International Journal of Soft Computing 3 (1): 78-83, 2008
ISSN: 1816-9503

The Robustness of the RST Controller Obtained VIA the Law of Generalized Predictive Control

[1]M. Sedraoui and [2]S. Gherbi
[1]Department of Electronics, Faculty of Science and Engineering, Constantine University, Road of Ain El Bey Constantine, Algeria
[2]Department of Electrical Engineering, Faculty of Science and Engineering, Skikda University, Algeria

Abstract: This study presents a contribution to the study of the robustness of RST controller obtained via the law of standard GPC (Generalized-Predictive-Control). The GPC is a very recent technique of control system, it is the subject of many research during these last years and allready applied successfully in industry. It is based on a predictive model CARIMA (Controlled Auto Regressive Integrated Moving Average), his role is to predict the future behavior of the system in an extended time horizon, thus it is based on a minimization of a quadratic criterion (performance criterion) to obtain a control law minimizing the error between the output of the system and the reference input. A considerable advantage of the law of GPC is that we can transform it in a RST polynomial form, which allow us to have the possibility of checking the robustness of the controller obtained via this law on the frequential domain. There are several methods of robust control which guarantee the robustness of stability and performances, among which: LQG: Linear-quadratic-Gaussian, the H_2 method, the H_∞ method, these methods give very effective results but in a limited frequential domain. In the following, we will stain to show that method RST-GPC can be as robust and powerful as that introduced by the robust control methods (in particular the method H_∞ in the example which will follow).

Key words: Generalized predictive control, robust control, RST-GPC controller, H_∞ control

INTRODUCTION

It is well-known that the behavior of the modelizing system always differs from the real model, according to a variation at least important (uncertainties of modeling) and all the methods of the control system always do not take account of this variation (Limit, 1993). Among the most recent methods but also most promising of control systems (Ogata, 1993; Dion *et al.*, 1993; M'saad and Chebassien, 1999), it there with method GPC and it would be thus very interesting to study the behavior of the systems controled by this method face of these uncertainties (we will take a practice example), we will thus speak about the robustness of this method. The obtained results will be to compare with those of another method introduced by the robust control theory and which is very popular to control this kind of systems: It is the H_∞ method; we will not be delayed on this method being given and we will use it as tools of comparison (Francis and Doyle, 1997).

PROBLEM FORMULATION

Let the following CARIMA model:

$$A(q^{-1})y(t) = B(q^{-1})u(t-1) + \frac{C(q^{-1})}{\Delta(q^{-1})}\zeta(t) \qquad (1)$$

With:
$$A(q^{-1}) = 1 + a_1 q^{-1} + ... + a_{na} q^{-na}$$
$$B(q^{-1}) = b_0 + b_1 q^{-1} + ... + b_{nb} q^{-nb}$$
$$C(q^{-1}) = 1 + c_1 q^{-1} + ... + c_{nc} q^{-nc}$$

Where, A, B and C are the monic polynomial, set C equal to the unity. $\zeta(t)$ is an uncorrelated random process and $\Delta(q^{-1}) = 1\text{-}q^{-1}$, this from enable to introduce an integrator in the control law. Y(t), $\mu(t)$ are the process output and control signal, respectively and q^{-1} is the backward shift operator.

The role of $\Delta(q^{-1})$ is to ensure integral action of controller in order to cancel the effect of step varying output. As in all receding horizon predictive control

Corresponding Author: M. Sedraoui, Department of Electronics, Faculty of Science and Engineering, Constantine University, Road of Ain El Bey Constantine, Algeria

strategies, the control law providing the control increment $\Delta\mu(t)$ is deduced minimising a classical GPC criterion defined as follow:

$$J=\left[\sum_{j=N_1}^{N_2}\left(w(t+j)-\hat{y}(t+j)\right)^T\left(w(t+j)-\hat{y}(t+j)\right)\right]+\left[\sum_{j=1}^{N_u}\lambda_j.(\Delta u(t+j-1))^T(\Delta u_i(t+j-1))\right] \quad (2)$$

The GPC must enable us to know with j steps in advance, the value of the output $\hat{y}(t)$ to do it, we must introduce and solve the following identity (Diophantine identity) (Clarck *et al*., 1995):

$$\begin{aligned}C(q^{-1})&=E_j(q^{-1})\Delta A(q^{-1})+q^{-j}F_j(q^{-1})\\E_j(q^{-1})B(q^{-1})&=G_j(q^{-1})C(q^{-1})+q^{-j}H_j(q^{-1})\end{aligned} \quad (3)$$

Using the Eq. 1 with 3, there will have the output predicator (Codron *et al*., 1993):

$$\hat{y}(t+j)=\frac{F_j}{C}y(t)+\frac{H_j}{C}\Delta u(t-1)+G_j\Delta u(t+j-1) \quad (4)$$

Replacing the Eq. 4 in 2 and minimizing the criterion compared to μ, we obtain the following GPC law:

$$u(t)=u(t-1)-n_1^T\left(\frac{1}{C}.IF.y(t)+\frac{1}{C}.IH.\Delta u(-1)-W\right) \quad (5)$$

Where:

$$W=\left[w(t+N_1)\ \ \dots\ \ w(t+N_2)\right]^T$$
$$IF=\left[F_1(q^{-1})\ \ \dots\ \ F_{N_2}(q^{-1})\right]^T$$
$$IH=\left[H_1(q^{-1})\ \ \dots\ \ H_{N_2}(q^{-1})\right]^T$$

And: $n=\left[n_1(q^{-1})\ \ \dots\ \ n_{N_u}\right]^T=\left(G^TG+\lambda.I_{N_u}\right)^{-1}G^T$

The Eq. 5 can be rewritten as following:

$$\Delta u(t)\left[C+n_1^T.IH.q^{-1}\right]=C.n_1^T.W-n_1^T.IF.y(t) \quad (6)$$

Equation 6 represents a linear controller and its polynomial from is found is the same way as if there were no constraints. Comparing this previous relation to the polynomial RST structure we want to deduce:

$$\Delta u(t).S(q^{-1})=T(q).w(t)-R(q^{-1}).y(t) \quad (7)$$

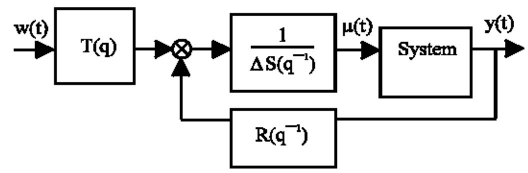

Fig. 1: Equivalent polynomial controller

The polynomials RST can be identified in the following form:

$$\begin{aligned}S(q^{-1})&=C(q^{-1})+n_1^T.IH.q^{-1}\\R(q^{-1})&=n_1^T.IF\\T(q)&=C(q^{-1}).n_1^T.\left[q^{-N_2+N_1}\ \ q^{-N_2+N_1+1}\ \ \dots\ \ 1\right]\end{aligned} \quad (8)$$

For the particular case $C(q^{-1})$; the parameters of the controller RST can be given by:

$$\begin{aligned}S_0(q^{-1})&=1+n_1^T.IH.q^{-1}\\R_0(q^{-1})&=n_1^T.IF\\T_0(q)&=n_1^T.\left[q^{-N_2+N_1}\ \ q^{-N_2+N_1+1}\ \ \dots\ \ 1\right]\end{aligned} \quad (9)$$

From the Fig. 1, we can be transformed without filter $C(q^{-1})$ by.

STUDY OF THE ROBUSTNESS OF STABILITY AND PERFORMANCES

Introduction: In all what preceded, we supposed that the system does not present disturbances (parametric or medelling errors, thus we dont have any idea on the reaction of the system faces of these uncertainties and in particular its capacity to reject them Thus we will study the robustness of such systems, to do it, we have draws up a comparative study between RST-GPC for the two cases of $C(q^{-1})$ and a method of the robust control (the H_∞ method), in what follows we will introduce notions of the robustness of the systems (Fig. 2).

Concepts of the robust control of the systems: There is always a difference between the system to be controled and the real system, this difference results from the modeling errors and of the parameters variations of the systems.

Figure 3 represents the system requirements in the presence of these disturbances:

Where, D_m is the multiplicative output uncertainty, it deduced by the difference between the real system and

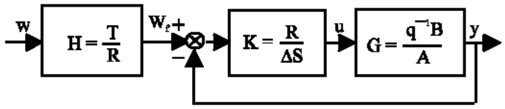

Fig. 2: The general form of the preceding structure

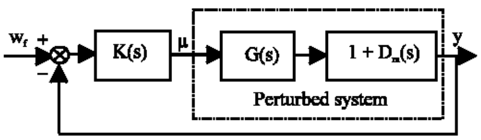

Fig. 3: Feedback system with multiplicative disturbance

the mathematic model which as to be controlled and (s) is the Laplace operator.

Denoted $G_p(s)$ the transfer function of the perturbed system given by:

$$G_p(s) = [1 + D_m(s)].G(s) \tag{10}$$

From the relation (10) the maximum standard of the disturbance $D_m(s)$ is given by:

$$|D_m(s)| = \left|\frac{G_p(s) - G(s)}{G(s)}\right| \tag{11}$$

ROBUSTNESS CONDITIONS OF STABILITY AND PERFORMANCES

Robustness of stability: By supposing that the feddback nominal system is stable when the disturbance $D_m(s)$ is null ($D_m(s) = 0$), then the perturbed system is stable if the following inequality is checked:

$$|S_c(s)| \prec \frac{1}{|W_t(s)|} \tag{12}$$

Where, $S_c(s)$ is the complementary sensitivity function given by:

$$S_c(s) = \frac{G(s).K(s)}{1 + G(s).K(s)} \tag{13}$$

And the transfer $D_m(s)$ should be verified the following inequality:

$$|D_m(s)| \leq |W_t(s)| \tag{14}$$

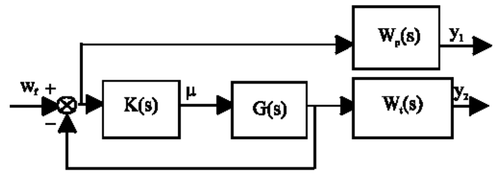

Fig. 4: The augmented system

$W_t(s)$: The weighting specification of stability.

The inequality (12) constitutes the robustness condition of the stability of the feddback system.

Performances robustness: Let $W_p(s)$ the weighting specification of system performances; the performances robustness condition of the feedback system is given by (Lewing, 1989; Chiang and Safonov, 1999):

$$|S_d(s)| \prec \frac{1}{|W_p(s)|} \tag{15}$$

$S_d(s)$ is the direct sensitivity function given by:

$$S_d(s) = \frac{1}{1 + G(s).K(s)} \tag{16}$$

The problem of the robust control is to design a controller K(s) such as the conditions of the robustness of stability and of the performances (12) and (15) are checked.

Let us notice that the transfer function $S_c(s)$ and sensitivity $S_d(s)$ are complementary:

$$S_c(s) + S_d(s) = 1 \tag{17}$$

In other words all change in the sensitivity function will have these consequences on stability and vice versa (dilemma stability-performances).

Standard H∞ Method: The goal of the method is to find a robust controller who minimizes the H∞ norm of the feedback system transfer function augmented by the specifications weightings on stability and performances (Fig. 4).

We call cost function the transfer $T_{y\mu}$ given by:

$$T_{y\mu} = \begin{bmatrix} W_p.S_d \\ W_t.S_c \end{bmatrix} \tag{18}$$

The optimization standard H_∞ problem is to find a controller who satisfies the following inequality:

$$\|T_{yu}\|_\infty \le 1 \tag{19}$$

Where, $||*||_\infty$ is the H_∞ norm of (*)

The K_∞ controller is generally found by using the software Hinf available in the Robust toolbox of Matlab®.

STUDY AND COMPARISON OF THE CONTROLLERS RST-GPC AND H∞ (PRACTICAL EXAMPLE)

With an aim of checking the robustness of the controller obtained by method RST_GPC and of comparing it with that obtained by the H_∞ method we will precede of a comparative study on a concrete example (a process used in (Matlab/Robust/actdemo. m) that we invite the reader to consult for more details).

Our objective being specified; we will not be delayed on the design of a controller by the H_∞ method (Lewing, 1989; Chiang *et al*., 1999).

Our study will be based on the checking of the satisfaction of the two robustness conditions.

The process we will study is described by the following data:

- The transfer function is:

$$G(s) = \frac{9000}{s^3 + 30s^2 + 700s + 1000}$$

- The weighting specification of performances $w_p(s)$ is given by:

$$W_p^{-1}(s) = \frac{0.01(1+s)^2}{\gamma\left(1+\left(\frac{s}{30}\right)^2\right)}$$

Where γ is the positive coefficient used in the H_∞ method

- The weighting specification of stability is given by:

$$W_t^{-1}(s) = \frac{3.16\left(1+\frac{s}{300}\right)}{1+\left(\frac{s}{10}\right)}$$

- The frequencies band is chosen of 10^{-4} Hz up to 10^{-4} Hz with sampling period $T_s = 0.01$ sec.

- The discrete model we should be controlled is given by:

$$\frac{q^{-1}.B(q^{-1})}{A(q^{-1})} = \frac{0.0014q^{-1} + 0.0051q^{-2} + 0.0012q^{-3}}{1 - 2.6802q^{-1} + 2.4219q^{-2} - 0.7408q^{-3}}$$

- The polynomial $C(q^{-1})$ is chosen as follows:

$$C(q^{-1}) = 1 - 2.5449q^{-1} + 2.3093q^{-2} - 1.0828q^{-3} + 0.5042q^{-4} - 0.1842q^{-5} + 0.0003q^{-6}$$

- The parameters of the GPC are selected like follow:

$$(N_2 \quad N_u \quad \lambda) = (6 \quad 1 \quad 0.04)$$

And the parameter of H_∞ method is selected: γ = 0.75.
For $C(q^{-1}) = 1$, the parameters of RST controller are:

$$R(q^{-1}) = 155.58q^{-1} - 365.52q^{-2} + 295.26q^{-3} - 81.73q^{-4}$$
$$S(q^{-1}) = 1 + 0.347q^{-1}$$
$$T(q) = 2.022q + 1.13q^2 + 0.51q^3 + 0.16q^4 + 0.02q^5$$

and for $C(q^{-1})$ chosen, the parameters of RST controller are:

$$R_1(q^{-1}) = 3.86q^{-1} - 10.35q^{-2} + 9.36q^{-3} - 2.86q^{-4}$$
$$S_1(q^{-1}) = 1 - 2.68q^{-1} + 2.21q^{-2} - 1.22q^{-3} + 0.80q^{-4} - 0.16q^{-5} - 0.09q^{-6}$$
$$T_1(q) = 2.022q - 4.28q^2 + 2.38q^3 + 0.015q^4 - 0.0043q^5 - 0.0405q^7 - 0.0691q^8 - 0.016q^9$$

The numerator and the denominator of K_∞ controller is:

$$numK_\infty = 5 - 12.14q^{-1} + 4.17q^{-2} + 10.64q^{-3} - 9.5q^{-4} + 1.14q^{-5} + 0.68q^{-6}$$

$$denK_\infty = 1 - 2.04q^{-1} + 0.87q^{-2} + 0.36q^{-3} - 0.16q^{-4} - 0.03q^{-5} + 0.001q^{-6}$$

RESULTS AND DISCUSSION

In the Fig. 5, for the value γ = 0.75. the robustness stability/performance is verified and we can note the satisfaction of the performances robustness, which leads in the temporal plan, to a fast stability with a very important rise time.

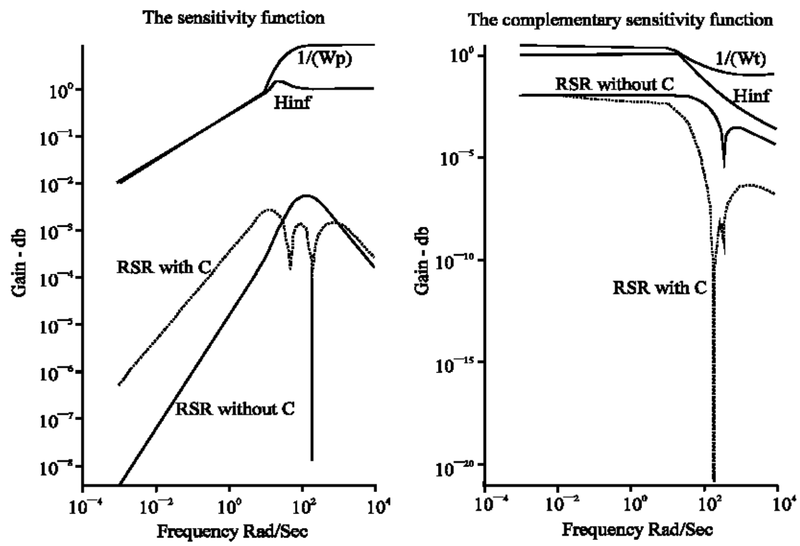

Fig. 5: The robustness stability/performance

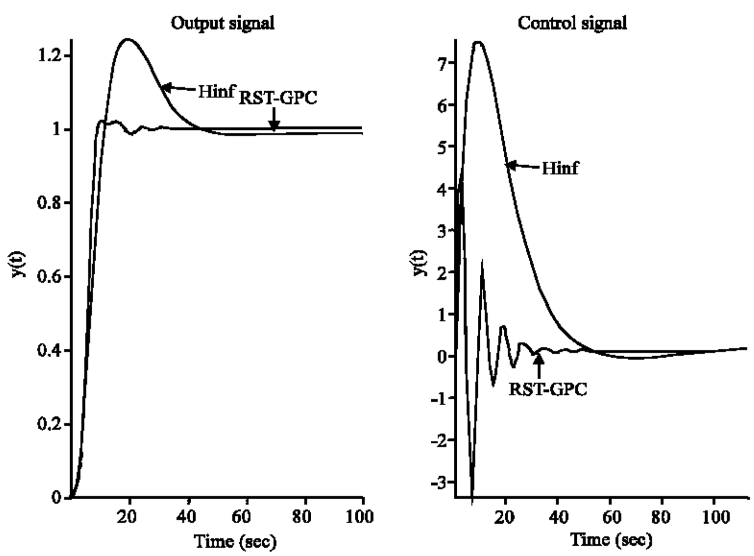

Fig. 6: The step response and the control signal

CONCLUSION

In the case of the system controlled by the H_∞ method, the robustness stability/performances is directly dependant on the good choice of the value of γ, this method offers very satisfactory results, but with a very large controller order.

For a system controlled by RST-GPC, the robustness stability/performances, depends directly on the choice on parameters of GPC (N_2, N_μ, λ), moreover we can note that the introduction of the polynomial $C(q^{-1})$, allow increased robustness of stability and performances (Fig. 6).

REFERENCES

Ansay, P. and M. Gevers, 1996. Enchancing the robustness of GPC via the youla parametrization. Service of Automatic, Cork University, Belgium.

Chiang, R.Y. and M.G. Safonov, 1999. Robust control tool Box. Users Guides, Ver. 2.

Clarke, D.W., C. Mohtadi and P.S. Tuffs, 1995. Predictive application of constrained receding horizon control to the benchmark problem. European Control Conf. Rome, Italy.

Codron, P. and P. Boucher, 1993. Multivariable generalized predictive control with multiple reference model: New choice for the reference model has, higher school of electricity, F91192 Gif-sur-Yvette (France).

Dion, J.M., L. Dugard and T. Nguyenminh, 1990. Some developments on the order adaptive multi-variable. Lab. of Car. Grenoble. (Uacnrs 228), 24-145-170, pp: 144-168.

Francis, B.A. and J.C. Doyle, 1997. Linear control theory with H_∞ optimally criterion. Control Opt., 25: 815-844.

Lewing, D.R., 1989. Stable robust performance specifications for uncertain siso systems. Int. J. Control, 53: 1263-1281.

Limit, P., 1993. Analysis and regulation of the industrial processes. Techniq Edn., 4: 05-42.

M'saad, M. and J. Chebassier, 1999. Adaptive Order of the systems. Techniques of the Engineer, Labo. D' Car. S7 426_1-S7 426-23.

Ogata, K, 1993. Discrete time control systems. University of Minnesota, USA, Flight II, pp: 478-600.

Richalet, J., 1999. Predictive order. Techniques of the Engineer, R7 423_1-R7 423_17.