

A Deterministic Approach to Process Noise Attenuation in a Communication Satellite Driven by White Noise Sequence

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Abstract: This study presents a qualitative evaluation of a communication satellite driven by a white noise sequence and proposes a finite-time filtering solution in which state estimation was addressed. The dynamics of the satellite assumed linear and reducible to an equivalent interconnection of subsystems enabled state estimation by least squares computational algorithms to be obtained by convergence of the iteration process. The significant practical benefits of using the stationary form of the Kalman filter in the computation were established.

Key words: Communication satellite, finite-time filtering solution, least squares computational algorithms, Kalman filter, unbiased estimates, white noise sequence

INTRODUCTION

For many space applications involving data communication, a large number of satellites have been launched into the earth orbit. For effectiveness in their operation, the attitude must be observable and the dynamics stabilizable (Flanagan, 1969; Sohn, 1959). By the former is meant that it should be possible to orient the satellites in a preferred or specified attitude and by the latter is meant that if the satellite is uncontrolled, it should be possible to obtain a time history of its attitude by suitable instrumentation and telemetry. Since measurement of the physical parameters made by satellite-borne instruments is strongly dependent on the orientation of the instrument, it is necessary to either control the attitude precisely or provide information regarding the attitude to enhance realistic interpretation of the results. In general, the satellites and the environment in which they operate are far from isotropic (Flanagan, 1969). In the case of communications satellites of active relay type, the effective power transmitted from the satellites depends on the satellite transmitter power, transmission efficiency and antenna gain (Flanagan, 1969). In such systems, many of the design techniques based on state variable approach assume that values are available for all the states for a given control vector. However, in most practical situations it is not possible to measure all the states and furthermore, the measurements

that are available often contain significant amounts of random noise and/or systematic errors. A near-earth satellite for example, orbiting in the attitude range of 150-450 km (Flanagan, 1969) encounters small but non-negligible aerodynamic forces due essentially to gravity waves set up in a stably stratified atmosphere (Obinabo, 1978). The influence of major environmental forces on the attitude response of gravity gradient satellites using essentially both analytical and numerical techniques presents a problem of major interest to the process engineer. Several aspects concerning formulation of the problem which rely on current measurements of the process variables and the ease with which detailed characterization of drag resistance effects of gravity waves in stably stratified atmosphere especially as it affects free and forced rotation of astronomical satellites through it have been addressed in the existing literature. In absence of correct measurement, any change in the control vector can hardly influence the dynamics of interconnected subsystems thereby making it difficult to establish directly the true value of the process data since all measurements are unavoidably subject to noise (Obinabo and Ojieabu, 2007). This method is entrenched in the estimation of parameters or states from a set of correlated data using least squares method and has a wide field of application which includes data smoothing and the problems concerned with identifying the parameters of noisy dynamic processes.

To this end, the Kalman filter has received considerable attention in the existing literature (Kalman, 1960; Hsiao and Wang, 2000) and has been applied successfully in the aerospace industry (Athans, 1971). Furthermore, there has been a number of theoretical investigations and simulation studies of its use in process control applications. Given a signal model that consists of a linear dynamic system driven by stochastic white noise processes, the Kalman filter (Kalman, 1960; Foster and Saffman, 1970) exploits a state space model for optimal filtration of noisy measurements. An interesting description of the history of Kalman filtering theory (Grimble, 1981; Ljung, 1979) and the applications include navigation and guidance, global positioning systems, target tracking, communications and signal processing and electrical machines. A precursor to the Kalman filter was the Weiner filter which was derived independently by Weiner and Kolmogorov and which gives a method of optimally attenuating noise in process measurements. However, the Weiner filter is limited to time-invariant problems involving stationary noise sequence. The filter algorithm is not computationally straightforward as the Kalman filter.

In this study, space applications involving data communication, noise attenuation can be uniquely sought by recourse to filter design (Grimble, 1981; Kalman, 1960) and control system modeling assuming all the noise processes are independent and the filter designed to give a minimum variance estimate. This reduced the problem to one of optimal control whereby the best estimate using the measured values of the input and output of the system are required. Because measurements invariably contain errors (Ahonsi and Gdula, 1993; Weber, 1971), the approach to the problem utilizes concepts of probability and statistics in which state estimation was addressed because noise is usually known to be correlated with the measured data (Dalley *et al.*, 1989; Esho, 1993).

FORMULATION OF THE STATE ESTIMATION PROBLEM

The procedure employed in least squares estimation of a process is usually carried out for a sequence of difference order process and noise models until the best and simplest possible model is obtained. The more general estimation problem can be formulated on the basis of maximum likelihood and Bayesian techniques (Athans, 1971; Kalman, 1960) using statistical information in terms of joint probability distribution functions. However, for the linear dynamic system with additive, zero-mean white Gaussian measurement noise defined in terms of mean values and variances which will be appropriate for

many practical problems, the least-squares solution formulated as a deterministic problem with appropriate weighting leads to the maximum likelihood estimate.

Estimation of the process (Oyediran, 2010) was based on the assumption that some or all of the parameters may be unknown even though the structure of the differential equation characterizing the system as well as the initial and boundary conditions may be available. This reduced the problem to one of optimal control whereby the best estimate using the measured values of the input and output the system are required.

Because measurements invariably contain errors, solution of the problem should utilize concepts of probability and statistics in which the problem should also address state estimation because noise is usually known to be correlated with the measured data. This must be estimated at the same time as the parameters during which filtration is mandatory in the processing of the data. Computation of the optimal estimates should consequently rely on convergence of the iteration employed which is accomplished through sequential filtration of the estimate. Here, we consider a time-series model of the form (Kochhar and Parnaby, 1978):

$$x(j) + \sum_{i=1}^m \alpha_i x(j-i) = \sum_{i=0}^m b_i u(j-i) \tag{1}$$

The partial fraction expansion required for obtaining a time solution to Eq. 1 is defined for distinct roots using the method of residues as:

$$Y(s) = \sum_{v=1}^k \sum_{k=1}^{m_v} \frac{\alpha_{vk}}{(s - \sigma_v)^k} \tag{2}$$

where, $\alpha_1, \dots, \alpha_k$ are the poles of the function $y(s)$ with multiplicities m_1, \dots, m_k . The inversion integral was then written in the form:

$$y(t) = \frac{1}{2\pi i} \int_{\sigma-i\infty}^{\sigma+i\infty} Y(s)e^{st} ds = \sum_{v=1}^k \sum_{k=1}^{m_v} \frac{C_{vk}}{(k-1)!} t^{k-1} e^{\sigma_v t}, t > 0 \tag{3}$$

so that the function $y(s)$ with contaminated noise becomes:

$$Y(s) = \left[\sum_{i=1}^n \frac{\beta_i}{s + \alpha_i} \right] U(s) + \text{white noise} \tag{4}$$

where, α_i, β_i are the parameters of the system and n is the order of the system. Using the shift operator z^{-1} defined by $z^{-1} x(j) = x(j-1)$, Eq. 1 becomes:

$$y(j) = \frac{B(z^{-1})}{A(z^{-1})}u(k) + \lambda \frac{c(z^{-1})}{A(z^{-1})}e(k), k = 1, \dots, N \quad (5)$$

Where:

- y (k) = The observed output signal
- u (k) = The applied input signal
- N = The number of samples
- e(t) = The noise sequence

The polynomial operators A, B and C are defined as follows:

$$\begin{aligned} A(z^{-1}) &= 1 + \sum_{i=1}^{NA} \alpha'_i z^{-i}, & B(z^{-1}) &= \sum_{i=1}^{NB} b'_i z^{-i} \\ C(z^{-1}) &= 1 + \sum_{i=1}^{NC} c'_i z^{-i} \end{aligned} \quad (6)$$

If the continuous model in Eq. 4 is discretized we shall obtain a discrete time model of the form:

$$y(k) = \sum_{i=1}^n \frac{b_i z^{-1}}{1 + a_i z^{-1}} u(k) + \lambda e(k) \quad (7)$$

where the parameter set a, b is related to the parameter set α, β via the relation:

$$a_i = -\exp(-\alpha_i T) \text{ and } b_i = \frac{\beta_i}{\alpha_i} (1 - \exp(-\alpha_i T)) \quad (8)$$

where, T is the sampling interval in seconds. The infinite noise of the discrete observations due to aliasing that may result from the above discretization process is negligible. This is justified if u(k) and e(s) are independent of all k and s. This is a reasonable assumption as long as the identification is performed for data acquired from experiments where u (k) is a priori known sequence. In some practical situations, this assumption is often violated when operating records are used because in such a case the input may depend on the output through feedback.

The canonical model in Eq. 5 was made equivalent to the discrete-time model of Eq. 7 by satisfying the conditions:

$$c'_i = a'_i \quad (9)$$

$$\frac{B(z^{-1})}{A(z^{-1})} = \sum_{i=1}^n \frac{b_i z^{-1}}{1 + a_i z^{-1}}$$

The statistical method of maximum likelihood was employed to optimize the probability of obtaining the expected result. Consequently, a loss function V(θ) was defined as follows:

$$V(\theta) = \frac{1}{2} \sum_{k=1}^N \epsilon^2(k) \quad (10)$$

which will be minimized with respect to the system parameter set $\theta = [a', b']$. The residues were defined by:

$$\epsilon(k) = y(k) - \frac{B(z^{-1})}{A(z^{-1})}u(k) \quad (11)$$

So that the values of the parameter set a and b that make V(θ) in Eq. 10 minimum will be the estimates of the parameters of the system. The approach considered is one of finding the coefficients of the prediction model:

$$\hat{y}\left(\frac{k}{k-1}\right) = \frac{B(z^{-1})}{C(z^{-1})}u(k) + \frac{C(z^{-1}) - A(z^{-1})}{c(z^{-1})}y(k) \quad (12)$$

so that the mean square prediction error:

$$V(\theta) = \sum_{k=1}^N y(k) - \hat{y}\left(\frac{k}{k-1}\right) = \sum_{k=1}^N \epsilon^2(k) \quad (13)$$

is as small as possible. By doing so the assumption of Gaussian distribution of the noise sequence $\epsilon(k)$ may be relaxed. In the model given in Eq. 7, the residues were obtained as:

$$\epsilon(k) = y(k) - \left(\sum_{i=1}^n \frac{b_i z^{-1}}{1 + a_i z^{-1}} \right) u(t) \quad (14)$$

For system order greater than n, $\epsilon(k)$ as given in Eq. 14 becomes computationally difficult and highly nonlinear whereas in the model of Eq. 7, the residues given by Eq. 11 may be computed quite easily for any given system order. Since the two models are equivalent via the transformations defined in Eq. 9, any of them are used. Thus the form of residues given in Eq. 11 is recommended and used in this study for parameter estimation and application of a filter for signal noise attenuation becomes evident.

MEASUREMENT NOISE FILTRATION

Consider a multivariable system with process:

$$x_{i+1} = ax_i + v_i \quad (15)$$

$$y_{i+1} = hx_{i+1} + w_{i+1} \quad (16)$$

and the predictor:

$$x_{i+1}^i = ax_i^i \quad (17)$$

$$y_{i+1}^i = hx_{i+1}^i \quad (18)$$

Now, if we ignore the control input and assume that at time $t_i = 0$ a best estimate x_i^i of the state is available then the best prediction of x at time t_{i+1} is expressed as:

$$x_{i+1}^i = ax_i^i \quad (19)$$

Where,

$$x_{i+1}^i \equiv x \left(\frac{i+1}{i} \right)$$

At time $t = t_{i+1}$, a measurement is available as follows:

$$y_{i+1} = hx_{i+1} + w_{i+1} \quad (20)$$

Now let:

$$\delta y_{i+1} = y_{i+1} - y_{i+1}^i = y_{i+1} - hx_{i+1}^i \quad (21)$$

An attempt to improve on the estimate x_{i+1}^i will require an addition of a proportion of δy^{i+1} that is:

$$x_{i+1}^{i+1} = x_{i+1}^i + k_{i+1} (\delta y_{i+1}), = ax_i^i + k_{i+1} (y_{i+1} - hx_{i+1}^i) \quad (22)$$

where, ax_i^i are the prediction and $k_{i+1}(y_{i+1}-hx_{i+1}^i)$ the desired correction. Define δx_{i+1} as follows:

$$\delta x_{i+1} = x_{i+1} - x_{i+1}^{i+1} \quad (23)$$

$$p_{i+1}^{i+1} = E[\delta x_{i+1}^2] \quad (24)$$

Then:

$$\delta x_{i+1} = (I + k_{i+1}h)a\delta x_i + (I - k_{i+1}h)v_i + k_{i+1}w_{i+1} \quad (25)$$

$$E[\delta x_{i+1}\delta x_{i+1}^T] = p_{i+1}^{i+1}$$

$$= (I - k_{i+1}h)ap_i^T(I - k_{i+1}h)^T + (I - k_{i+1}h)GQG^T(I - k_{i+1}h)^T + k_{i+1}Rk_{i+1}^T \quad (26)$$

We required k_{i+1} such that is p_{i+1}^{i+1} a minimum. We did this by evaluating the differential:

$$\frac{\partial}{\partial k_{i+1}} p_{i+1}^{i+1} = 0 \quad (27)$$

and then substituting:

$$p_{i+1}^i = ap_i^i a^T + GQG^T \quad (28)$$

to obtain:

$$k_{i+1} = p_{i+1}^i h^T (hp_{i+1}^i h^T + R)^{-1} \quad (29)$$

Now with the process given by:

$$x_{i+1} = ax_i + v_i \quad (30)$$

$$y_{i+1} = hx_{i+1} + w_{i+1} \quad (31)$$

With the predictor:

$$x_{i+1}^i = ax_i^i \quad (32)$$

$$y_{i+1}^i = hx_{i+1}^i \quad (33)$$

The time-varying gain k_{i+1} was evaluated by defining the estimation error:

$$\delta x_{i+1} = x_{i+1} - x_{i+1}^{i+1} \quad (34)$$

The mean value of $\delta x_{i+1} = 0$ and the variance of δx_{i+1} is given as:

$$E[(\delta x_{i+1}^2)] = p_{i+1}^{i+1} \quad (35)$$

Which is a function of k_{i+1} . From the process and filter equations, it is obtained:

$$\begin{aligned} \delta x_{i+1} &= x_{i+1} - x_{i+1}^{i+1} \\ &= ax_i + v_i - [ax_i^i + k_{i+1}(y_{i+1} - hx_{i+1}^i)] \\ &= ax_i + v_i - ax_i^i - k_{i+1}[h(ax_i + v_i) + w_{i+1} - h(ax_i^i)] \quad (36) \\ &= a(x_i - x_i^i) - k_{i+1}ha(x_i - x_i^i) + v_i + k_{i+1}hv_i + kw_{i+1} \\ &= (1 - k_{i+1}h)a\delta x_i + (1 - k_{i+1}h)v_i + k_{i+1}w_{i+1} \end{aligned}$$

Therefore;

$$E[\delta_{i+1}^2] = p_{i+1}^{i+1} = (1 - k_{i+1}h)^2 a^2 p_i^i + (1 - k_{i+1}h)^2 Q_i + k_{i+1}^2 R_{i+1} \quad (37)$$

Since;

$$E[\delta x_i w_{i+1}] = E[v_i w_{i+1}] = E[\delta x_i v_i] = 0 \quad (38)$$

All noise processes are assumed to be independent and the filter designed to give a minimum variance estimate of x_{i+1}^{i+1} is as follows:

$$\frac{\partial}{\partial x_{i+1}}(p_{i+1}^{i+1}) = -2(1 - k_{i+1}h)ha^2p_i^i - 2 \quad (39)$$

$$(1 - k_{i+1}h)hQ_i + 2k_{i+1}R_{i+1} = 0$$

Let,

$$a^2p_i^i = p_{i+1}^i \quad (40)$$

Then the predicted variance is:

$$k_{i+1} = p_{i+1}^i h (h^2 p_{i+1}^i + R_{i+1})^{-1} \quad (41)$$

and

$$p_{i+1}^{i+1} = (1 - k_{i+1}h)p_{i+1}^i \quad (42)$$

$$\frac{d}{dk_{i+1}}(p_{i+1}^{i+1}) = -2(1 - k_{i+1}h)ha^2p_i^i - 2(1 - k_{i+1}h)hQ_i + 2k_{i+1}R_{i+1}$$

$$= -2(1 - k_{i+1}h)hp_{i+1}^i + 2k_{i+1}R_{i+1} \quad (43)$$

Where:

$$p_{i+1}^i \triangleq a^2p_i^i + Q_i = E[\delta x_{i+1}^2] \quad (44)$$

and

$$E[v_i^2] = Q_i \quad (45)$$

Example, we consider the dynamics of the satellite (Fig. 1) assumed linear and reducible to an equivalent interconnection defined for a single-input single-output system as follows:

Time (i)	0.0	1.00	2.00	3.00	4.00
Input u (i)	1.0	-1.00	-1.00	1.00	1.00
Output y (i+1)	-0.1	0.56	-0.34	-0.63	0.42

The system is identified using a discrete weighting sequence:

$$y(i+1) = \sum_{j=0}^1 h(j)u(i-1) + v(i-1)$$

We now formulate a sequential least squares procedure for $\hat{h}(0)$ and $\hat{h}(1)$ as follows:

$$\begin{bmatrix} y(1) \\ y(2) \\ y(3) \\ y(4) \\ y(5) \end{bmatrix} = \begin{bmatrix} u(0) & 0 \\ u(1) & u(0) \\ u(2) & u(1) \\ u(3) & u(2) \\ u(4) & u(3) \end{bmatrix} \begin{bmatrix} h(0) \\ h(1) \end{bmatrix} + \begin{bmatrix} V(1) \\ V(2) \\ V(3) \\ V(4) \\ V(5) \end{bmatrix}$$

$$\hat{h}_{k+1} = \hat{h}_k + P_{k+1}H(k+1)^T [y(k+1) - H(k+1)\hat{h}_k]$$

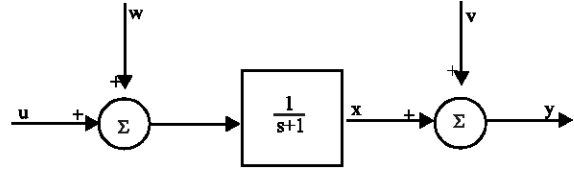


Fig. 1: The dynamics of satellite

$$\hat{h}_0 = \begin{bmatrix} u(0) & 0 \\ u(1) & u(0) \end{bmatrix}^{-1} \begin{bmatrix} y(1) \\ y(2) \end{bmatrix} = \begin{bmatrix} -0.1 \\ 0.46 \end{bmatrix}$$

$$P_0 = (H_0^T H_0)^{-1} = \begin{bmatrix} 11 \\ 12 \end{bmatrix}, P_1 = \begin{bmatrix} \frac{1}{3} & 0 \\ 0 & \frac{1}{2} \end{bmatrix}, \hat{h}_1 = \begin{bmatrix} -0.1067 \\ 0.45 \end{bmatrix}$$

$$P_2 = \begin{bmatrix} \frac{4}{5} & 0.1 \\ 0.1 & \frac{7}{20} \end{bmatrix}, \hat{h}_2 = \begin{bmatrix} -0.1189 \\ 0.4683 \end{bmatrix}, \hat{h}_3 = \begin{bmatrix} -0.106 \\ 0.488 \end{bmatrix}, P_3 = \begin{bmatrix} \frac{1}{5} & 0 \\ 0 & \frac{1}{4} \end{bmatrix}$$

which gives an unbiased estimate of the sampled data.

CONCLUSION

A stochastic time-invariant model has been developed for linear filtering and prediction of state in oil and gas seismic data acquisition. The model provides a processing technique under operator control for eliminating the interference from signals generated by seismic wave reflections further down in the earth's crust. There are no useful convergence results available in existing literature for the plant situation described in this research which do not operate on the basis of data filtration. The plant itself is represented by a stable, linear, parameter-dependent state model. The filtration algorithm guarantees stable generation of the residuals. However, extensive simulation results must be obtained to establish the convergence properties of the algorithm for many applications.

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