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Measurement Noise Filtration and State Estimation of a Discrete-Time Stochastic Process

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Abstract: This study presents a stochastic time-invariant model for linear filtering and prediction of state of a classical problem of known system in which it is required to calculate the output for all t>0. Signal filtration in this application provides additional processing techniques under operator control to obtain accurate results in noisy measurements. This was enhanced by inclusion of an analogue filter to reduce the interference from signals generated in the measurements and to ensure that accurate signal values could be detected in the majority of the cases.

Key words: Signal filtration, noisy measurements, discrete-time stochastic process, kalman filter, maximum likelihood, Ekpoma, Nigeria

INTRODUCTION

The problem of predicting a discrete-time stochastic process correlated with a measurement noise sequence is of fundamental importance in modern control theory. Extensive developments using the concepts of finite-dimensional vector spaces and principles of orthogonal projection in Hilbert space have taken place since the original Wiener-Kolmogolov work on steady state least-squares filtering for stationary stochastic processes.

The control design problem will usually require identification of the process dynamics for predicting the responses of the system to control and disturbance inputs. Elements within the system which contribute to measurement errors are numerous, they range from problems of infrastructure to errors introduced in transmission of data especially from a remote collation site. The response of any system to a shown input function is clearly dependent on the nature of the input (Karimi *et al*., 2004; Obinabo and Ojieabu, 2007) as well as the dynamic characteristics of the system. In practice, these errors are collated as additional inputs and thereby modify the respondent of the system in a random manner.

Such inputs, generally referred to as noise when they consist of random fluctuations about a mean value have a negligible effect in many situations and can also be a variable causing an alteration in the system datum operating point and hence in the parameter of the system equations (Sage, 1967; Ho, 1962). The method of least squares provides a basis for the solution of the estimation, identification and control problems associated with unconstrained linear dynamic systems. State estimation of noisy dynamic process variables from a set of correlated data is an optimal prediction problem of fundamental interest to operators of power generating plants.

Several aspects concerning state space formulation which rely on current measurements of the process variables which control the ease with which generation is adjusted in response to demand (Bacher *et al*., 1981; Grimble, 1981; Ljung, 1979) have been addressed in the existing literature. An important deficiency in the state space description of the problem is that estimates of all the process variables must be known. Unfortunately, only a few of the states are monitored instantaneously because of sensor cost and time delays caused by the need for intermittent processing of data. In absence of correct measurement, any change in the control input can hardly influence the dynamics of interconnected subsystems.

Estimation of parameter by least squares (Hsiao and Wang, 2000) causes a smoothing of a shown set of data and eliminates errors in observation, recording transmission and conversion and as well as other types of random errors which may somehow become introduced in the data except of course bias errors due to a fixed offset, say in the measurements which cannot be eliminated by any smoothing techniques. In the light of these therefore, this study provides a basis for search for an optimum solution to the problem of state and parameter estimation of a noisy process.

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MATERIALS AND METHODS

Parametric Identification: The problem was formulated from the state vector differential equation:

$$q_n \frac{d^n x(t)}{dt^n} + ... + q_1 \frac{dx(t)}{dt} + x(t) = p_m \frac{d^m u(t)}{dt^m} + ... + p_o u(t) \quad (1)$$

First, an appropriate substitution is made as follows:

$$\frac{dx(t)}{dt} = \frac{x(t) - x(t-T)}{T} = \frac{x(t)(1-z^{-1})}{T} \quad (2)$$

where, z^{-1} is a backward shift operator and is defined as follows:

$$z^{-1}x(t) = x(t-1) \quad (3)$$

$$z^{-m}x(t) = x(t-m) \text{ and } z^{-1}u(k) = (k-1) \quad (4)$$

Here, a time-series model of the form was considered:

$$x(j) + \sum_{i=1}^{m} \alpha_i x(j-i) = \sum_{i=0}^{m} b_i u(j-i) \quad (5)$$

The partial fractions required for obtaining a time solution can for the case of distinct roots be obtained by the method of residues. If $\alpha_1 ... \alpha_k$ are the poles of the function y (s) with multiplicities $m_1 ... m_k$, we can obtain the partial fraction expansion:

$$Y(s) = \sum_{v=1}^{k} \sum_{k=1}^{m\sigma} \frac{\alpha_{vk}}{(s-\sigma_v)^k} \quad (6)$$

The inversion integral will then have the form

$$y(t) = \frac{1}{2\pi i} \int_{\sigma - i\infty}^{\sigma + i\infty} Y(s) e^{st}\, ds = \sum_{v=1}^{k} \sum_{k=1}^{m_v} \frac{C_{vk}}{(k-1)!} t^{k-1} e^{\sigma_v t}, t > 0 \quad (7)$$

For the case of distinct roots:

$$Y(s) = \left[\sum_{i=1}^{n} \frac{\beta_i}{S + \alpha_i} \right] U(s) + \text{white noise} \quad (8)$$

where, α_i, β_i are the parameters of the system and n is the order of the system whose output-input relation can be put in the form Eq. 8:

$$x(j) + \sum_{i=1}^{m} a_i x(j-i) = \sum_{i=0}^{m} b_i u(j-i) \quad (9)$$

using the shift operator z^{-1} defined by z^{-1} x(j) = x(j-1) in Eq. 9 becomes:

$$y(j) = \frac{B(z^{-1})}{A(z^{-1})} u(k) + \lambda \frac{c(z^{-1})}{A(z^{-1})} e(k), k = 1, ..., N \quad (10)$$

Where:

y (k) = The observed output signal
u (k) = The applied input signal
N = The number of samples and
e (t) = The noise sequence.

The polynomial operators A, B and C are defined as follows:

$$A(z^{-1}) = 1 + \sum_{i=1}^{NA} \alpha_i' z^{-i}$$
$$B(z^{-1}) = \sum_{i=1}^{NB} b_i' z^{-i} \quad (11)$$
$$C(z^{-1}) = 1 + \sum_{i=1}^{NC} C_i' z^{-i}$$

If the continuous model in Eq. 5 is discretized, a discrete time model of the form will obtain:

$$y(k) = \sum_{i=1}^{n} \frac{b_i z^{-1}}{1 + a_i z^{-1}} u(k) + \lambda e(k) \quad (12)$$

where the parameter set a, b is related to the parameter set α, β via the relation

$$a_i = -\exp(-\alpha_i T)$$

and

$$b_i = \frac{\beta_i}{\alpha_i}(1 - \exp(-\alpha_i T)) \quad (13)$$

T is the sampling interval in seconds. The infinite noise of the discrete observations due to aliasing that may result from the above discretization process is negligible. This is justified if u(k) and e(s) are independent of all k and s, this is a reasonable assumption as long as the identification is performed for data acquired from experiments where u (k) is a priori known sequence. In some practical situations this assumption is often violated when operating records are used because in such a case the input may depend on the output through feedback. It is also assumed that is u(k) persistently exciting. The canonical model in Eq. 6 can be made equivalent to the discrete-time model of Eq. 12 if the following conditions are satisfied:

$$c'_i = a'_i$$
$$\frac{B(z^{-1})}{A(z^{-1})} = \sum_{i=1}^{n} \frac{b_i z^{-1}}{1 + a_i z^{-1}} \quad (14)$$

If the method of maximum likelihood is employed to estimate the system, a loss function $V(\theta)$ is defined as follows:

$$V(\theta) = \frac{1}{2}\sum_{k=1}^{N} \varepsilon^2(k) \quad (15)$$

which will be minimized with respect to the system parameter set θ = (a', b'). The residual numbers are defined by:

$$\varepsilon(k) = y(k) - \frac{B(z^{-1})}{A(z^{-1})} u(k) \quad (16)$$

So that the values of the parameter set a' and b' that make $V(\theta)$ in Eq. 15 minimum will be the estimates of the parameters of the system. The approach considered in the foregoing can be interpreted as one of finding the coefficients of the prediction model so that the mean square prediction error is as small as possible.

$$\hat{y}\left(\frac{k}{k-1}\right) = \frac{B(z^{-1})}{C(z^{-1})} u(k) + \frac{C(z^{-1}) - A(z^{-1})}{C(z^{-1})} y(k) \quad (17)$$

$$V(\theta) = \sum_{k=1}^{N} y(k) - \hat{y}\left(\frac{k}{k-1}\right)^2 = \sum_{k=1}^{N} \varepsilon^2(k) \quad (18)$$

By doing so the assumption of Gaussian distribution of the noise sequence ε (k) can be relaxed but on the other hand, the nice stochastic properties of the estimates are lost. In the model, the residues are obtained as:

$$\varepsilon(k) = y(k) - \left(\sum_{i=1}^{n} \frac{b_i z^{-1}}{1 + a_i z^{-1}}\right) u(t) \quad (19)$$

Now write the difference equation is then written as:

$$y(k) = \frac{b_1 z^{-1} + b_2 z^{-2} + \ldots + b_n z^{-n}}{1 + a_1 z^{-1} + \ldots + a_n z^{-n}} u(k) = \frac{Bz^{-1}}{Az^{-1}} u(k) \quad (20)$$

where is the $z = e^{Ts}$ transform operator. Multiplying out gives:

$$y(k)(1 + a_1 z^{-1} + \ldots + a_n z^{-n}) = (1 + b_1 z^{-1} + \ldots + b_n z^{-n}) u(k) \quad (21)$$

$$y(k+n)\left(1 + a_1 z^{-1} + \ldots + a_n z^{-n}\right) = \left(b_1 z^{-1} + \ldots + b_n z^{-n}\right) u(k+n) \quad (22)$$

Here z^{-1} simply refers to z^{-1} (k) and $z^{-1}u$ (k) = u (k-1). Substituting this into Eq. 22 gives:

$$y(k+n) = b_1 u(k+n-1) + b_2 u(k+n-2) + \ldots + b_n u(k) - a_1 y(k+n-1) - a_2 y(k+n-2) - \ldots - a_n y(k) \quad (23)$$

For system order n <1, ε(k) as shown above becomes computationally difficult and highly nonlinear. Whereas in the model of Eq. 8, the residues shown by Eq. 12 are easier and quicker to compute for any system order. Since the two models are equivalent via the transformations defined in Eq. 10, thus the form of residues shown in Eq. 12 is recommended and used in this study for parameter estimation.

RESULTS AND DISCUSSION

Signal filtration and the state estimation problem: First, we consider the representation of linear dynamical systems whose input-output behaviour is governed by set differential or difference equations and analyze the behaviour of motion of the defining state variables in state space.

The linear system may also be defined in terms of transfer function components in the frequency domain and analyzed using transformed methods of solutions which provide a basis for many important designed methods. The overall system control problem will in general be concerned with identifying the dynamics of the process and with designing a controller to achieve some desired performance. The system may also contain noise-contaminated measurements as shown in Fig. 1.

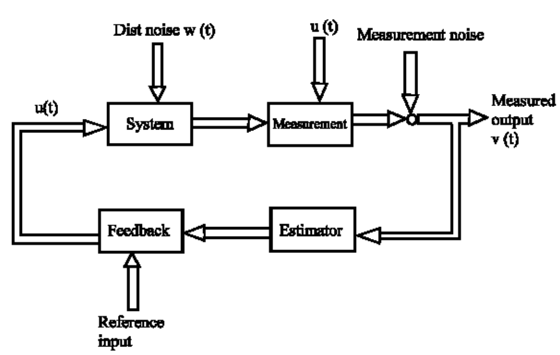

Fig. 1: Noise-contaminated measurement

The controller in the feedback system is normally designed to minimize the differece between the output and it is designed to transfer the system from some initial state to a required terminal state with desired performance defined in terms of minimum time or a minimum error function. Here, the requirement is to estimate the parameters of the process directly from its input-output relationship. In this system we employ a discrete-time sampler of input/output by choosing a correct value for say, β so that the predicted output is exactly equal to the actual output y and hence gives a zero error. In practice this is not realizable since all measurements are unavoidably subject to errors. Elements within the system which contribute to these errors are numerous; they range from the infrastructures to errors introduced in transmission of data especially from a remote collation site to the central monitoring station.

Signal filtration in this application provides additional processing techniques, under operator control to obtain accurate results in noisy measurements. This is enhanced by inclusion of an analogue filter to reduce the interference from signals generated in the measurements and to ensure that accurate signal values could be detected in the majority of the cases. This is markedly enhanced by increasing the signal-to-noise ratio in the recorded data. One method of doing this is to filter measured values to reject undesirably high frequency components. This is achieved by passing the measurements through simple low-pass filters before connecting to the input. The cut-off frequency of the filter would then be selected in conjunction with the sampling and conversion rates in order to satisfy a shown sampling theorem while preserving the signal components of interest. This method (Obinabo and Ojieabu, 2007) is entrenched in the estimation of parameters or states from a set of correlated data using least squares method and has a wide field of application which includes data smoothing and the problems concerned with identifying the parameters of noisy dynamic processes.

The procedure employed in least squares estimation of a process is usually carried out for a sequence of different order process and noise models until the best and simplest possible model is obtained. The more general estimation problem can be formulated on the basis of maximum likelihood and Bayesian techniques (Athans, 1971; Kalman, 1960) using statistical information in terms of joint probability distribution functions. However, for the linear dynamic system with additive, zero-mean white Gaussian measurement noise defined in terms of mean values and variances which will be appropriate for many practical problems, the least-squares solution formulated as a deterministic problem with appropriate weighting leads to the maximum likelihood estimate. Estimation of the process was based on the assumption that some or all of the parameters may be unknown even though the structure of the differential equation characterizing the system as well as the initial and boundary conditions may be available. This reduced the problem to one of optimal control whereby the best estimate using the measured values of the input and output the system are required.

Because measurements invariably contain errors, solution of the problem should utilize concepts of probability and statistics in which the problem should also address state estimation because noise is usually known to be correlated with the measured data. This must be estimated at the same time as the parameters during which filtration is mandatory in the processing of the data. Computation of the optimal estimates should consequently rely on convergence of the iteration employed which is accomplished through sequential filtration of the estimate.

Here, noise attenuation can be uniquely sought by recourse to filter design and control system modeling assuming all the noise processes are independent and the filter designed to give a minimum variance estimate. This should reduce the problem to one of optimal control whereby the best estimate using the measured values of the input and output of the system are required.

Because measurements invariably contain errors (Ahonsi and Gdula, 1993; Weber, 1971); the approach to the problem utilizes concepts of probability and statistics in which state estimation is addressed because noise is usually known to be correlated with the measured data (Dalley *et al*., 1989; Esho, 1993).

Example: A system is described by the difference equations:

$$x(k) = \frac{bz^{-1}}{1+az^{-1}} \times u(k)$$

$$y(k) = x(k) + n(k)$$

where, u(k) and y(k) are the measured input and output sequences, respectively and n(k) is a discrete white noise sequence. Using the data sequences shown in Table 1, it determined a simple least squares estimate of the unknown parameters (a, b), estimates of (a, b) after one step of a generalized least squares procedure with $y^F(0) = 6.225$ and $u^F(0) = -2.0$ where u^F and y^F represent the initial values of the filtered input and output, respectively.

Table 1: Data sequence

k	1	2	3	4
u (k)	1	1	–1	1
y (k)	–4	5	0	–2

$$x(k) = \frac{bz^{-1}}{1+az^{-1}}u(k) \tag{24}$$

$$y(k) = x(k) + n(k) \tag{23}$$

From Eq. 24 and 25,

$$y(k) = \frac{bz^{-1}}{1+az^{-1}}u(k) + n(k)$$
$$y(k)(1+az^{-1}) = bz^{-1}u(k) + n(k)(1+az^{-1})$$
$$y(k) = bu(k-1) - ay(k-1) + n(k)(1+az^{-1})$$
$$y(k+m) = bu(k+m-1) - ay(k+m-1) + n(k+m)(1+az^{-1})$$

$$\phi = \begin{bmatrix} -y(m) & -y(1)u(m) & u(1) \\ - & - & - \\ - & - & - \\ -y(M-1) & -y(M-m)u(M-1) & u(M-m) \end{bmatrix}$$

$$\hat{\beta} = \begin{bmatrix} a \\ b \end{bmatrix} \text{that is } \hat{\beta} = \left(\phi^T\phi\right)^{-1}\phi^T Y$$

From the Table 1:

$$y = \begin{bmatrix} 5 \\ 0 \\ 2 \end{bmatrix} \text{and } \phi = \begin{bmatrix} 4.........1 \\ -5.......1 \\ 0......-1 \end{bmatrix}$$

$$Y = y(k+m)$$

CONCLUSION

A stochastic time-invariant model has been developed for linear filtering and prediction of state for application to a classical problem of known system in which it is required to calculate the output state for all t>0. The model provides a processing technique under operator control for eliminating the interference from signals generated by seismic wave reflections further down in the earth's crust.

There are no useful convergence results available in existing literature for the plant situation described in this research which do not operate on the basis of data filtration. The plant itself is represented by a stable, linear, parameter-dependent state model. The filtration algorithm guarantees stable generation of the residuals However, extensive simulation results must be obtained to establish the convergence properties of the algorithm for many applications.

In conclusion we indicate that extensive simulation results must be obtained to establish the convergence properties of the algorithms for applications to load frequency in electric power systems. Finally, the error generated in the estimation procedure is practically zero since it is analytical and as the entire method relied on an original heat balance that correctly describes the problem considered, the largest uncertainty can only be those associated with the physical data. The measured load frequency data over a specified time interval reflects this conclusion.

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