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Weight of Interval Type-2 Fuzzy Rasch Model in Decision Making Approach: Ranking Causes Lead of Road Accident Occurrence

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Abstract: An extension of Interval Type-2 Fuzzy Sets (IT2FS), to a group decision environment which is fuzzy TOPSIS is investigated where the new weighting part of this method is developed. The objective of this study is to develop a new weighting method based on the idea of IT2FS in fuzzy TOPSIS with the Rasch Model. Rasch Model is one of the simplest methods in Item Response Theory (IRT). The Rating Scale Model (RSM) devised by Andrich in 1978 apples Rasch's Model to polytomous rating scale instruments which include the five-point Likert scale. Thus, ranking of the highest causes of road accidents are adapted to demonstrate the feasibility of the new method. Results show that this method provides us with a useful way to handle the fuzzy multiple attribute group decision-making problems in a more flexible and more intelligent manner due to the fact that it uses interval type-2 fuzzy sets rather than traditional type-1 fuzzy sets to represent the evaluating values and the weights of attributes.

Key words: Technique for Order Preference by Similarity to Ideal Solution (TOPSIS), Interval Type-2 Fuzzy Sets (IT2FS), fuzzy rasch, road accidents, intelligent, criteria

INTRODUCTION

One of the famous methods in Multiple-Criteria Decision Attribute (MCDA) is Technique for Order Preference by Similarity to Ideal Solution (TOPSIS). The concept of TOPSIS was first developed by Hwang and Yoon (1981) as the method for chosen the shortest distance of the alternative from the positive ideal solution and the farthest distance from the negative ideal solution. Since, that a lot of research has been done to develop and enrich the TOPSIS Method (Opricovic and Tzeng, 2004; Jahanshahloo *et al*., 2006; Ruta and Leonas, 2010; Wang and Xin, 2011; Ju and Wang, 2012; Yusuf, 2012). However, there are necessary steps in utilizing TOPSIS involving numerical measures of the relative importance of criteria and the performance of each alternative on these criteria. Besides, exact data may be difficult to be precisely determined since human judgments are often vague under many conditions (Chen, 2011). Thus, an extension of TOPSIS to the fuzzy environment is a natural generalization of TOPSIS Methods (Jahanshahloo *et al*., 2006; Berentsen *et al*., 2008). Since, fuzzy numbers were applied to establish a prototype fuzzy TOPSIS (Chen and Hwang, 1992; Negi, 1989), many researches on fuzzy TOPSIS have been investigated (Wang and Elhag, 2006; Gokhan *et al*., 2011; Renato and Vinicius, 2011; Tan, 2011; Buyukozkan and Cifci, 2012; Chen, 2012; Saeed *et al*., 2012). However, fuzzy TOPSIS is still believed not suitable to represent the uncertainties. Thus in 1975, Zadeh was developed a new fuzzy set method known as type-2 fuzzy set (Zadeh, 1975). Then, Mendel *et al*. (2006) improved this type-2 fuzzy set into an interval type-2 fuzzy set. Apart from this, Chen and Lee (2010) developed fuzzy TOPSIS based on interval type-2 fuzzy set. The used of interval type-2 fuzzy sets in handling fuzzy group decision-making problems is believed to give more room for flexibility due to the fact that interval type-2 fuzzy sets are more suitable to represent uncertainties than type-1 fuzzy sets (Chen and Lee, 2010).

The most important part in fuzzy TOPSIS is the weighting part. Weight in fuzzy TOPSIS can be divided into two groups which are subjective weight and objective weight. Subjective weight can reflect the subjective judgment or intuition of the Decision Makers (DM) and they can be obtained based on preference information of the attributes given by the DM through interviews, questionnaires or trade-off interrogation directly (Chena *et al*., 2011). Objective weight can be obtained from the objective information such as decision matrix through mathematics Models (Hwang and Yoon, 1981). Many of literatures mentioned about the weighting

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part in fuzzy TOPSIS. Until now, there are a lot of studies discussed on the improvement of weight in fuzzy TOPSIS. For example, Wang and Lee (2009) proposed the subjective weights assigned by Decision Makers (DM) and normalized it into a comparable scale. They adopted end-user ratings as an objective weight based on Shannon's Entropy Theory. A closeness coefficient was defined to determine the ranking order of alternatives by calculating the distances to both ideal and negative-ideal solutions. A case study was performed showing how the propose method can be used for a software outsourcing problem. This method provided decision makers more information to make more subtle decisions. Besides, Singh and Benyoucef (2011) proposed a fuzzy TOPSIS based methodology along with a mechanism for determination of fuzzy linguistic value of each attribute. Entropy method wass utilised to enumerate the weights of various attributes automatically without involvement of decision makers. An illustrative example was presented to demonstrate the applicability of the proposed methodology. Furthermore, Yue (2011) presented a new approach for determining weights of DMs in group decision environment based on an extended TOPSIS (Technique for Order Preference by Similarity to an Ideal Solution) Method. Yue defined the positive ideal solution as the average of group decision. The negative ideal solution includes two parts: left and right negative ideal solutions which were the minimum and maximum matrixes of group decision, respectively. Moreover, Huang and Peng (2011) proposed a novel approach, the Fuzzy Rasch Model which combines Item Response Theory (IRT) and Fuzzy Set Theory. This study applied the Fuzzy Rasch Model in Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) to analyse the Tourism Destination Competitiveness (TDC) of nine Asian countries, China, Hong Kong, Japan, Korea, Malaysia, Singapore, Taiwan, Thailand and the Philippines. However, this model is still unaccurate although it is used fuzzy numbers because it is due to the fact that type-2 fuzzy sets are more suitable to represent uncertainties than type-1 fuzzy sets. Motivated from the idea of Huang and Peng (2011) this study proposes a new weight based on the combination between Fuzzy Rasch Model and interval type-2 fuzzy set. This new weight then apply into interval type-2 fuzzy TOPSIS. Thus, to demonstrate the feasibility of the new method, the study road accidents in Malaysia are apply into a new method.

The last few years, road accidents have also become a serious problem in Malaysia (Hizal Hanis and Sharifah Allyana, 2009). The fatality rate caused by road accident is very alarming. The United Nations has ranked Malaysia 30th among countries with the highest number of fatal road accidents, registering an average of 4.5 deaths per 10,000 registered vehicles. The report from Royal Malaysian Police showed that traffic accident in Malaysia have been increasing at the average rate of 9.7% per annum over the last three decades. Compared to the earlier days, total number of road accidents had increased from 24,581 cases in 1974-363,319 cases in 2007. The number of fatalities (death within 30 days after accident) also increased but at slower rate compared to total road accident from 2,303 in 1974- 6,282 in 2007. Since, that road accidents have become a hot topic of discussions among public and definitely a concern for all countries. Therefore, based on the new method this study analyses the causes that lead to road accidents.

BASIC CONCEPTS

In the following, researchers recall basic notations and definitions of interval type-2 fuzzy sets and Rasch Model.

Interval type-2 fuzzy sets: This study is briefly reviews some definitions of type-2 fuzzy sets and interval type-2 fuzzy sets from Mendel *et al*. (2006).

Definition: A type-2 fuzzy set $\tilde{\tilde{A}}$ in the universe of discourse X can be represented by a type-2 membership function $\mu_{\tilde{\tilde{A}}}$ shown as follows (Mendel *et al*., 2006):

$$\tilde{\tilde{A}} = \left\{ \left((x,u), \mu_{\tilde{\tilde{A}}}(x,u) \right) \middle| \begin{array}{l} \forall x \in X, \\ \forall u \in J_x \subseteq [0,1], \\ 0 \le \mu_{\tilde{\tilde{A}}}(x,u) \le 1 \end{array} \right\} \quad (1)$$

where, J_x denotes an interval in [0, 1]. Moreover, the type-2 fuzzy set $\tilde{\tilde{A}}$ also can be represented as follows:

$$\tilde{\tilde{A}} = \int_{x \in X} \int_{u \in J_x} \mu_{\tilde{\tilde{A}}}(x,u) / (x,u) \quad (2)$$

where, $J_x \subseteq [0,1]$ and $\iint$ denotes the union over all admissible x and u.

Definition: Let $\tilde{\tilde{A}}$ be a type-2 fuzzy set in the universe of discourse X represented by the type-2 membership function $\mu_{\tilde{\tilde{A}}}$. If all $\mu_{\tilde{\tilde{A}}} = 1$ then A is called an interval type-2 fuzzy sets. An interval type-2 fuzzy set $\tilde{\tilde{A}}$ can be regarded as a special case of a type-2 fuzzy set, represented as follows (Mendel *et al*., 2006):

$$\tilde{\tilde{A}} = \int_{x \in X} \int_{u \in J_x} 1/(x,u) \quad (3)$$

where, $J_x \subseteq [0,1]$

Rasch Model: The Rating Scale Model (RSM) devised by Andrich (1978) apples Rasch's Model to polytomous rating scale instruments which include the five-point Likert scale. The Rasch Model is based on the concept that the probability of correctly obtaining an item is a function of a latent trait or ability (Kastrin and Peterlin, 2010). Notably, the Rasch Model is also known as the 1-Parameter Logistic Model (1PL). The Rasch Model converts raw data from a rating scale to an equal interval scale measured in logits (log odd units) (Belvedere and De Morton, 2010) which reflect both the difficulty of the item and individual ability (Bond and Fox, 2007).

Since, Andrich (1978) developed RSM, it has been extensively adopted by scholars to assess the values of item and person parameters as shown in Eq. 4:

$$\log\left(\frac{P_{nij}}{P_{ni(j-1)}}\right) = \theta_n - \left(\delta_i + \tau_j\right) \quad (4)$$

In Eq. 1, P_{nij} and $P_{ni\,(j-1)}$ represent the probability that the item n obtains j and j-1 scores from the expert i. θ_n represents the measure score (i.e., item difficulty) of the item n, δ_i represents the measure score (i.e., individual ability) of expert i, τ_j and represents the step difficulty (i.e., threshold difficulty) of category j. The step difficulty of RSM is identical for all items (Wright and Masters, 1982). Thus, the RSM is useful if the psychological distances between categories are identical for all items (Kim and Hong, 2004) as is the case for the Likert scales:

$$\delta_{ij} = \delta_i + \tau_j \quad (5)$$

In Eq. 2, i = 1,..., E and E represents the number of experts. j = 1,..., m and m represents the number of linguistic scales which range from very unimportant to very improtant.

A METHOD OF FUZZY MULTIPLE GROUP DECISION-MAKING BASED ON INTERVAL TYPE-2 FUZZY SETS

TOPSIS to the fuzzy environment is the main non-invasive method that very suitable for solving the group decision making problem. A multi-criteria decision making problem is the process of finding the best opinion from all of the feasible alternatives. Fuzzy TOPSIS Method with interval type-2 fuzzy set was decided as a method in this study because it is one of the best methods in decision making besides there is a room for more flexibility due to the fact that interval type-2 fuzzy sets are more suitable to represent uncertainties than type-1 fuzzy sets. To represent more uncertainties, a new weighting method which is an Interval Type-2 Fuzzy Rasch (IT2FR) weight is used. This weighting method is believes that the model is not only feasible but can also rectify the inaccuracy of the fuzzy numbers assigned by the individual experts for the specific criteria. Therefore, general process of interval type-2 fuzzy TOPSIS with IT2FR weight is listed as follows:

Assume that there is a set X of alternatives where, $X = \{x_1, x_2, ..., x_n\}$ and assume that there is a set F attributes where, $F = \{f_1, f_2, ..., f_m\}$. Assume that there are k decision-makers D1, D2 ... and D_k. The set F of attributes can be divided into two sets F_1 and F_2, F_1 where denotes the set of benefit attributes, F_2 denotes the set of cost attributes, F_2.

Step 1 construct the matrix: Construct the design matrix Y_p of the pth decision-maker and construct the average decision matrix, respectively shown as follows:

$$Y_p = \left(\tilde{\tilde{f}}_{ij}^p\right)_{m\times n} = \begin{array}{c} \\ f_1 \\ f_2 \\ \vdots \\ f_m \end{array} \begin{array}{c} \begin{array}{cccc} x_1 & x_2 & \cdots & x_n \end{array} \\ \begin{bmatrix} \tilde{\tilde{f}}_{11}^p & \tilde{\tilde{f}}_{12}^p & \cdots & \tilde{\tilde{f}}_{1n}^p \\ \tilde{\tilde{f}}_{21}^p & \tilde{\tilde{f}}_{22}^p & \cdots & \tilde{\tilde{f}}_{2n}^p \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{\tilde{f}}_{m1}^p & \tilde{\tilde{f}}_{m2}^p & \cdots & \tilde{\tilde{f}}_{mn}^p \end{bmatrix} \end{array} \quad (6)$$

$$Y = \left(\tilde{\tilde{f}}_{ij}\right)_{m\times n} \quad (7)$$

Where:

$$\tilde{\tilde{f}}_{ij} = \left(\frac{\tilde{\tilde{f}}_{ij}^1 \oplus \tilde{\tilde{f}}_{ij}^2 \oplus \ldots \oplus \tilde{\tilde{f}}_{ij}^k}{k}\right)$$

$\tilde{\tilde{f}}_{ij}$ is an interval type 2 fuzzy set, $1 \le i \le m$, $1 \le j \le n$, $1 \le p \le k$ and k denotes the number of decision-makers.

Weighting method (Step 2-6)

Step 2: Appoint the linguistic variables for the Likert Rating scale. Appoint the degree of each Likert Rating scale using the linguistic variables in interval type-2 fuzzy sets concept. This degree of each scale is indicates by selected expert opinions. There are five points of scale and each point is adapted with the concept of interval type-2 fuzzy sets.

Step 3: Determine the degree of the importance for each criteria/indices. Selected experts indicate the degree of importance of each criteria using the Likert Rating scale ranging from 1-5 that has pointed in Step 2.

Step 4: Calculate the step parameters (δ_{ij}). Calculate the step parameters (δ_{ij}) to generate the weight of Interval

Type-2 Fuzzy Rasch (IT2FR) Model. Adapting the concept of triangular interval type-2 fuzzy sets in fuzzy Rasch Model weight then this new concept is used to substitute the values of importance calculated in Step 3.

Step 5: Calculate the arithmetic average. Use an arithmetic average to integrate the new weight of each expert. This study uses an arithmetic average to integrate the fuzzy weight of each expert. The concept of triangular interval type-2 fuzzy sets of arithmetic average is adapted as follows:

$$\tilde{\tilde{w}}_j = 1/FOU(\tilde{\tilde{w}}_j) = \left(\left(\underline{\frac{1}{E}\left[\sum_{j=1}^{E}\tilde{W}_{ijc}\right]}\right),\left(\overline{\frac{1}{E}\left[\sum_{j=1}^{E}\tilde{W}_{ijc}\right]}\right)\right) = \left(\left(\underline{\delta}^L_{ijc},\underline{\delta}^M_{ijc},\underline{\delta}^U_{ijc};1\right),\left(\overline{\delta}^L_{ijc},\overline{\delta}^M_{ijc},\overline{\delta}^U_{ijc};1\right)\right) \quad (8)$$

Step 6 (weight): Calculate the weight of attributes by using the weight equation. We use w_j to represent the outcome of weight value of attribute j and it can be defined as:

$$\tilde{\tilde{w}}_j = 1/FOU(\tilde{\tilde{w}}_j) = \left[\underline{\tilde{\tilde{w}}}_j, \overline{\tilde{\tilde{w}}}_j\right] \quad (9)$$

$$\left(\underline{\tilde{\tilde{w}}}_j, \overline{\tilde{\tilde{w}}}_j\right) = \left[\left(\underline{\delta}^L_{ijc},\underline{\delta}^M_{ijc},\underline{\delta}^U_{ijc};1\right),\left(\overline{\delta}^L_{ijc},\overline{\delta}^M_{ijc},\overline{\delta}^U_{ijc};1\right)\right] \quad (10)$$

$\tilde{\tilde{w}}_j$ represents the Interval Type-2 Fuzzy Rasch (IT2FR) weight of expert i to criteria c.

Step 7: Construct the weighted decision matrix $\overline{Y}_w$:

$$\overline{Y}_w = \left(\tilde{\tilde{v}}_{ij}\right)_{m\times n} = \begin{matrix} & x_1 & x_2 & \cdots & x_n \\ f_1 & \tilde{\tilde{v}}_{11} & \tilde{\tilde{v}}_{12} & \cdots & \tilde{\tilde{v}}_{1n} \\ f_2 & \tilde{\tilde{v}}_{21} & \tilde{\tilde{v}}_{22} & \cdots & \tilde{\tilde{v}}_{2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ f_m & \tilde{\tilde{v}}_{m1} & \tilde{\tilde{v}}_{m2} & \cdots & \tilde{\tilde{v}}_{mn} \end{matrix} \quad (11)$$

where, $\tilde{\tilde{v}}_{ij} = \tilde{\tilde{w}}_i \otimes \tilde{\tilde{f}}_{ij}$, $1 \le i \le m$ and $1 \le j \le n$

Step 8 (ranking value): Calculate the ranking value $Rank(\tilde{\tilde{v}}_{ij})$ of the interval type-2 fuzzy set $\tilde{\tilde{v}}_{ij}$ where, $1 \le j \le n$. Construct the ranking weighted decision matrix $\overline{Y}^*_w$:

$$\overline{Y}^*_w = \left(Rank\left(\tilde{\tilde{v}}_{ij}\right)\right)_{m\times n} \quad (12)$$

where, $1 \le i \le m$ and $1 \le j \le n$. The ranking value $Rank(\tilde{\tilde{A}}_i)$ of the trapezoidal interval type-2 fuzzy set $\tilde{\tilde{A}}_i$ is defined as follows (Lee and Chen, 2008):

$$\begin{aligned} Rank\left(\tilde{\tilde{A}}_i\right) = {} & M_1\left(\tilde{A}_i^U\right) + M_1\left(\tilde{A}_i^L\right) + M_2\left(\tilde{A}_i^U\right) + \\ & M_2\left(\tilde{A}_i^L\right) + M_3\left(\tilde{A}_i^U\right) + M_3\left(\tilde{A}_i^L\right) - \\ & \frac{1}{4}\left(S_1\left(\tilde{A}_i^U\right) + S_1\left(\tilde{A}_i^L\right) + S_2\left(\tilde{A}_i^U\right) + S_2\left(\tilde{A}_i^L\right) + \right. \\ & \left. S_3\left(\tilde{A}_i^U\right) + S_3\left(\tilde{A}_i^L\right) + S_4\left(\tilde{A}_i^U\right) + S_4\left(\tilde{A}_i^L\right)\right) + \\ & H_1\left(\tilde{A}_i^U\right) + H_1\left(\tilde{A}_i^L\right) + H_2\left(\tilde{A}_i^U\right) + H_2\left(\tilde{A}_i^L\right) \end{aligned} \quad (13)$$

where, $M_p(\tilde{A}_i^j)$ denoted the average of the elements a^j_{ip} and:

$$a^j_{i(p+1)}\, M_p\left(\tilde{A}_i^j\right) = \left(a^j_{ip} + a^j_{i(p+1)}\right)/2, 1 \le p \le 3, S_q\left(\tilde{A}_i^j\right)$$

denotes the standard deviation of the elements a^j_{iq} and $a^j_{i(q+1)}$:

$$S_p\left(\tilde{A}_i^j\right) = \sqrt{\frac{1}{2}\sum_{k=q}^{q+1}\left(a^j_{ik} - \frac{1}{2}\sum_{k=q}^{q+1} a^j_{ik}\right)^2}$$

$1 \le q \le 3$, $s_q(\tilde{A}_i^j)$ denotes the standard deviation of the elements a^j_{i1}, a^j_{i2}, a^j_{i3}, a^j_{i4}:

$$S_4\left(\tilde{A}_i^j\right) = \sqrt{\frac{1}{4}\sum_{k=1}^{4}\left(a^j_{ik} - \frac{1}{4}\sum_{k=1}^{4} a^j_{ik}\right)^2}$$

$H_p(\tilde{A}_i^j)$ denotes the the membership value of the element $a^j_{i(p+1)}$ in the trapezoidal membership function $\tilde{A}_i^j$, $1 \le p \le 2$, $j \in \{U, L\}$ and $1 \le i \le n$.

Step 9: Positive ideal solution and negative ideal solution. Determine the positive ideal solution $x^+ = (v_1^+, v_2^+, \ldots, v_m^+)$ and the negative-ideal solution $x^- = (v_1^-, v_2^-, \ldots, v_m^-)$ where:

$$v_i^+ = \begin{cases} \max\limits_{1\le j\le n}\left\{Rank\left(\tilde{\tilde{v}}_{ij}\right)\right\}, & \text{if } f_i \in F_1 \\ \min\limits_{1\le j\le n}\left\{Rank\left(\tilde{\tilde{v}}_{ij}\right)\right\}, & \text{if } f_i \in F_2 \end{cases} \quad (14)$$

and

$$v_i^- = \begin{cases} \min\limits_{1\le j\le n}\left\{Rank\left(\tilde{\tilde{v}}_{ij}\right)\right\}, & \text{if } f_i \in F_1 \\ \max\limits_{1\le j\le n}\left\{Rank\left(\tilde{\tilde{v}}_{ij}\right)\right\}, & \text{if } f_i \in F_2 \end{cases} \quad (15)$$

where, F_1 denotes the set of benefit attributes, F_2 denotes the set of cost attributes and $1 \le i \le m$.

Step 10 (distance of each alternative): Calculate the distance $d^+(x_j)$ between each alternative x_j and the positive ideal solution x^+ as shown as follows:

$$d^{+}(x_j) = \sqrt{\sum_{i=1}^{m}\left(\mathrm{Rank}\left(\tilde{\tilde{v}}_{ij}\right) - v_i^{+}\right)^2} \quad (16)$$

where, $1 \le j \le n$. Calculate the distance $d^{-}(x_j)$ between each alternative x_j and the negative-ideal solution x^{-} shown as follows:

$$d^{-}(x_j) = \sqrt{\sum_{i=1}^{m}\left(\mathrm{Rank}\left(\tilde{\tilde{v}}_{ij}\right) - v_i^{-}\right)^2} \quad (17)$$

where, $1 \le j \le n$.

Step 11 (relative degree of closeness): Calculate the relative degree of closeness $C(x_j)$ of x_j with respect to the positive ideal solution x^{+} shown as follow:

$$C(x_j) = \frac{d^{-}(x_j)}{d^{+}(x_j) + d^{-}(x_j)} \quad (18)$$

Step 12 (sort the values of $C(x_j)$): Sort the values of $C(x_j)$ in a decending sequence where $1 \le j \le n$. The larger the value of $C(x_j)$, the higher the preference of the alternatives x_j where $1 \le j \le n$.

RANKING OF ROAD ACCIDENT CAUSES

This study discussed the flow chart of this section which is the decision to choose risks factors of road accidents. The evaluation procedure of this study consists of four main steps as shown in Fig. 1.

First, identify the causes of road accidents selection (evaluation) alternatives that are considered as the most important to the user. Second, identify the criteria of road accidents selection that are considered as the most important option to the alternative risk factors problem. After constructing the evaluation criteria hierarchy, calculating the criteria weights by applying objective weights. Lastly, conduct the interval type-2 TOPSIS Method to achieve the final ranking results.

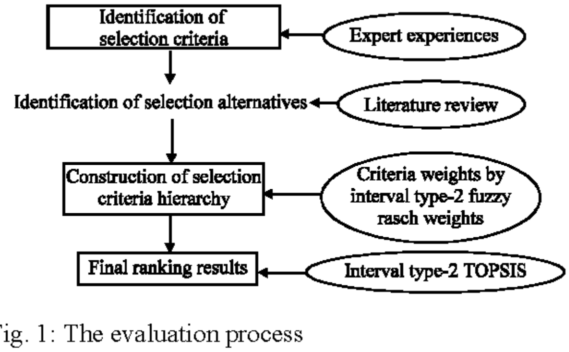

Fig. 1: The evaluation process

EVALUATION CRITERIA AND ALTERNATIVES

In reviewing the literature, data for criteria and alternatives is identified. This study set out four criteria based on the statistics from. There are four vehicles that most involved in road accidents, there are motorcycle (C_1), car (C_2), bus (C_3) and lorry (C_4).

One of the main situations that need to be considered when to decrease the number of road accidents in Malaysia is to recognize the factors that lead to road accidents. Therefore, five subjective alternatives have been highlighted based on the literature.

The five of them are speeding behaviour (x_1), reckless driving (x_2), driver's health (x_3), road condition (x_4) and road environment (x_5).

Speeding behaviour (x_1) is a socially acceptable with many thinking that their peers approve of their behaviour and that there is little chance of either being apprehended by the police or causing a collision (Holland and Conner, 1996).

Besides, reckless driving (x_2) is a threat appeal may lead people who are high in sensation seeking to perceive driving as a source of thrill and then to engage in risky driving (Orit *et al*., 2000).

Whereas, examples of driver's health (x_3) are such stroke, heart attack and so on (Hejar *et al*., 2005; Hassan and Mohamed, 2002).

Moreover, road condition (x_4) can also be divided into three parts where road type (i.e., one-way road, divided and undivided two-ways, wet roads, mud roads, dry roads, loose sands and gravel roads), location of accidents (i.e., rural road and urban road) and road light condition (i.e., day, dusk, dark lit condition, dark unit and dawn condition). Moreover, road environment (x_5) is defined as rainy weather, foggy and sleet freezing rain.

Furthermore, a committee of three decision-makers or experts, d_1-d_3 has been identified to seek reliable data over the accidents. Data in form of linguistics variables were collected through interviewing of three authorised personnel from three Malaysian Government agencies.

The interview was conducted in three separated sessions to elicit the information about causes that regularly lead to accident. Three decision-makers were:

- Vice Director of Fire Brigade Department of Kuala Terengganu (d_1)
- Sergeant of Administration from Police Traffic Department of Kuala Terengganu (d_2)

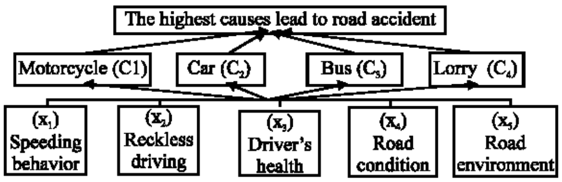

Fig. 2: Hierarchical structure of the decision problem

- Manager of Road Transport from Road Transport Department of Kuala Terengganu (d_3). The hierarchical structure of this experiment can be shown in Fig. 2

NUMERICAL PART

In this study, numerical parts of finding highest causes of road accidents are presented to illustrate the fuzzy multiple attributes group decision-making process of the proposed method. Table 1 shows the linguistic terms Very Poor (VP), Poor (P), Medium Poor (MP), Fair (F), Medium Good (MG), Good (G), Very Good (VG) and their corresponding type-2 fuzzy sets. While Table 2 is the linguistic of the decision matrix table that has been taken from the experts.

Step 1: Based on Table 1 and MCDM matrix 4, the decision matrices Y_1-Y_3 of the alternatives x_1-x_5 are constructed, respectively where:

$$Y_1 = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \end{array} \begin{array}{c} \begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & x_5 \end{array} \\ \begin{bmatrix} VG & VG & F & MG & MG \\ VG & VG & G & G & VG \\ VG & VG & VG & VG & G \\ VG & VG & VG & MG & G \end{bmatrix} \end{array}$$

$$Y_2 = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \end{array} \begin{array}{c} \begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & x_5 \end{array} \\ \begin{bmatrix} G & G & G & G & MG \\ G & G & MG & MG & G \\ G & G & G & G & MG \\ G & G & MG & G & MG \end{bmatrix} \end{array}$$

$$Y_3 = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \end{array} \begin{array}{c} \begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & x_5 \end{array} \\ \begin{bmatrix} G & P & P & P & VG \\ G & VG & MP & MP & MP \\ G & VG & MP & MP & MP \\ G & VG & MP & F & F \end{bmatrix} \end{array}$$

It should be noted that in Table 1, the universe of discourse of the linguistic terms for the rating is [0, 1].

Table 1: Linguistic terms for the ratings and their corresponding type-1 fuzzy sets

Linguistic terms	Type-1 fuzzy sets
Very Poor (VP)	((0, 0, 0.1; 1), (0, 0, 0.1; 1))
Poor (P)	((0, 0.1, 0.3; 1), (0, 0.1, 0.3; 1))
Medium Poor (MP)	((0.1, 0.3, 0.5; 1), (0.1, 0.3, 0.5; 1))
Fair (F)	((0.3, 0.5, 0.7; 1), (0.3, 0.5, 0.7; 1))
Medium Good (MG)	((0.5, 0.7, 0.9; 1), (0.5, 0.7, 0.9; 1))
Good (G)	((0.7, 0.9, 1; 1), (0.7, 0.9, 1; 1))
Very Good (VG)	((0.9, 1, 1; 1), (0.9, 1, 1; 1))

Table 2: Linguistic of decision matrix

		Decision makers		
Criteria	Alternatives	d_1	d_2	d_3
C_1	x_1	VG	G	G
	x_2	VG	G	P
	x_3	F	G	P
	x_4	MG	G	P
	x_5	MG	MG	VG
C_2	x_1	VG	G	G
	x_2	VG	G	VG
	x_3	G	MG	MP
	x_4	G	MG	MP
	x_5	VG	G	MP
C_3	x_1	VG	G	G
	x_2	VG	G	VG
	x_3	VG	G	MP
	x_4	VG	G	MP
	x_5	G	MG	MP
C_4	x_1	VG	G	G
	x_2	VG	G	VG
	x_3	VG	MG	MP
	x_4	MG	G	F
	x_5	G	MG	F

Therefore, the decision matrices Y_1-Y_3 of the alternatives x_1-x_5 are reconstructed, respectively, where:

$$Y_1 = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \end{array} \begin{array}{c} \begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & x_5 \end{array} \\ \begin{bmatrix} \frac{VG}{10} & \frac{VG}{10} & \frac{F}{10} & \frac{MG}{10} & \frac{MG}{10} \\ \frac{VG}{10} & \frac{VG}{10} & \frac{G}{10} & \frac{G}{10} & \frac{VG}{10} \\ \frac{VG}{10} & \frac{VG}{10} & \frac{VG}{10} & \frac{VG}{10} & \frac{G}{10} \\ \frac{VG}{10} & \frac{VG}{10} & \frac{VG}{10} & \frac{MG}{10} & \frac{G}{10} \end{bmatrix} \end{array}$$

$$Y_2 = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \end{array} \begin{array}{c} \begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & x_5 \end{array} \\ \begin{bmatrix} \frac{G}{10} & \frac{G}{10} & \frac{G}{10} & \frac{G}{10} & \frac{MG}{10} \\ \frac{G}{10} & \frac{G}{10} & \frac{MG}{10} & \frac{MG}{10} & \frac{G}{10} \\ \frac{G}{10} & \frac{G}{10} & \frac{G}{10} & \frac{G}{10} & \frac{MG}{10} \\ \frac{G}{10} & \frac{G}{10} & \frac{MG}{10} & \frac{G}{10} & \frac{MG}{10} \end{bmatrix} \end{array}$$

Table 3: The linguistic variables

Fuzzy number	Very low	Low	Medium	High	Very high
Expert 1	((-9, -9, -1; 1), (-9, -9, -1; 1))	((-8, -0.5, 3; 1), (-8, -0.5, 3; 1))	((-1, 3.5, 7.3; 1), (-1, 3.5, 7.3; 1))	((2.4, 5.6, 7.8; 1), (2.4, 5.6, 7.8; 1))	((1, 8.4, 8.4; 1), (1, 8.4, 8.4; 1))
Expert 2	-	((-7.8, -.8, 1.5; 1), (-7.8, -7.8, 1.5; 1))	((-5.5, 0, 5.5; 1), (-5.5, 0, 5.5; 1))	((3, 6.1, 7.3; 1), (3, 6.1, 7.3; 1))	((1.5, 9, 9; 1), (1.5, 9, 9; 1))
Expert 3	-	-	((-5.5, -5.5, 1.5; 1), (-5.5, -5.5, 1.5; 1))	((-1, 5, 7; 1), (-1, 5, 7; 1))	((0, 8, 8; 1), (0, 8, 8; 1))

$$Y_3 = \begin{matrix} & x_1 & x_2 & x_3 & x_4 & x_5 \\ C_1 & \frac{G}{10} & \frac{P}{10} & \frac{P}{10} & \frac{P}{10} & \frac{VG}{10} \\ C_2 & \frac{G}{10} & \frac{VG}{10} & \frac{MP}{10} & \frac{MP}{10} & \frac{MP}{10} \\ C_3 & \frac{G}{10} & \frac{VG}{10} & \frac{MP}{10} & \frac{MP}{10} & \frac{MP}{10} \\ C_4 & \frac{G}{10} & \frac{VG}{10} & \frac{MP}{10} & \frac{F}{10} & \frac{F}{10} \end{matrix}$$

Based on Eq. 6, we can get the average decision matrix $\bar{Y}$ shown as follows:

$$\bar{Y} = \begin{matrix} & x_1 & x_2 & x_3 & x_4 & x_5 \\ C_1 & \tilde{\tilde{f}}_{11} & \tilde{\tilde{f}}_{12} & \tilde{\tilde{f}}_{13} & \tilde{\tilde{f}}_{14} & \tilde{\tilde{f}}_{15} \\ C_2 & \tilde{\tilde{f}}_{21} & \tilde{\tilde{f}}_{22} & \tilde{\tilde{f}}_{23} & \tilde{\tilde{f}}_{24} & \tilde{\tilde{f}}_{25} \\ C_3 & \tilde{\tilde{f}}_{31} & \tilde{\tilde{f}}_{32} & \tilde{\tilde{f}}_{33} & \tilde{\tilde{f}}_{34} & \tilde{\tilde{f}}_{35} \\ C_4 & \tilde{\tilde{f}}_{41} & \tilde{\tilde{f}}_{42} & \tilde{\tilde{f}}_{43} & \tilde{\tilde{f}}_{44} & \tilde{\tilde{f}}_{45} \end{matrix}$$

for example for $\tilde{\tilde{f}}_{11}$ is stated as follows:

$$\tilde{\tilde{f}}_{11} = \begin{pmatrix} (0.7667, 0.9333, 0.9333, 1.0; 1; 1), \\ (0.7667, 0.9333, 0.9333, 1.0; 1; 1) \end{pmatrix}$$

$$\tilde{\tilde{f}}_{12} = \begin{pmatrix} (0.5333, 0.6667, 0.6667, 0.7667; 1; 1), \\ (0.5333, 0.6667, 0.6667, 0.7667; 1; 1) \end{pmatrix}$$

$$\tilde{\tilde{f}}_{13} = \begin{pmatrix} (0.3333, 0.5, 0.5, 0.6667; 1; 1), \\ (0.3333, 0.5, 0.5, 0.6667; 1; 1) \end{pmatrix}$$

$$\tilde{\tilde{f}}_{14} = \begin{pmatrix} (0.4, 0.5667, 0.5667, 0.7333; 1; 1), \\ (0.4, 0.5667, 0.5667, 0.7333; 1; 1) \end{pmatrix}$$

$$\tilde{\tilde{f}}_{15} = \begin{pmatrix} (0.6333, 0.8, 0.8, 0.9333; 1; 1), \\ (0.6333, 0.8, 0.8, 0.9333; 1; 1) \end{pmatrix}$$

Step 2: Appoint the linguistic variables for the Likert Rating scale. The degree of each Likert Rating scale is appointed using the linguistic variables in interval type-2 fuzzy sets concept. Table 3 shows the linguistic variables from very low to very high obtained from the three experts. Based on Table 3 this study finds as shown as follows:

Table 4: Degree of importance of four criteria (scale ranges from 1-5)

Criteria	Expert 1	Expert 2	Expert 3
C_1	5	5	5
C_2	4	4	4
C_3	3	3	3
C_4	2	2	3

Table 5: Linguistic of triangular interval type-2 fuzzy sets

Criteria	Expert 1	Expert 2	Expert 3
C_1	((1, 8.4, 8.4; 1), (1, 8.4, 8.4; 1))	((1.5, 9, 9; 1), (1.5, 9, 9; 1))	((0, 8, 8; 1), (0, 8, 8; 1))
C_2	((2.4, 5.6, 7.8; 1), (2.4, 5.6, 7.8; 1))	((3.6, 1, 7.3; 1), (3.6, 1, 7.3; 1))	((-1, 5, -7; 1), (-1, 5, -7; 1))
C_3	((-1, 3.5, 7.3; 1), (-1, 3.5, 7.3; 1))	((-5.5, 0, 5.5; 1), (-5.5, 0, 5.5; 1))	((-5.5, -5.5, 1.5; 1), (-5.5, -5.5, 1.5; 1))
C_4	(-8, -0.5, 3; 1), (-8, -0.5, 3; 1)	((-7.8, -7.8, 1.5; 1), (-7.8, -7.8, 1.5; 1))	((-1, 5, 7; 1), (-1, 5, 7; 1))

$$\tilde{\tilde{w}}_j = \left[\left(\underline{\delta}^L_{ijc}, \underline{\delta}^M_{ijc}, \underline{\delta}^U_{ijc}; 1\right), \left(\bar{\delta}^L_{ijc}, \bar{\delta}^M_{ijc}, \bar{\delta}^U_{ijc}; 1\right)\right]$$

Step 3: Determine the degree of the importance for each criteria/indices. This study assesses highest causes that lead to road accidents by instructing three experts to indicate the degree of importance of four criteria on a Likert Rating scale ranging from 1-5 (from very low to very high) as shown in Table 4.

Step 4: Calculate the step parameters (δ_{ij}) to generate the weight of Interval Type-2 Fuzzy Rasch (IT2FR) Model. The concept of triangular interval type-2 fuzzy sets in Fuzzy Rasch Model weight is adapted. Table 5 shows the linguistic of triangular type-2 fuzzy sets for all the criteria.

Step 5 (calculate the arithmetic average): An arithmetic average is used to integrate the new weight of each expert.

Step 6 (weight): The weight of attributes is calculated by using the weight formula. Thus, the results for weight of interval type-2 Rasch Model are shown in Table 6.

Step 7: Based on Table 6, we can get the weighting matrices W_1, W_2 and W_3, respectively where:

$$W_1 = \begin{matrix} C_1 & C_2 & C_3 & C_4 \\ [VH & H & M & L] \end{matrix}$$

Table 6: Weight of interval type-2 Rasch Model

Weight	C_1	C_2	C_3	C_4
$\overline{W}$	((0.8333, 8.4667, 8.4667; 1), (0.8333, 8.4667, 8.4667; 1))	((1.6667, 3.8667, 7.3667; 1), (1.6667, 3.8667, 7.3667; 1))	((-4, -0.6667, 4.7667; 1), (-4, -0.6667, 4.7667; 1))	((-5.7667, -3.3, 3.83333; 1), (-5.7667, -3.3, 3.83333; 1))

$$W_2 = \begin{matrix} C_1 & C_2 & C_3 & C_4 \\ [VH & H & M & L] \end{matrix}$$

$$W_3 = \begin{matrix} C_1 & C_2 & C_3 & C_4 \\ [VH & H & M & L] \end{matrix}$$

$$\overline{W} = \begin{matrix} C_1 & C_2 & C_3 & C_4 \\ [\tilde{\tilde{w}}_1 & \tilde{\tilde{w}}_2 & \tilde{\tilde{w}}_3 & \tilde{\tilde{w}}_4] \end{matrix}$$

where for $\tilde{\tilde{w}}_1$ is describe as follows:

$$\tilde{\tilde{w}}_1 = \begin{pmatrix} (0.8333, 8.4667, 8.4667; 1), \\ (0.8333, 8.4667, 8.4667; 1) \end{pmatrix}$$

$$\tilde{\tilde{w}}_2 = \begin{pmatrix} (1.6667, 3.8667, 7.3667; 1), \\ (1.6667, 3.8667, 7.3667; 1) \end{pmatrix}$$

$$\tilde{\tilde{w}}_3 = \begin{pmatrix} (-4, -0.6667, 4.7667; 1), \\ (-4, -0.6667, 4.7667; 1) \end{pmatrix}$$

$$\tilde{\tilde{w}}_4 = \begin{pmatrix} (-5.7667, -3.3, 3.83333; 1), \\ (-5.7667, -3.3, 3.83333; 1) \end{pmatrix}$$

Step 8: Based on Eq. 11, we can get the weighted decision matrix $\overline{Y}_w$ shown as follows:

$$\overline{Y}_w = \begin{matrix} & x_1 & x_2 & x_3 & x_4 & x_5 \\ C_1 & \tilde{\tilde{v}}_{11} & \tilde{\tilde{v}}_{12} & \tilde{\tilde{v}}_{13} & \tilde{\tilde{v}}_{14} & \tilde{\tilde{v}}_{15} \\ C_2 & \tilde{\tilde{v}}_{21} & \tilde{\tilde{v}}_{22} & \tilde{\tilde{v}}_{23} & \tilde{\tilde{v}}_{24} & \tilde{\tilde{v}}_{25} \\ C_3 & \tilde{\tilde{v}}_{31} & \tilde{\tilde{v}}_{32} & \tilde{\tilde{v}}_{33} & \tilde{\tilde{v}}_{34} & \tilde{\tilde{v}}_{35} \\ C_4 & \tilde{\tilde{v}}_{41} & \tilde{\tilde{v}}_{42} & \tilde{\tilde{v}}_{43} & \tilde{\tilde{v}}_{44} & \tilde{\tilde{v}}_{45} \end{matrix}$$

Where:

$$\tilde{\tilde{v}}_{11} = \begin{pmatrix} (0.69, 0.9333, 1.0; 1), \\ (0.69, 0.9333, 1.0; 1) \end{pmatrix}$$

$$\tilde{\tilde{v}}_{12} = \begin{pmatrix} (0.48, 0.6667, 0.7667; 1), \\ (0.48, 0.6667, 0.7667; 1) \end{pmatrix}$$

$$\tilde{\tilde{v}}_{13} = \begin{pmatrix} (0.3, 0.5, 0.6667; 1), \\ (0.3, 0.5, 0.6667; 1) \end{pmatrix}$$

$$\tilde{\tilde{v}}_{14} = \begin{pmatrix} (0.36, 0.5667, 0.7333; 1), \\ (0.36, 0.5667, 0.7333; 1) \end{pmatrix}$$

$$\tilde{\tilde{v}}_{15} = \begin{pmatrix} (0.3, 0.8, 0.9333; 1), \\ (0.3, 0.8, 0.9333; 1) \end{pmatrix}$$

$$\tilde{\tilde{v}}_{21} = \begin{pmatrix} (0.5367, 0.84, 1.0; 1), \\ (0.5367, 0.84, 1.0; 1) \end{pmatrix}$$

$$\tilde{\tilde{v}}_{22} = \begin{pmatrix} (0.5833, 0.87, 1.0; 1), \\ (0.5833, 0.87, 1.0; 1) \end{pmatrix}$$

$$\tilde{\tilde{v}}_{23} = \begin{pmatrix} (0.3033, 0.57, 0.8; 1), \\ (0.3033, 0.57, 0.8; 1) \end{pmatrix}$$

$$\tilde{\tilde{v}}_{24} = \begin{pmatrix} (0.3033, 0.57, 0.8; 1), \\ (0.3033, 0.57, 0.8; 1) \end{pmatrix}$$

$$\tilde{\tilde{v}}_{25} = \begin{pmatrix} (0.3967, 0.66, 0.8333; 1), \\ (0.3967, 0.66, 0.8333; 1) \end{pmatrix}$$

$$\tilde{\tilde{v}}_{31} = \begin{pmatrix} (0.23, 0.4667, 0.7; 1), \\ (0.23, 0.4667, 0.7; 1) \end{pmatrix}$$

$$\tilde{\tilde{v}}_{32} = \begin{pmatrix} (0.25, 0.4834, 0.7; 1), \\ (0.25, 0.4834, 0.7; 1) \end{pmatrix}$$

$$\tilde{\tilde{v}}_{33} = \begin{pmatrix} (0.17, 0.3667, 0.5833; 1), \\ (0.17, 0.3667, 0.5833; 1) \end{pmatrix}$$

$$\tilde{\tilde{v}}_{34} = \begin{pmatrix} (0.17, 0.3667, 0.5833; 1), \\ (0.17, 0.3667, 0.5833; 1) \end{pmatrix}$$

$$\tilde{\tilde{v}}_{35} = \begin{pmatrix} (0.13, 0.3167, 0.4433; 1), \\ (0.13, 0.3167, 0.4433; 1) \end{pmatrix}$$

$$\tilde{\tilde{v}}_{41} = \begin{pmatrix} (0.0, 0.0933, 0.3; 1), \\ (0.0, 0.0933, 0.3; 1) \end{pmatrix}$$

$$\tilde{\tilde{v}}_{42} = \begin{pmatrix} (0.0, 0.0967, 0.3; 1), \\ (0.0, 0.0967, 0.3; 1) \end{pmatrix}$$

$$\tilde{\tilde{v}}_{43} = \begin{pmatrix} (0.0, 0.0667, 0.24; 1), \\ (0.0, 0.0667, 0.24; 1) \end{pmatrix}$$

$$\tilde{\tilde{v}}_{44} = \begin{pmatrix} (0.0, 0.07, 0.26; 1), \\ (0.0, 0.07, 0.26; 1) \end{pmatrix}$$

$$\tilde{\tilde{v}}_{45} = \begin{pmatrix} (0.0, 0.07, 0.26; 1), \\ (0.0, 0.07, 0.26; 1) \end{pmatrix}$$

Step 9: Based on Eq. 13, the ranking values $\text{Rank}(\tilde{\tilde{v}}_{ij})$ of the interval type-2 fuzzy set $\tilde{\tilde{v}}_{ij}$ can be calculated where, $1 \le i \le 5$ and $1 \le j \le 3$ shown as follows:

$$\text{Rank}\left(\tilde{\tilde{v}}_{11}\right) = M_1\left(\tilde{A}_i^U\right) + M_1\left(\tilde{A}_i^L\right) + M_2\left(\tilde{A}_i^U\right) + M_2\left(\tilde{A}_i^L\right) - \frac{1}{3}\begin{pmatrix} S_1\left(\tilde{A}_i^U\right) + S_1\left(\tilde{A}_i^L\right) + S_2\left(\tilde{A}_i^U\right) \\ +S_2\left(\tilde{A}_i^L\right) + S_3\left(\tilde{A}_i^U\right) + S_3\left(\tilde{A}_i^L\right) \end{pmatrix} + H_1\left(\tilde{A}_i^U\right) + H_1\left(\tilde{A}_i^L\right)$$

$$\text{Rank}\left(\tilde{\tilde{v}}_{11}\right) = 3.9705 + 3.9705 + 8.1844 + 8.1844 - \frac{1}{3}\begin{pmatrix} 3.1352 + 3.1352 + 1.4046 \\ +1.4046 + 1.7306 + 1.7306 \end{pmatrix} + 1 + 1 = 22.1295$$

In the same way, we can get:

$$\begin{aligned}
&\text{Rank}\left(\tilde{\tilde{v}}_{12}\right) = 17.3393, \text{Rank}\left(\tilde{\tilde{v}}_{13}\right) = 13.9971,\\
&\text{Rank}\left(\tilde{\tilde{v}}_{14}\right) = 15.4772, \text{Rank}\left(\tilde{\tilde{v}}_{15}\right) = 20.4817,\\
&\text{Rank}\left(\tilde{\tilde{v}}_{21}\right) = 15.338, \text{Rank}\left(\tilde{\tilde{v}}_{22}\right) = 15.7663,\\
&\text{Rank}\left(\tilde{\tilde{v}}_{23}\right) = 11.3024, \text{Rank}\left(\tilde{\tilde{v}}_{24}\right) = 11.3024,\\
&\text{Rank}\left(\tilde{\tilde{v}}_{25}\right) = 12.5738, \text{Rank}\left(\tilde{\tilde{v}}_{31}\right) = -0.9373,\\
&\text{Rank}\left(\tilde{\tilde{v}}_{32}\right) = -1.3225, \text{Rank}\left(\tilde{\tilde{v}}_{33}\right) = -0.0201,\\
&\text{Rank}\left(\tilde{\tilde{v}}_{34}\right) = -0.0201, \text{Rank}\left(\tilde{\tilde{v}}_{35}\right) = 0.7256,\\
&\text{Rank}\left(\tilde{\tilde{v}}_{41}\right) = -8.6399, \text{Rank}\left(\tilde{\tilde{v}}_{42}\right) = -10.4393,\\
&\text{Rank}\left(\tilde{\tilde{v}}_{43}\right) = -5.0953, \text{Rank}\left(\tilde{\tilde{v}}_{44}\right) = -5.2189,\\
&\text{Rank}\left(\tilde{\tilde{v}}_{45}\right) = -5.2189
\end{aligned}$$

Based on Eq. 12, we can construct the ranking weighted decision matrix $\bar{Y}_w^*$:

$$\bar{Y}_w^* = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \end{array}\begin{array}{c} \begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & x_5 \end{array} \\ \begin{bmatrix} \text{Rank}(\tilde{\tilde{v}}_{11}) & \text{Rank}(\tilde{\tilde{v}}_{12}) & \text{Rank}(\tilde{\tilde{v}}_{13}) & \text{Rank}(\tilde{\tilde{v}}_{14}) & \text{Rank}(\tilde{\tilde{v}}_{15}) \\ \text{Rank}(\tilde{\tilde{v}}_{21}) & \text{Rank}(\tilde{\tilde{v}}_{22}) & \text{Rank}(\tilde{\tilde{v}}_{23}) & \text{Rank}(\tilde{\tilde{v}}_{24}) & \text{Rank}(\tilde{\tilde{v}}_{25}) \\ \text{Rank}(\tilde{\tilde{v}}_{31}) & \text{Rank}(\tilde{\tilde{v}}_{32}) & \text{Rank}(\tilde{\tilde{v}}_{33}) & \text{Rank}(\tilde{\tilde{v}}_{34}) & \text{Rank}(\tilde{\tilde{v}}_{35}) \\ \text{Rank}(\tilde{\tilde{v}}_{41}) & \text{Rank}(\tilde{\tilde{v}}_{42}) & \text{Rank}(\tilde{\tilde{v}}_{43}) & \text{Rank}(\tilde{\tilde{v}}_{44}) & \text{Rank}(\tilde{\tilde{v}}_{45}) \end{bmatrix} \end{array}$$

$$\bar{Y}_w^* = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \end{array}\begin{array}{c} \begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & x_5 \end{array} \\ \begin{bmatrix} 22.1295 & 17.3393 & 13.9971 & 15.4772 & 20.4817 \\ 15.338 & 15.7663 & 11.3024 & 11.3024 & 12.5738 \\ -0.9373 & -1.3225 & -0.0201 & -0.0201 & 0.7256 \\ -8.6399 & -10.4393 & -5.0953 & -5.2189 & -5.2189 \end{bmatrix} \end{array}$$

Step 10: Based on Eq. 14 and 15, we can get the positive ideal solution x^+ and the negative-ideal solution x^-, respectively.

Step 11: Based on Eq. 16 and 17, we can calculate the distance $d^+(x_j)$ between each alternative x^+ and the ideal solution and we can calculate the distance $d^-(x_j)$ between each alternative x_j and the negative ideal solution x^-, respectively where $1 \le j \le 5$ as shown as follows:

$$x^- = \left(v_1^-, v_2^-, \ldots, v_m^-\right) = \begin{pmatrix} \min\left(\text{Rank}(\tilde{\tilde{v}}_{11}), \text{Rank}(\tilde{\tilde{v}}_{12}), \text{Rank}(\tilde{\tilde{v}}_{13}), \text{Rank}(\tilde{\tilde{v}}_{14}), \text{Rank}(\tilde{\tilde{v}}_{15})\right), \\ \min\left(\text{Rank}(\tilde{\tilde{v}}_{21}), \text{Rank}(\tilde{\tilde{v}}_{22}), \text{Rank}(\tilde{\tilde{v}}_{23}), \text{Rank}(\tilde{\tilde{v}}_{24}), \text{Rank}(\tilde{\tilde{v}}_{25})\right), \\ \min\left(\text{Rank}(\tilde{\tilde{v}}_{31}), \text{Rank}(\tilde{\tilde{v}}_{32}), \text{Rank}(\tilde{\tilde{v}}_{33}), \text{Rank}(\tilde{\tilde{v}}_{34}), \text{Rank}(\tilde{\tilde{v}}_{35})\right), \\ \min\left(\text{Rank}(\tilde{\tilde{v}}_{41}), \text{Rank}(\tilde{\tilde{v}}_{42}), \text{Rank}(\tilde{\tilde{v}}_{43}), \text{Rank}(\tilde{\tilde{v}}_{44}), \text{Rank}(\tilde{\tilde{v}}_{45})\right) \end{pmatrix}$$
$$= \left(\text{Rank}(\tilde{\tilde{v}}_{13}), \text{Rank}(\tilde{\tilde{v}}_{23}), \text{Rank}(\tilde{\tilde{v}}_{35}), \text{Rank}(\tilde{\tilde{v}}_{43})\right) = (13.9971, 11.3024, -1.3225, -10.4393)$$

$$x^+ = \left(v_1^+, v_2^+, \ldots, v_m^+\right) = \begin{pmatrix} \max\left(\text{Rank}(\tilde{\tilde{v}}_{11}), \text{Rank}(\tilde{\tilde{v}}_{12}), \text{Rank}(\tilde{\tilde{v}}_{13}), \text{Rank}(\tilde{\tilde{v}}_{14}), \text{Rank}(\tilde{\tilde{v}}_{15})\right), \\ \max\left(\text{Rank}(\tilde{\tilde{v}}_{21}), \text{Rank}(\tilde{\tilde{v}}_{22}), \text{Rank}(\tilde{\tilde{v}}_{23}), \text{Rank}(\tilde{\tilde{v}}_{24}), \text{Rank}(\tilde{\tilde{v}}_{25})\right), \\ \max\left(\text{Rank}(\tilde{\tilde{v}}_{31}), \text{Rank}(\tilde{\tilde{v}}_{32}), \text{Rank}(\tilde{\tilde{v}}_{33}), \text{Rank}(\tilde{\tilde{v}}_{34}), \text{Rank}(\tilde{\tilde{v}}_{35})\right), \\ \max\left(\text{Rank}(\tilde{\tilde{v}}_{41}), \text{Rank}(\tilde{\tilde{v}}_{42}), \text{Rank}(\tilde{\tilde{v}}_{43}), \text{Rank}(\tilde{\tilde{v}}_{44}), \text{Rank}(\tilde{\tilde{v}}_{45})\right) \end{pmatrix}$$
$$= \left(\text{Rank}(\tilde{\tilde{v}}_{11}), \text{Rank}(\tilde{\tilde{v}}_{22}), \text{Rank}(\tilde{\tilde{v}}_{32}), \text{Rank}(\tilde{\tilde{v}}_{42})\right) = (22.1295, 15.7663, 0.7256, -5.0953)$$

$$\begin{aligned}
d^+(x_1) &= \sqrt{\textstyle\sum_{i=1}^4 \left(\text{Rank}\left(\tilde{\tilde{v}}_{i1}\right) - v_i^+\right)^2} = 3.9386,\\
d^-(x_1) &= \sqrt{\textstyle\sum_{i=1}^4 \left(\text{Rank}\left(\tilde{\tilde{v}}_{i1}\right) - v_i^-\right)^2} = 9.2633,\\
d^+(x_1) &= \sqrt{\textstyle\sum_{i=1}^4 \left(\text{Rank}\left(\tilde{\tilde{v}}_{i1}\right) - v_i^+\right)^2} = 7.4632,\\
d^-(x_1) &= \sqrt{\textstyle\sum_{i=1}^4 \left(\text{Rank}\left(\tilde{\tilde{v}}_{i1}\right) - v_i^-\right)^2} = 5.5764,\\
d^+(x_1) &= \sqrt{\textstyle\sum_{i=1}^4 \left(\text{Rank}\left(\tilde{\tilde{v}}_{i1}\right) - v_i^+\right)^2} = 9.3069,\\
d^-(x_1) &= \sqrt{\textstyle\sum_{i=1}^4 \left(\text{Rank}\left(\tilde{\tilde{v}}_{i1}\right) - v_i^-\right)^2} = 5.5004,\\
d^+(x_1) &= \sqrt{\textstyle\sum_{i=1}^4 \left(\text{Rank}\left(\tilde{\tilde{v}}_{i1}\right) - v_i^+\right)^2} = 8.0468,\\
d^-(x_1) &= \sqrt{\textstyle\sum_{i=1}^4 \left(\text{Rank}\left(\tilde{\tilde{v}}_{i1}\right) - v_i^-\right)^2} = 5.5803,\\
d^+(x_1) &= \sqrt{\textstyle\sum_{i=1}^4 \left(\text{Rank}\left(\tilde{\tilde{v}}_{i1}\right) - v_i^+\right)^2} = 3.5948,\\
d^+(x_1) &= \sqrt{\textstyle\sum_{i=1}^4 \left(\text{Rank}\left(\tilde{\tilde{v}}_{i1}\right) - v_i^+\right)^2} = 8.6669
\end{aligned}$$

Step 12: Based on Eq. 18, we can calculate the relative degree of closeness $C(x_j)$ of each alternative x_j with respect to the positive ideal solution x^+ where $1 \le j \le 5$ shown as follows:

$$C(x_1) = \frac{d^-(x_1)}{d^+(x_1) + d^-(x_1)} = 0.7017$$

$$C(x_2) = \frac{d^-(x_2)}{d^+(x_2) + d^-(x_2)} = 0.4277$$

$$C(x_3) = \frac{d^-(x_3)}{d^+(x_3) + d^-(x_3)} = 0.3715$$

$$C(x_4) = \frac{d^-(x_4)}{d^+(x_4) + d^-(x_4)} = 0.4095$$

$$C(x_5) = \frac{d^-(x_5)}{d^+(x_5) + d^-(x_5)} = 0.7068$$

Step 13: Because $C(x_5) > C(x_1) > C(x_2) > C(x_4) > C(x_3)$, the preferred order of the alternatives x_1-x_5 is: $x_5 > x_1 > x_2 > x_4 > x_3$. That is the best alternative among x_1-x_5 is x_5.

CONCLUSION

This study successfully combines interval type-2 fuzzy TOPSIS with interval type-2 fuzzy rasch weight to rank the causes of road accidents. This study conducted five alternatives to be evaluated with four criteria using three different expert opinions. Expert opinions were taken from three different agencies. Determining the weight of the evaluation criteria with interval type-2 fuzzy rasch weight appears to be feasible tasks. Doing so can rectify the inaccuracy of the real fuzzy numbers assigned by the expert opinions for the specific criteria. Additionally, this weight methodology represents not only an innovative attempt to evaluate causes of road accidents but also a practical application of the MCDM Method to study the causes of road accidents. This method can be dealt with both quantitative and qualitative assessment of multiple causes of accident and multiple motor vehicles. Last results showed that the first rank went to road environment at 0.7068 while the last rank went to driver's health at 0.3715. The evidence from this study suggests that the five factors need to be thoroughly investigated in order to reduce the accidents. Therefore, this method provides us with a useful way to handle the fuzzy multiple attribute group decision-making problems in a more flexible and more intelligent manner due to the fact that it uses interval type-2 fuzzy sets rather than traditional type-1 fuzzy sets to represent the evaluating values and the weights of attributes. Besides, this method believes that the model is not only feasible but can also rectify the inaccuracy of the fuzzy numbers assigned by the individual experts for the specific criteria. On the other hand, this model is more suitable to represent uncertainties because it is involve end-users into the whole weighting process. For further research, a decision based on linguistic judgment is made after taking into account the multiple factors. The evidence from this study also suggests that to consider more criteria. Besides, it seems that adding sub-criteria of fuzzy TOPSIS will have the best closeness coefficients.

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