleavers for such schemes is a code design problem and is also an ongoing area of research.

The golden section has applications in many mathematical problems. It has been used for the interleaver design in turbo codes and shows good properties (Crozier *et al.*, 1999). To the best of our knowledge, there is no report on the application of the golden interleaving method to BICM-ID. To utilize the good properties of chaos and golden section, a novel golden interleaving scheme based on chaos theory is proposed in the study, which outperforms both the golden interleaver and the random interleaver when used in BICM-ID.

SYSTEM MODEL

We consider the BICM system with iterative decoding depicted in Fig. 1. A block of data bits u is convolutionally encoded, then the coded bit sequence b' is bitwise interleaved. The interleaved sequence is divided into subsequences of m consecutive bits $b = (b_0, b_1, \dots, b_{m-1})$. Each subsequence b is mapped to a complex symbol $x = \lambda(b)$ chosen from the M -ary signal constellation χ , where λ is the labeling map and $m = \log_2 M$.

The channel is assumed to be described by $y = \rho x + n$, where ρ denotes the fading coefficient with $E(\rho^2) = 1$ (for the AWGN channel $\rho = 1$) and n is the complex zero-mean Gaussian noise with one-sided spectral density N_0 .

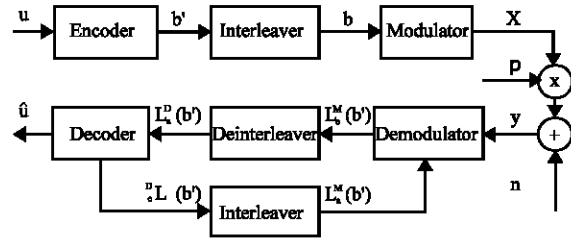


Fig. 1: BICM-ID system model

At the receiver the signal is iteratively decoded by mutually exchanging soft information between the inner demapper and the outer decoder. The demapper takes channel observations y and a priori knowledge $L_a^M(b)$ and computes the extrinsic information $L_e^M(b)$ for each of m bits per modulated symbol. The extrinsic output $L_e^M(b)$ are deinterleaved and applied as a priori input to the outer soft-input soft-output (SISO) decoder which calculates the extrinsic information of the coded bits (Benedetto *et al.*, 1997; Xuelan and Guangzeng, 2002). Then $L_e^D(b)$ is re-interleaved and fed back as a priori knowledge $L_a^M(b)$ to the inner demapper where it is exploited to reduce bit error rate (BER) in further iterative decoding steps. During the initial demapping step, $L_a^M(b)$ are set to zeros. After the last iteration, the final decoded outputs are the hard decisions based on the extrinsic bit probability, which is the total a posteriori probability because the a priori information of the information bits is unused in a BICM-ID system as shown in Fig. 1.

THE NEW INTERLEAVING DESIGN METHOD

Interleaving is a standard signal processing technique used in a variety of communication systems. An interleaver is a device that takes sequences of symbols in a fixed alphabet at the input and produces an output sequence over the same alphabet, which is identical to the input sequence except for the order. As shown in Fig. 2, an interleaver is described by the function:

$$\pi : Z \rightarrow Z \tag{1}$$

It shows that interleaving is a permutation on the integers Z , where $\pi(i)$ is the output of the interleaver at time i . There are 2 classical types of interleaving, commonly referred to as block and convolutional interleaving. The random interleaver used in BICM-ID is actually the pseudorandom block interleaver, which is a variation of the classical block interleaver (Heegard and

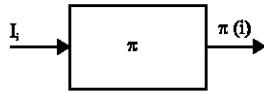


Fig. 2: An interleaver

Wicker, 1999). In this study the random interleaver is generated using the permutation that puts a noise vector in a sorted order and the noise vector is generated by a uniformly distributed noise generator.

The golden interleaver is different from the random interleaver, which is generated from the sorted real-valued numbers derived from the golden section value (Crozier *et al.*, 1999). The real-valued golden vector e is calculated as follows:

$$e(k) = s + kc \text{ mod } N, k = 0, 1, \dots, N - 1 \quad (2)$$

where, s is any real starting value and N is the interleaver length. c is calculated by

$$c = N(g^m + j)/r \quad (3)$$

In the expression of c , g is the golden section value ($g = (\sqrt{5} - 1)/2 \approx 0.618$, m is any integer greater than zero, r is the index spacing and j is any integer modulo r . The preferred values of j , m and r are 0, 1 and 1, respectively. After finding the vector e , it is possible to find the index vector z . Let vector a denote the sorted version of the vector e , then the index vector z and the sorted vector a will be related as:

$$a(k) = e(z(k)), k = 0, 1, \dots, N - 1 \quad (4)$$

Then the golden interleaver indexes are computed by the following expression:

$$\pi(z(k)) = k, k = 0, 1, \dots, N - 1 \quad (5)$$

From the above description, it is clear that the golden interleaver belongs to the field of algebraic interleavers. Simulation results in BICM-ID system show that when applied to the BICM-ID system, the golden interleaver outperforms the random interleaver when SNR is high both in AWGN and Rayleigh fading channels. However, the performance of the golden interleaver is worse than that of the random interleaver when SNR is low, which is different from the result in turbo codes. In general, the performance of a random interleaver is better than that of a traditional block interleaver and interleavers with some randomness tend to perform better than completely

structured interleavers. Considering that chaotic maps can generate high performance random sequences, we combine the chaotic sequence with the generation of the golden vector e to add the randomness to the golden interleaver. Such an interleaver is called the golden-chaotic interleaver.

A chaotic map is a dynamical discrete-time continuous-value equation which describes the relation between the present state and next state of a chaotic system. In this study we focus on one-dimensional chaotic maps. The Logistic map is a typical example of one-dimensional chaotic maps and will be used in the rest of the study. The Logistic map is defined as (Petitgen *et al.*, 1992):

$$w_{i+1} = \mu w_i(1 - w_i) \quad 0 < w_i < 1 \text{ and } 0 < \mu < 4 \quad (6)$$

$$i = 0, 1, 2, \dots$$

where, μ is the bifurcation parameter. When μ increases up to values close to 4, the Logistic map enters chaotic state and the sequence that iteration produces is chaotic. The new interleaver is generated as follows:

- Generate the chaotic sequence w according to Eq. 6 and find the index vector d . The index vector d and the vector \tilde{w} which is the sorted version of the vector w are related as:

$$\tilde{w}(k) = w(d(k)), k = 0, 1, \dots, N - 1 \quad (7)$$

- Add the index vector d to the generation of the golden vector to get the modified golden vector \tilde{e}

$$\tilde{e}(k) = s + kc + d(k) \text{ mod } N, k = 0, 1, \dots, N - 1 \quad (8)$$

Other design steps are the same with the process of the golden interleaver, following Eq. 3-5 to get the final interleaving indexes. Note that compared with the golden interleaver the additional complexity to generate the golden-chaotic interleaver is the calculation of the chaotic vector w and the index vector d . The reason that we choose the index vector d instead of the chaotic sequence w is that the effect of d on the generation of the modified golden vector \tilde{e} is more than that of w , as shown in Eq. 2 and 6-8. Simulation results in our experiment also show that the performance of the system using the vector d is much better than that directly using the chaotic sequence w .

To illustrate the effectiveness of the proposed approach, the analysis of correlation is first provided and

the BER performance comparisons among the 3 interleaving schemes including the random interleaver, the golden interleaver and the golden-chaotic interleaver in BICM-ID system are performed by simulation.

PERFORMANCE EVALUATION

Comparison of correlation: The concept of interleaver correlation is proposed in (Pingyi *et al.*, 1998). Suppose that the input data sequence to the interleaver is:

$$S_0^i, S_1^i, \dots, S_{N-2}^i, S_{N-1}^i$$

and the output data sequence is

$$S_0^o, S_1^o, \dots, S_{N-2}^o, S_{N-1}^o$$

The correlation of interleaver is defined as (Pingyi *et al.*, 1998):

$$R = \sum_{k=0}^{N-1} (2s_k^i - 1)(2s_k^o - 1) \quad (9)$$

Which is the correlation between the input data sequence and the output data sequence of the interleaver. According to the principle of the statistics we know that if the input data sequence is independent of the output sequence, the corresponding correlation is zero. However, there practically exists some relation between the input data sequence and the output data sequence. It could be calculated by averaging a number of tested data. Moreover, data correlation occurs when there is an immovable point in the interleaving sequence, namely, for an input datum its position in the input data sequence before interleaving is the same with the position in the output sequence after interleaving. Therefore, the corresponding correlation is

$$R_m + N_imm / N \quad (10)$$

where, R_m is the mean of the correlation, N_imm denotes the number of the immovable points of the interleaving method and N is the length of the interleaving sequence. An interleaver with lower correlation means less dependence between the input data sequence and the output data sequence of the interleaver and generally a better performance. Table 1 is the comparison of the correlation of the 3 interleaving methods with different interleaver length, where R_m is averaged by 10000 data. Table 1 shows that for all the interleaving methods correlation decreases with the increase of the interleaver length, which coincides with the conclusion that there is

Table 1: Correlation comparison of different interleaving methods

Interleaver length		100	1000	2004	4000
N_i	Rand int	1.0124	1.0210	1.0063	0.9978
	Gold int	2.0000	2.0000	1.0000	1.0000
	Gold-chao int	0.9996	1.0016	0.9934	0.9930
R_m	Rand int	1.0124	1.0210	5.0215	2.4945
		e-002	e-003	e-004	e-004
	Gold int	2.0000	2.0000	4.9900	2.5000
		e-002	e-003	e-004	e-004
	Gold-chao int	9.9960	1.0016	4.9571	2.4825
		e-002	e-003	e-004	e-004

a performance improvement when increasing the interleaver length (Li *et al.*, 2002). Table 1 also shows that for a given interleaver length the golden-chaotic interleaver has the lowest correlation among the 3 algorithms and the correlation of the golden interleaver is higher than that of the random interleaver. According to the analysis of correlation, it could be conjectured that the golden-chaotic interleaver has the best performance among the 3 methods.

Simulation results: Now we provide the simulation results for BICM-ID. We focus on a BICM-ID system with rate-1/2, 4-state convolutional encoder and 8-PSK modulation with SP (set partitioning) labeling map. The interleaver length is chosen to be 2004 bits and the SISO decoder for all the simulation results uses the log-MAP decoding algorithm with iteration number to be 8. The bifurcation parameter is chosen to be $\mu = 3.7$ which ensures that the Logistic map enters chaotic state and the sequence that iteration produces is chaotic sequence. And j , m and r are 0, 1 and 1, respectively.

Figure 3 is the performance comparison between the random interleaver and the chaotic interleaver in AWGN and Rayleigh fading channels. The chaotic interleaver is generated by the sorted chaotic sequence which is generated according to Eq. 6. Figure 3 indicates that the chaotic interleaver outperforms the random interleaver both in AWGN and Rayleigh fading channels. There are 2 reasons that we choose a chaotic sequence instead of a pseudorandom sequence. Firstly, a chaotic sequence is determined by the initial conditions and the chaotic maps, for example the Logistic map, therefore, only a few parameters are needed to be transferred from the transmitter to the receiver. Secondly, due to the good features including ergodic, confusion and deterministic properties, chaos based interleaver provides promising methods that show good performance. As shown in Fig. 3, the interleaver generated from a chaotic sequence is better than the interleaver generated from a pseudorandom sequence (i.e. the random interleaver).

Figure 4 gives the performance comparison among the 3 interleaving schemes in AWGN channel. Figure 4 shows that at low signal to noise ratio (SNR) the random

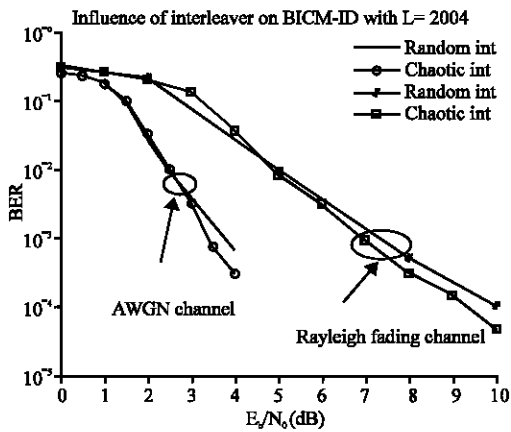


Fig. 3: Performance comparison of BICM-ID with random interleaver and chaotic interleaver both in AWGN and Rayleigh fading channels. 4-state, rate-1/2 convolutional code, 8-PSK modulation with 1002 information bits per block

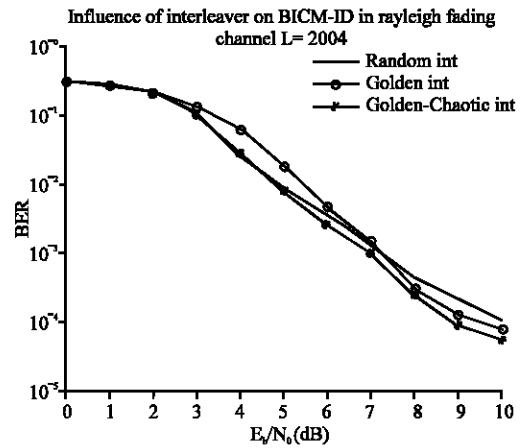


Fig. 5: Effect of interleaving methods on the performance of BICM-ID in Rayleigh fading channel. 4-state, rate-1/2 convolutional code, 8-PSK modulation with 1002 information bits per block

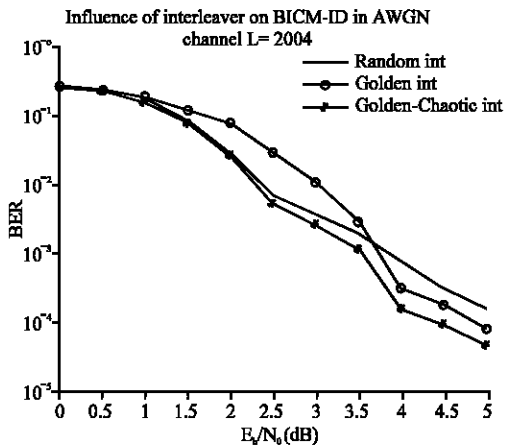


Fig. 4: Effect of interleaving methods on the performance of BICM-ID in AWGN channel. 4-state, rate-1/2 convolutional code, 8-PSK modulation with 1002 information bits per block

interleaver outperforms the golden interleaver. Whereas, when SNR is more than 3.7 dB, the golden interleaver shows the better performance and is approximately 0.15 dB better than random interleaver at BER of 10^{-3} . The golden-chaotic interleaver has the best overall performance among the 3 interleavers. There is a clear performance improvement between the golden-chaotic interleaver and the golden interleaver and a performance gain of approximately 0.4 dB is observed at BER of 10^{-4} .

Figure 5 shows the effect of the 3 interleavers on the performance of BICM-ID in Rayleigh fading channel. From Fig. 5 we can see that the performance of the golden

interleaver is better than the random interleaver when SNR is more than 7.2 dB in Rayleigh fading channel. Therefore, the golden interleaver only outperforms the random interleaver when SNR is high both in AWGN and Rayleigh fading channels. The golden-chaotic interleaver has the best performance among the 3 interleavers in Rayleigh fading channel. The performance of the golden-chaotic interleaver is approximately 1.1 dB better than the random interleaver and 0.7 dB better than the golden interleaver, respectively at BER of 10^{-4} . The results coincide with the conclusion derived from the analysis of correlation.

The increased complexity for the design of the golden-chaotic interleaver compared with the golden interleaver is the calculation of the chaotic sequence and the index vector. Because the proposed approach has the best overall performance for all the SNR values, the additional complexity is acceptable. Compared with random interleaver, the golden-chaotic interleaver inherits advantages from the golden interleaver including reduction of storage requirements, few parameters needed to be transferred from the transmitter to the receiver. Because of the significant performance gain and advantages which are attractive in the practical applications, the chaotic-golden interleaver has promising application in BICM-ID.

CONCLUSION

The interleaver design is an important parameter for the high performance of BICM-ID. Recently, chaos theory has been widely applied to signal processing and digital

communications. In this study, we investigate the application of chaotic theory as a novel coding technique in the iterative decoding system. The correlation of the interleaver as a design parameter is also investigated. Simulation results show that the golden interleaver only outperforms the random interleaver when SNR is high both in AWGN and Rayleigh fading channels. Based on chaos theory, we propose the golden-chaotic interleaving scheme to improve the performance of the golden interleaver in BICM-ID. Numerical results show that the proposed approach has the lowest correlation among the 3 interleaving schemes. In comparison to the golden interleaver, the golden-chaotic interleaver exhibits a significant performance improvement and has the best overall performance for all the SNR values with an acceptable complexity addition. The application of the proposed interleaving method is not limited to BICM-ID and it could be used in other systems where an interleaver is required. Obviously, the chaotic technique provides a new method to further improve the interleaver's performance and implementation. At the same time, it also shows a promising application of the chaos theory for channel coding in digital communication systems.

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