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The Total Completion Time: An Important Performance Measure in Scheduling

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Abstract: Given the problem of scheduling n jobs with release dates on a single machine, we prove that any solution method that minimizes the total completion time (C_{tot}) also minimizes total flow time (F_{tot}), total lateness (L_{tot}), average completion time (C_{avg}), average flow time (F_{avg}) and average lateness (L_{avg}). Two approaches (Analytical and Experimental approaches) were adopted to prove this conjecture.

Key words: Scheduling, Heuristics, total completion time, single-machine, release date

INTRODUCTION

In scheduling, performance measures are the criteria by which we evaluate schedules and ultimately solution methods (Oluleye and Oyetunji, 1999; Oyetunji and Oluleye, 2007). One crucial point to note is the understanding of the interactions among performance measures (scheduling criteria) (Patchrawat, 2000). The problem of minimizing the total completion time on single machine with release date has been studied by many researchers and a number of approximation algorithms have been proposed for the problem (Afrati *et al*., 1999; Chakrabarti *et al*., 1996; Chekuri *et al*., 1997; Hoogeveen, 1992; Hoogeveen and Van de Velde, 1995; Lawler *et al*., 1993; Philips *et al*., 1998; Patchrawat, 2000).

The problem is described as follows: A set of n independent jobs with release date or ready time is to be scheduled on a single machine that is continuously available from time zero onwards. It can process at most one job at a time. Each job J_i has a positive processing time p_i. We assume that pre-emtion is not allowed and that the problem is static and deterministic (The number of jobs, their processing times and ready times are all known in advance and fixed). The primary objective is to minimize the total completion time (C_{tot}) while the secondary objectives are minimization of total flow time (F_{tot}), total lateness (L_{tot}), average completion time (C_{avg}), average flow time (F_{avg}) and average lateness (L_{avg}). The aim of this study is to show that any solution method that minimizes C_{tot} also minimizes F_{tot}, L_{tot}, C_{avg}, F_{avg} and L_{avg}. To do this 2 approaches (analytical and experimental approaches) were employed.

Analytical (mathematical) approach: The mathematical expressions for all the 6 performance measures being considered are developed as follows:

Lemma 1: F_{tot}, L_{tot}, C_{avg}, F_{avg} and L_{avg} criteria are all optimal when C_{tot} criterion is optimal.

Proof: Let:

C_i = The completion time of the ith scheduled job.
F_i = The flow time of the ith scheduled job.
L_i = The lateness of the ith scheduled job.
r_i = The ready time of the ith scheduled job.
d_i = The due date (expected delivery date) of the ith scheduled job.

- Total Completion time (C_{tot}), by definition, is the sum of the completion time of all the jobs.

$$C_{tot} = C_1 + C_2 + ... + C_n = \sum_{i=1}^{n} C_i \quad (1)$$

- Total Flow time (F_{tot}), by definition, is the sum of the flow time of all the jobs.

$$F_{tot} = F_1 + F_2 + ... + F_n = \sum_{i=1}^{n} F_i \quad (2)$$

But, the flow time of each job is defined as:

$$F_i = C_i - r_i \quad (3)$$

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Now, substituting Eq. (3) in Eq. (2), we have

$$F_{tot} = \sum_{i=1}^{n}(C_i - r_i) = \sum_{i=1}^{n} C_i - \sum_{i=1}^{n} r_i \quad (4)$$

- Total Lateness (L_{tot}), by definition, is the sum of the lateness of all the jobs.

$$L_{tot} = L_1 + L_2 + ... + L_n = \sum_{i=1}^{n} L_i \quad (5)$$

But, the lateness of a job is defined as:

$$L_i = C_i - d_i \quad (6)$$

Substituting Eq. (6) in Eq. (5), we have

$$L_{tot} = \sum_{i=1}^{n}(C_i - d_i) = \sum_{i=1}^{n} C_i - \sum_{i=1}^{n} d_i \quad (7)$$

A careful examination of Eq. 4 and 7 (which are expressions for toal flow time and total lateness criteria, respectively) shows that they are functions of the total completion time. Hence, the total flowtime and total lateness are directly proportional to the total completion time. This means that the 2 criteria (F_{tot} and L_{tot}) are optimal when C_{tot} is optimal.

The expressions for the average completion time (C_{avg}), average flow time (F_{avg}) and average lateness (L_{avg}) follows from (i), (ii) and (iii) above.

- Average completion time (C_{avg}), by definition, is the sum of the completion time of all the jobs divided by the number of job (n).

$$C_{avg} = \frac{1}{n}\sum_{i=1}^{n} C_i \quad (8)$$

- Average flow time (F_{avg}), by definition, is the sum of the flow time of all the jobs divided by the number of job (n).

$$F_{avg} = \frac{1}{n}\sum_{i=1}^{n} F_i = \frac{1}{n}\sum_{i=1}^{n} C_i - \frac{1}{n}\sum_{i=1}^{n} r_i \quad (9)$$

- Average lateness (L_{avg}), by definition, is the sum of the lateness of all the jobs divided by the number of job (n).

$$L_{avg} = \frac{1}{n}\sum_{i=1}^{n} L_i = \frac{1}{n}\sum_{i=1}^{n} C_i - \frac{1}{n}\sum_{i=1}^{n} d_i \quad (10)$$

Since, C_{avg}, F_{avg} and L_{avg} are C_{tot}, F_{tot} and L_{tot} divided by a constant, respectively, they are also optimal when C_{tot} is optimal.

MATERIALS AND METHODS

The problem of scheduling n jobs with release dates on a single machine with the aim of minimizing the total completion time of job is NP-hard (Hoogeveen, 1992; Karger *et al*., 1997). Since, we are unaware of an optimal solution method to this problem, three widely reported heuristics and found to perform well were selected as test heuristics.

A number of random problems as shown in Table 1 were generated and solved using the three test heuristics. The values of the total completion time (C_{tot}), total flow time (F_{tot}), total lateness (L_{tot}), average completion time (C_{avg}), average flow time (F_{avg}) and average lateness (L_{avg}) obtained by each solution method were computed for all the 500 random problems. A program was written in Microsoft Visual Basic 6.0 to carry out the above.

The data obtained was exported into Statistical Analysis System (SAS) version 9.1 where the mean value of each criterion was computed for all the ten problem sizes considered using the SAS means procedure. The aim is to see the trend (ranking) of the heuristics with respect to the 6 criteria under different problem sizes.

Test heuristics: The three heuristics selected from the literature are as follows:

AEO: The basic idea in this heuristic consists of choosing a job Ji with the least processing time among the set of jobs that have arrived and are available for processing at time t until all the jobs have been scheduled. The AEO heuristic cleverly selects the job to process each time the machine becomes idle or a new job arrives. This heuristic was proposed by Oyetunji and Oluleye (2007).

Table 1: Classification of problems solved

Problem size	N. of problems solved
6×1	50
8×1	50
10×1	50
12×1	50
15×1	50
20×1	50
25×1	50
30×1	50
40×1	50
50×1	50
Total	500

HR1: The HR1 heuristic schedules jobs according to the ascending order of the sum of the processing time and release date of the job (p_i+r_i). This heuristic was also proposed by Oyetunji and Oluleye (2007).

BestA: BestA constructs pre-emptive schedule by 1st constructing a non-pre-emptive schedule using Shortest Remaining Processing Time (SRPT) heuristic. The non-pre-emptive schedule is then converted to the pre-emptive schedule by list scheduling in the ascending order of the time at which an aplha (alpha takes n different values from 0-1) portion of the job has been completed in the pre-emptive schedule. This heuristic is proposed by Chekuri *et al*. (1997) and evaluated by Eric and Patchrawat (1999) and Patchrawat (2000).

RESULTS

Table 2-7 gives the mean values of total completion time (C_{tot}), total flow time (F_{tot}), total lateness (L_{tot}), average completion time (C_{avg}), average flow time (F_{avg}) and average lateness (L_{avg}), respectively.

Based on the mean value of the total completion time (C_{tot}), the ranking order AEO, BestA and HR1 was obtained when the number of jobs (n) = 6, 10, 12, 15, 20, 25, 30, 40 and 50. However, when n = 8 the ranking order changed to BestA, AEO and HR1 (Table 2). Similar ranking order was obtained for total flowtime (Table 3), total lateness (Table 4), average completion time (Table 5), average flowtime (Table 6) and average lateness (Table 7).

Table 2: Mean of total completion time obtained from the heuristics

	Mean of total completion time		
Problem size	AEO	BestA	HR1
6×1	886.16	888.98	896.10
8×1	1517.26	1501.42	1524.88
10×1	2219.48	2231.88	2288.28
12×1	2991.58	3049.12	3147.12
15×1	4601.14	4721.42	4882.12
20×1	8191.60	8272.64	9116.56
25×1	11920.16	12178.68	13802.60
30×1	16079.30	16393.18	19108.40
40×1	28765.44	29668.08	35145.38
50×1	45646.62	46142.70	58089.84

Sample size = 50

Table 3: Mean of total flow time obtained from the heuristics

	Mean of total flow time		
Problem size	AEO	BestA	HR1
6×1	798.46	801.28	808.50
8×1	1351.86	1336.02	1360.68
10×1	1965.48	1977.88	2037.02
12×1	2673.14	2730.68	2828.68
15×1	4042.28	4162.56	4327.16
20×1	7156.22	7237.26	8091.14
25×1	10324.60	10583.12	12213.74
30×1	13878.24	14192.12	16913.84
40×1	24693.46	25596.10	31084.46
50×1	39302.60	39798.68	51759.46

Sample size = 50

Table 4: Mean of total Lateness obtained from the heuristics

	Mean of total lateness		
Problem size	AEO	BestA	HR1
6×1	149.14	151.96	159.18
8×1	443.28	427.44	452.10
10×1	824.20	836.60	895.74
12×1	1328.28	1385.82	1483.82
15×1	2315.62	2435.90	2600.50
20×1	4545.48	4626.52	5480.40
25×1	7013.10	7271.62	8902.24
30×1	9826.24	10140.12	12861.84
40×1	18796.72	19699.36	25187.72
50×1	31235.98	31732.06	43692.84

Sample size = 50

Table 5: Mean of average completion time obtained from the heuristics

	Mean of average completion time		
Problem size	AEO	BestA	HR1
6×1	147.69	148.16	149.35
8×1	189.66	187.68	190.61
10×1	221.95	223.19	228.83
12×1	249.30	254.09	262.26
15×1	306.74	314.76	325.47
20×1	409.58	413.63	455.83
25×1	476.81	487.15	552.10
30×1	535.98	546.44	636.95
40×1	719.14	741.70	878.63
50×1	912.93	922.85	1161.79

Sample size = 50

Table 6: Mean of average flow time obtained from the heuristics

	Mean of average flow time		
Problem size	AEO	BestA	HR1
6×1	133.08	133.55	134.75
8×1	168.98	167.00	170.09
10×1	196.55	197.79	203.70
12×1	222.76	227.56	235.72
15×1	269.49	277.50	288.48
20×1	357.81	361.86	404.56
25×1	412.98	423.32	488.55
30×1	462.61	473.07	563.79
40×1	617.34	639.90	777.11
50×1	786.05	795.97	1035.19

Sample size = 50

Table 7: Mean of average lateness obtained from the heuristics

	Mean of average lateness		
Problem size	AEO	BestA	HR1
6×1	24.86	25.33	26.53
8×1	55.41	53.43	56.51
10×1	82.42	83.66	89.57
12×1	110.69	115.49	123.65
15×1	154.38	162.39	173.37
20×1	227.27	231.33	274.02
25×1	280.52	290.86	356.09
30×1	327.54	338.00	428.73
40×1	469.92	492.48	629.69
50×1	624.72	634.64	873.86

Sample size = 50

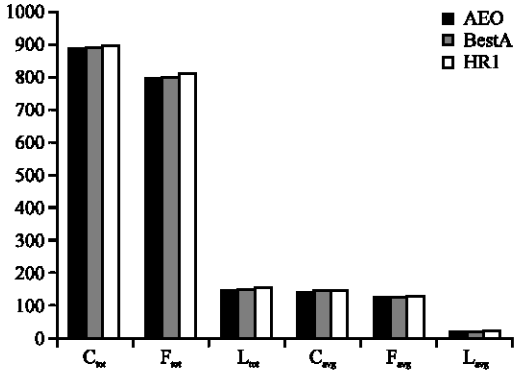

Fig. 1: Heuristic's ranking by performance measure for 6×1 problem size

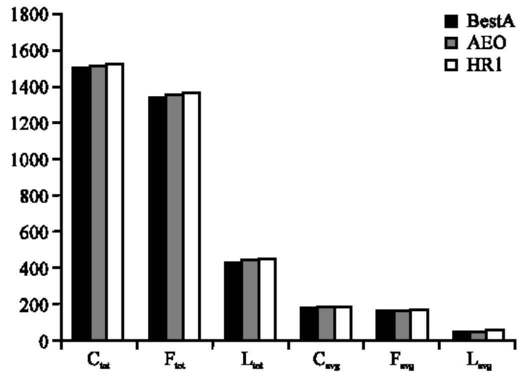

Fig. 2: Heuristic's ranking by performance measure for 8×1 problem size

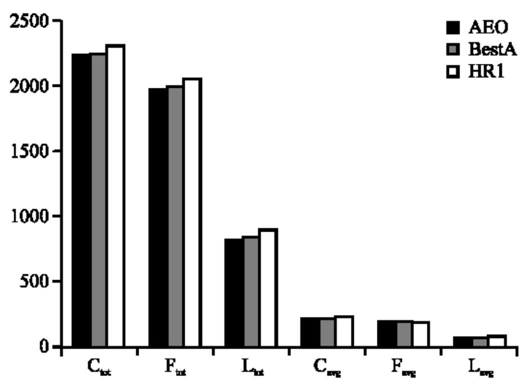

Fig. 3: Heuristic's ranking by performance measure for 10×1 problem size

The above results show that the heuristics that yield the minimum value of the total completion time (AE0 for n = 6 and n > 8 and BestA for n = 8) also yield the minimum values for F_{tot}, L_{tot}, C_{avg}, F_{avg} and L_{avg}.

To correctly interprete Table 2-7, for each problem size, observe the ranking of the heuristics in Table 2,

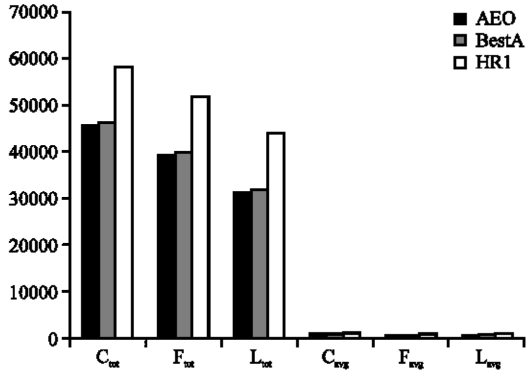

Fig. 4: Heuristic's ranking by performance measure for 50×1 problem size

proceed to Table 3-7. For the same problem size, you will observe that the heuristic's ranking is the same. Repeat the above for all the ten problem sizes. Figure 1-4 show the comparisons of the heuristic's ranking by performance measures for 6×1, 8×1, 10×1 and 50×1 problem sizes, respectively. The experimental results presented below gave similar result to Lemma 1 which was proved in the study.

CONCLUSION

We had a conjecture that minimizing the total completion time (C_{tot}) criterion also minimizes five other scheduling criteria namely: total flow time (F_{tot}), total lateness (L_{tot}), average completion time (C_{avg}), average flow time (F_{avg}) and average lateness (L_{avg}). In proving this conjecture, 2 approaches (analytical and experimental approaches) were adopted. Three heuristics designed to minimize the total completion time (C_{tot}) criterion were selected from the literature and used for experimental purposes.

Experimental results which was obtained from the 500 random problems of different sizes (6-50 jobs) solved show similar pattern, thus making the total completion time an important performance measure in single machine scheduling problems with release dates.

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