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Sliding Controller Design of Hybrid Synchronization of Four-Wing Chaotic Systems

[1]V. Sundarapandian and [2]S. Sivaperumal
[1]Research and Development Centre,
Vel Tech Dr. RR and Dr. SR Technical University, Avadi, 600-062 Chennai, India
[2]Institute of Technology, CMJ University, Shillong, 793-003 Meghalaya, India

Abstract: This study investigates the sliding controller design of hybrid synchronization of Four-Wing Chaotic Systems. In this study, researchers derive new results based on the Sliding Mode Control (SMC) for the hybrid synchronization of identical Qi 3D Four-Wing Chaotic Systems (2008) and identical Liu 3D Four-Wing Chaotic Systems (2009). The stability results for the hybrid synchronization schemes derived in this paper using SMC are established using the Lyapunov Stability theory. Since, the Lyapunov exponents are not required for these calculations, the sliding controller design is very effective and convenient to achieve global hybrid synchronization of the identical Qi Four-Wing Chaotic Systems and the identical Liu Four-Wing Chaotic Systems. Numerical simulations are presented to demonstrate the effectiveness of the synchronization results derived in this study.

Key words: Sliding control, chaos, hybrid synchronization, Chaotic Systems, Qi Four-Wing Systems, Liu Four-Wing Systems

INTRODUCTION

Chaotic Systems are Nonlinear Dynamical Systems which are highly sensitive to initial conditions. This sensitivity is popularly known as the butterfly effect (Alligood *et al*., 1997). Chaos is an interesting nonlinear phenomenon and has been studied well in the last three decades. Chaos theory has wide applications in several fields like physical systems (Lakshmanan and Murali, 1996), chemical systems (Han *et al*., 1995), ecological systems (Blasius *et al*., 1999), secure communications (Cuomo and Oppenheim, 1993; Kocarev and Parlitz, 1995; Murali and Lakshmanan, 2003), etc.

Chaos synchronization is a phenomenon that may occur when two or more chaotic oscillators are coupled or a chaotic oscillator drives another chaotic oscillator. Because of the butterfly effect which causes the exponential divergence of the trajectories of two identical chaotic systems started with nearly the same initial conditions, synchronizing two chaotic systems is seemingly a very challenging problem. In most of the chaos synchronization approaches, the master-slave or drive-response formalism is used. If a particular Chaotic System is called the Master or Drive System and another Chaotic System is called the Slave or Response System then the idea of the chaos synchronization is to use the output of the Master System to control the Slave System so that the output of the Slave System tracks the output of the Master System asymptotically. Since, the seminal research by Pecora and Carroll (1990), chaos synchronization problem has been studied intensively and extensively in the literature (Pecora and Carroll, 1990; Ott *et al*., 1990; Ho and Hung, 2002; Huang *et al*., 2004; Chen, 2005; Sundarapandian, 2011a-f; Lu *et al*., 2004; Chen and Lu, 2002; Park and Kwon, 2003; Xiau-Qun and Jun-An, 2003; Park, 2006; Vincent, 2007; Lee *et al*., 2010; Wang and Guan, 2006; Qiang, 2007; Sarasu and Sundarapandian, 2011; Slotine and Sastry, 1983).

In the last two decades, various schemes have been successfully applied for chaos synchronization such as OGY Method (Ott *et al*., 1990), Active Control Method (Ho and Hung, 2002; Huang *et al*., 2004; Chen, 2005; Sundarapandian, 2011), Adaptive Control Method (Lu *et al*., 2004; Chen and Lu, 2002; Sundarapandian, 2011f, g) Time-Delay Feedback Method (Park and Kwon, 2003), Backstepping Design Method (Xiau-Qun and Jun-An, 2003; Park, 2006; Vincent, 2007) Sampled-Data Deedback Synchronization Method (Lee *et al*., 2010), etc.

So far, many types of synchronization phenomenon have been presented such as complete synchronization (Pecora and Carroll, 1990), generalized synchronization (Wang and Guan, 2006), anti-synchronization

Corresponding Author: V. Sundarapandian, Research and Development Centre,
Vel Tech Dr. RR and Dr. SR Technical University, Avadi, 600-062 Chennai, India

(Sundarapandian, 2011d), projective synchronization (Qiang, 2007), generalized projective synchronization (Sarasu and Sundarapandian, 2011), etc. Complete Synchronization (CS) is characterized by the equality of state variables evolving in time while Anti-Synchronization (AS) is characterized by the disappearance of the sum of relevant state variables evolving in time.

Projective Synchronization (PS) is characterized by the fast that the Master and Slave Systems could be synchronized up to a scaling factor. In Generalized Projective Synchronization (GPS), the responses of the synchronized dynamical states synchronize up to a constant scaling matrix α. It is easy to see that the complete synchronization and anti-synchronization are special cases of the generalized projective synchronization where the scaling matrix $\alpha = I$ and $\alpha = -I$, respectively.

In hybrid synchronization of two Chaotic Systems (Sundarapandian, 2011a-c) one part of the systems is completely synchronized and the other part is anti-synchronized so that the Complete Synchronization (CS) and Anti-Synchronization (AS) co-exist in the systems.

In control theory, sliding mode control or SMC is a Nonlinear Control Method that alters the dynamics of a nonlinear system by application of a discontinuous control signal that forces the system to slide along a cross-section of the system's normal behaviour. The state-feedback control law is not a continuous function of time. Instead, it can switch from one continuous structure to another continuous structure based on the current position in the state space. Hence, sliding mode control is a variable structure control method.

In Robust Control Systems, sliding mode control is often adopted due to its inherent advantages of easy realization, fast response and good transient performance as well as its insensitivity to parameter uncertainties and disturbances. In this study, researchers derive new results based on the sliding mode control (Slotine and Sastry, 1983; Utkin, 1993; Vaidyanathan and Sampath, 2011) for the hybrid chaos synchronization of identical Qi Four-Wing Chaotic Systems (Qi *et al*., 2008) and identical Liu Four-Wing Chaotic Systems (Liu, 2009). The stability results have been established using the Lyapunov Stability theory (Hahn, 1967).

PROBLEM STATEMENT AND THE METHODOLOGY USING SLIDING MODE CONTROL

Researchers discuss the master-slave synchronization of identical Chaotic Systems and the methodology of achieving hybrid synchronization using Sliding Mode Control (SMC). Consider the Chaotic System described by the dynamics:

$$\dot{x} = Ax + f(x) \quad (1)$$

Where:

$x \in R^n$ = The state of the system
A = The $n \times n$ matrix of the system parameters
$f: R^n \to R^n$ = The nonlinear part of the system

We consider the system (1) as the master or drive system. As the slave or response system, we consider the following chaotic system described by the dynamics:

$$\dot{y} = Ay + f(y) + u \quad (2)$$

Where:

$y \in R^n$ = The state of the system
$u \in R^m$ = The controller to be designed

In hybrid synchronization, we define the synchronization error so that the odd states of the systems (1) and (2) are completely synchronized and the even states of the systems (1) and (2) are anti-synchronized. Thus, we define the hybrid synchronization error as:

$$e_i = \begin{cases} y_i - x_i, & \text{if i is odd} \\ y_i + x_i, & \text{if i is even} \end{cases} \quad (3)$$

Then, the error dynamics can be expressed in the form:

$$\dot{e} = Ae + \eta(x, y) + u \quad (4)$$

The objective of the global chaos synchronization problem is to find a controller u such that:

$$\lim_{t\to\infty} \|e(t)\| = 0 \text{ for all } e(0) \in R^n \quad (5)$$

To solve this problem, we first define the control u as:

$$u = -\eta(x, y) + Bv \quad (6)$$

where, B is a constant gain vector selected such that (A, B) is controllable. Substituting Eq. 6 into Eq. 4, the error dynamics simplifies to:

$$\dot{e} = Ae + Bv \quad (7)$$

Which is a linear time-invariant control system with single input v. Thus, the original hybrid chaos synchronization problem can be replaced by an equivalent problem of stabilizing the zero solution e = 0 of the system (7) by a suitable choice of the sliding mode control. In the sliding mode control, we define the variable:

$$s(e) = Ce = c_1e_1 + c_2e_2 + \cdots + c_ne_n \quad (8)$$

Where:

$$C = [c_1 \quad c_2 \quad \cdots \quad c_n]$$

is a constant vector to be determined. In the sliding mode control, we constrain the motion of the system (7) to the sliding manifold defined by:

$$S = \{x \in R^n \mid s(e) = 0\}$$

which is required to be invariant under the flow of the error dynamics (7). When in sliding manifold S, the system (7) satisfies the following conditions:

$$s(e) = 0 \quad (9)$$

which is the defining equation for the manifold S and:

$$\dot{s}(e) = 0 \quad (10)$$

which is the necessary condition for the state trajectory e (t) of (7) to stay on the sliding manifold S. Using Eq. 7 and 8, the Eq. 10 can be rewritten as:

$$\dot{s}(e) = C[Ae + Bv] = 0 \quad (11)$$

Solving Eq. 11 for v, we obtain the equivalent control law:

$$v_{eq}(t) = -(CB)^{-1}CA\,e(t) \quad (12)$$

where, C is chosen such that CB ≠ 0. Substituting Eq. 12 into the error dynamics (7), we obtain the closed-loop dynamics as:

$$\dot{e} = \left[I - B(CB)^{-1}C\right]Ae \quad (13)$$

The row vector C is selected such that the system matrix of the controlled dynamics $[I-B(CB)^{-1}C]$ is Hurwitz i.e., it has all eigenvalues with negative real parts. Then the controlled system (13) is globally asymptotically stable. To design the sliding mode controller for Eq. 7, we apply the constant plus proportional rate reaching law:

$$\dot{s} = -q\,\mathrm{sgn}(s) - ks \quad (14)$$

where, sgn (.) denotes the sign function and the gains q>0, k>o are determined such that the sliding condition is satisfied and sliding motion will occur. From Eq. 11 and 14, we can obtain the control v (t) as:

$$v(t) = -(CB)^{-1}\left[C(kI + A)e + q\,\mathrm{sgn}(s)\right] \quad (15)$$

which yields:

$$v(t) = \begin{cases} -(CB)^{-1}\left[C(kI + A)e + q\right], & \text{if } s(e) > 0 \\ -(CB)^{-1}\left[C(kI + A)e - q\right], & \text{if } s(e) < 0 \end{cases} \quad (16)$$

Theorem 1: The Master System (1) and the Slave System (2) are globally and asymptotically hybrid-synchronized for all initial conditions x (0), y (0)$\in R^n$ by the feedback control law:

$$u(t) = -\eta(x, y) + Bv(t) \quad (17)$$

where, v (t) is defined by Eq. 15 and B is a column vector such that (A, B) is controllable. Also, the sliding mode gains k, q are positive.

Proof: First, we note that substituting Eq. 17 and 15 into the error dynamics (4), we obtain the closed-loop error dynamics as:

$$\dot{e} = Ae - B(CB)^{-1}\left[C(kI + A)e + q\,\mathrm{sgn}(s)\right] \quad (18)$$

To prove that the error dynamics (18) is globally asymptotically stable, we consider the candidate Lyapunov function defined by the equation:

$$V(e) = \frac{1}{2}s^2(e) \quad (19)$$

which is a positive definite function on R^n. Differentiating V along the trajectories of Eq. 18 or the equivalent dynamics (14), we get:

$$\dot{V}(e) = s(e)\dot{s}(e) = -ks^2 - q\,\mathrm{sgn}(s)s \quad (20)$$

which is a negative definite function on R^n. This calculation shows that V is a globally defined, positive definite, Lyapunov function for the error dynamics (18) which has a globally defined, negative definite time derivative $\dot{V}$. Thus, by Lyapunov Stability theory (Hahn, 1967), it is immediate that the error dynamics (18) is globally asymptotically stable for all initial conditions e (0)$\in R^n$. This means that for all initial conditions e (0)$\in R^n$ we have:

$$\lim_{t\to\infty} \|e(t)\| = 0$$

Hence, it follows that the Master System (1) and the Slave System (2) are globally and asymptotically hybrid synchronized for all initial conditions x (0), y (0)$\in R^n$. This completes the proof.

HYBRID SYNCHRONIZATION OF IDENTICAL QI FOUR-WING CHAOTIC SYSTEMS

Theoretical results: We apply the sliding mode control results derived in study for the hybrid synchronization of identical Qi Four-Wing Chaotic Systems (Qi *et al*., 2008). Thus, the Master System is described by the Qi dynamics:

$$\begin{aligned} \dot{x}_1 &= a(x_2 - x_1) + \varepsilon x_2 x_3 \\ \dot{x}_2 &= cx_1 + dx_2 - x_1 x_3 \\ \dot{x}_3 &= -bx_3 + x_1 x_2 \end{aligned} \tag{21}$$

where, x_1-x_3 are state variables and a-d, ε are constant, real parameters of the system with a>0, b>0 and d>0. The Slave System is also described by the controlled Qi dynamics:

$$\begin{aligned} \dot{y}_1 &= a(y_2 - y_1) + \varepsilon y_2 y_3 + u_1 \\ \dot{y}_2 &= cy_1 + dy_2 - y_1 y_3 + u_2 \\ \dot{y}_3 &= -by_3 + y_1 y_2 + u_3 \end{aligned} \tag{22}$$

where, y_1-y_3 are state variables and u_1-u_3 are the controllers to be designed. The Qi Systems (21) and (22) are chaotic when:

a = 14, b = 43, c = -1, d = 16 and ε = 4

Figure 1 shows the four-wing strange attractor of the Qi Chaotic System (21). The hybrid synchronization error is defined by:

$$e_1 = y_1 - x_1 \;\;,\;\; e_2 = y_2 + x_2 \;\;,\;\; e_3 = y_3 - x_3 \tag{23}$$

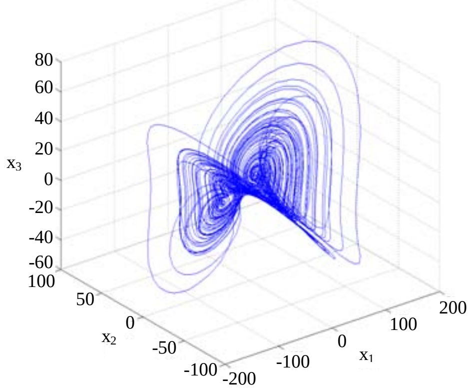

Fig. 1: Strange attractor of the Qi Four-Wing System

The error dynamics is easily obtained as:

$$\begin{aligned} \dot{e}_1 &= a(e_2 - e_1) - 2ax_2 + \varepsilon(y_2 y_3 - x_2 x_3) + u_1 \\ \dot{e}_2 &= ce_1 + de_2 + 2cx_1 - y_1 y_3 - x_1 x_3 + u_2 \\ \dot{e}_3 &= -be_3 + y_1 y_2 - x_1 x_2 + u_3 \end{aligned} \tag{24}$$

We write the error dynamics (24) in the matrix notation as:

$$\dot{e} = Ae + \eta(x, y) + u \tag{25}$$

Where:

$$A = \begin{bmatrix} -a & a & 0 \\ c & d & 0 \\ 0 & 0 & -b \end{bmatrix} \tag{26}$$

$$\eta(x,y) = \begin{bmatrix} -2ax_2 + \varepsilon(y_2 y_3 - x_2 x_3) \\ 2cx_1 - y_1 y_3 - x_1 x_3 \\ y_1 y_2 - x_1 x_2 \end{bmatrix} \tag{27}$$

And:

$$u = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \tag{28}$$

The sliding mode controller design is carried out as detailed in study. First, we set u as:

$$u = -\eta(x, y) + Bv \tag{29}$$

where, B is chosen such that (A, B) is controllable. We take B as:

$$B = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \tag{30}$$

In the chaotic case, the parameter values are:

a = 14, b = 43, c = -1, d = 16 and ε = 4

The sliding mode variable is selected as:

$$s = Ce = \begin{bmatrix} 2 & 8 & 1 \end{bmatrix} e \tag{31}$$

which makes the sliding mode state equation asymptotically stable. We choose the sliding mode gains as k = 4 and q = 0.2. We note that a large value of k can cause chattering and an appropriate value of q is chosen

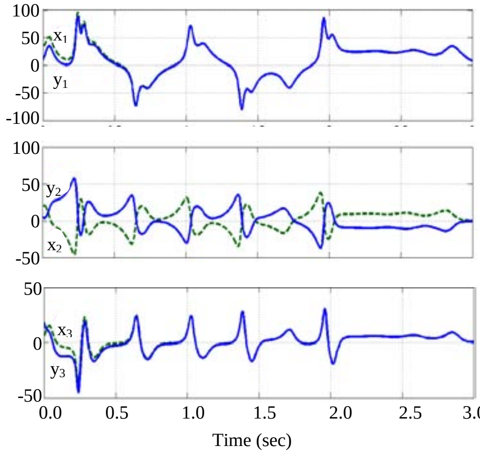

Fig. 2: Hybrid synchronization of the identical Qi Four-Wing Chaotic Systems

to speed up the time taken to reach the sliding manifold as well as to reduce the system chattering. From Eq. 15, we can obtain v (t) as:

$$v = 2.546e_1 - 17.091e_2 + 3.546e_3 - 0.018\,\mathrm{sgn}(s) \tag{32}$$

Thus, the required sliding mode controller is obtained as:

$$u = -\eta(x, y) + Bv \tag{33}$$

where, η (x, y) and v (t) are defined as in the Eq. 27, 30 and 32. By Theorem 1, we obtain the following result.

Theorem 3: The identical Qi Four-Wing Chaotic Systems (21) and (22) are globally hybrid-synchronized for all initial conditions with the sliding controller u defined by Eq. 33.

Numerical results: For the numerical simulations, the fourth-order Runge-Kutta method with time-step h = 10^{-8} is used to solve the Qi Four-Wing Chaotic Systems (21) and (22) with the sliding mode controller u given by (33) using MATLAB. The initial values of the Master System (21) are taken as:

$$x_1(0) = 32, x_2 = (0), x_3 = (0) = 4$$

The initial values of the slave system (22) are taken as:

$$y_1(0) = 10, y_2(0) = 5, y_3(0) = 18$$

Figure 2 shows the hybrid synchronization of the identical Qi Four-Wing Chaotic Systems (21) and (22). Figure 3 shows the time-history of the synchronization error e.

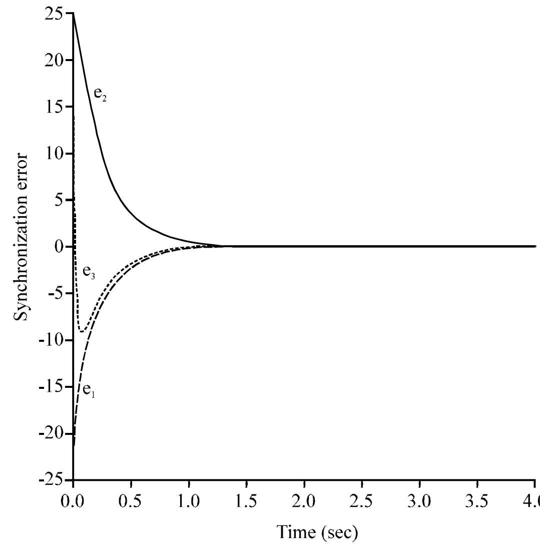

Fig. 3: Time history of the synchronization error

HYBRID SYNCHRONIZATION OF IDENTICAL LIU FOUR-WING CHAOTIC SYSTEMS

Theoretical results: We apply the sliding mode control results derived in this study for the hybrid synchronization of identical Liu Four-Wing Chaotic Systems (Liu, 2009). Thus, the Master System is described by the Liu dynamics:

$$\begin{aligned} \dot{x}_1 &= a(x_2 - x_1) + x_2 x_3^2 \\ \dot{x}_2 &= b(x_1 + x_2) - x_1 x_3^2 \\ \dot{x}_3 &= -cx_3 + dx_2 + x_1 x_2 x_3 \end{aligned} \tag{34}$$

where, x_1-x_3 are state variables and a-d are constant, real parameters of the system. The Slave System is also described by the controlled Liu dynamics:

$$\begin{aligned} \dot{y}_1 &= a(y_2 - y_1) + y_2 y_3^2 + u_1 \\ \dot{y}_2 &= b(y_1 + y_2) - y_1 y_3^2 + u_2 \\ \dot{y}_3 &= -cy_3 + dy_2 + y_1 y_2 y_3 + u_3 \end{aligned} \tag{35}$$

where, y_1-y_3 are state variables and u_1-u_3 are the controllers to be designed. The Liu Systems (34) and (35) are chaotic when:

$$a = 50, b = 13, c = 13 \text{ and } d = 6$$

Figure 4 shows the four-wing strange attractor of the Liu chaotic system (34). The hybrid synchronization error is defined by:

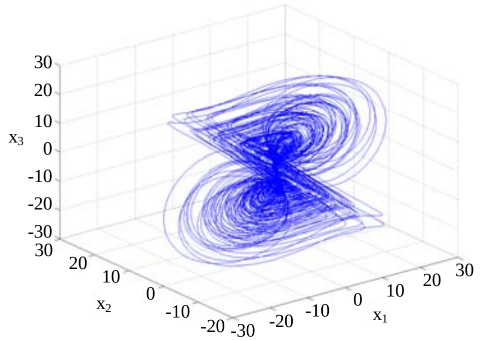

Fig. 4: Strange attractor of the Liu Four-Wing System

$$e_1 = y_1 - x_1 \;,\; e_2 = y_2 + x_2 \;,\; e_3 = y_3 - x_3 \tag{36}$$

The error dynamics is easily obtained as:

$$\begin{aligned} \dot{e}_1 &= a(e_2 - e_1) - 2ax_2 + y_2y_3^2 - x_2x_3^2 + u_1 \\ \dot{e}_2 &= b(e_1 + e_2) + 2bx_1 - y_1y_3^2 - x_1x_3^2 + u_2 \\ \dot{e}_3 &= -ce_3 + de_2 - 2dx_2 + y_1y_2y_3 - x_1x_2x_3 + u_3 \end{aligned} \tag{37}$$

We write the error dynamics (37) in the matrix notation as:

$$\dot{e} = Ae + \eta(x, y) + u \tag{38}$$

Where:

$$A = \begin{bmatrix} -a & a & 0 \\ b & b & 0 \\ 0 & d & -c \end{bmatrix} \tag{39}$$

$$\eta(x,y) = \begin{bmatrix} -2ax_2 + y_2y_3^2 - x_2x_3^2 \\ 2bx_1 - y_1y_3^2 - x_1x_3^2 \\ -2dx_2 + y_1y_2y_3 - x_1x_2x_3 \end{bmatrix} \tag{40}$$

And:

$$u = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \tag{41}$$

The sliding mode controller design is carried out as shown in study. First, we set u as:

$$u = -\eta(x, y) + Bv \tag{42}$$

where, B is chosen such that (A, B) is controllable.
We take B as:

$$B = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \tag{43}$$

In the chaotic case, the parameter values are:

$$a = 50,\; b = 13,\; c = 13 \text{ and } d = 6$$

The sliding mode variable is selected as:

$$s = Ce = \begin{bmatrix} 2 & 8 & 1 \end{bmatrix} e \tag{44}$$

which makes the sliding mode state equation asymptotically stable. We choose the sliding mode gains as $k = 4$ and $q = 0.2$.

We note that a large value of q can cause chattering and an appropriate value of q is chosen to speed up the time taken to reach the sliding manifold as well as to reduce the system chattering. From Eq. 15, we can obtain v (t) as:

$$v = -1.091e_1 - 21.455e_2 + 0.818e_3 - 0.018\,\text{sign}(s) \tag{45}$$

Thus, the required sliding mode controller is obtained as:

$$u = -\eta(x, y) + Bv \tag{46}$$

where, η (x, y), Band v (t) are defined as in the Eq. 40, 43 and 45. By Theorem 1, we obtain the following result.

Theorem 4: The identical Liu Four-Wing Chaotic Systems (34) and (35) are globally hybrid-synchronized for all initial conditions with the sliding controller u defined by Eq. 46.

Numerical results: For the numerical simulations, the ourth-order Runge-Kutta method with time-step $h = 10^{-8}$ is used to solve the Qi Four-Wing Chaotic Systems (34) and (35) with the sliding mode controller u given by Eq. 46 using MATLAB. The initial values of the Master System (34) are taken as:

$$x_1(0) = 14,\; x_2(0) = 3,\; x_3(0) = 26$$

The initial values of the Slave System (35) are taken as:

$$y_1(0) = 7,\; y_2(0) = 35,\; y_3(0) = 5$$

Figure 5 shows the hybrid synchronization of the identical Liu Four-Wing Chaotic Systems (34) and (35). Figure 6 shows the time-history of the synchronization error e.

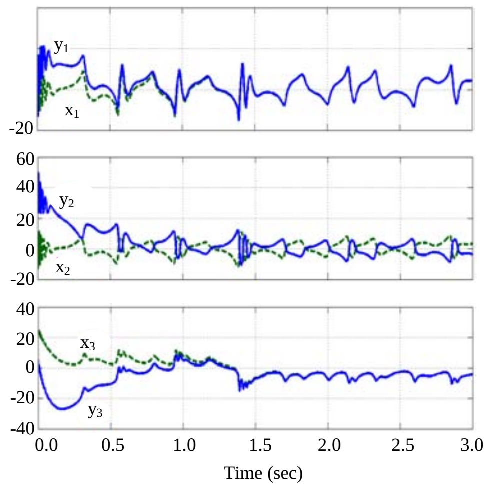

Fig. 5: Hybrid synchronization of the identical Liu Four-Wing Chaotic Systems

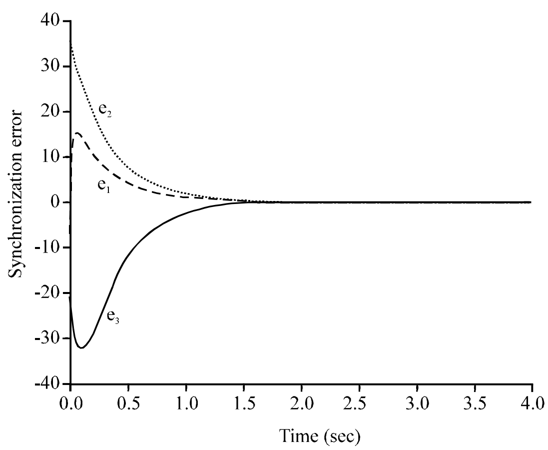

Fig. 6: Time history of the synchronization error

CONCLUSION

In this study, we have designed sliding controllers to achieve hybrid synchronization for the identical Qi Four-Wing Chaotic Systems (Qi *et al*., 2008) and Liu Four-Wing Chaotic Systems (Liu, 2009). The synchronization results have been proved using the Lyapunov Stability theory. Numerical simulations are also shown to validate synchronization results derived in this study.

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