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Global Chaos Synchronization of Hyperchaotic Pang and Hyperchaotic Wang Systems via Adaptive Control

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Abstract: This study investigates the global chaos synchronization of identical hyperchaotic Wang Systems, identical hyperchaotic Pang Systems and non-identical hyperchaotic Wang and hyperchaotic Pang Systems via Adaptive Control Method. Hyperchaotic Pang System and hyperchaotic Wang System are recently discovered hyperchaotic systems. Adaptive Control Method is deployed in this study for the general case when the system parameters are unknown. Sufficient conditions for global chaos synchronization of identical hyperchaotic Pang Systems, identical hyperchaotic Wang Systems and non-identical hyperchaotic Pang and Wang Systems are derived via Adaptive Control Theory and Lyapunov Stability Theory. Since, the Lyapunov exponents are not required for these calculations, the Adaptive Control Method is very convenient for the global chaos synchronization of the hyperchaotic systems discussed in this study. Numerical simulations are presented to validate and demonstrate the effectiveness of the proposed synchronization schemes.

Key words: Adaptive control, hyperchaos, synchronization, hyperchaotic Pang System, hyperchaotic Wang System

INTRODUCTION

Chaotic Systems are dynamical systems that are highly sensitive to initial conditions. The sensitive nature of chaotic systems is commonly called as the butterfly effect (Alligood *et al.*, 1997). Since, chaos phenomenon in weather models was first observed by Lorenz (1963) a large number of chaos phenomena and chaos behaviour have been discovered in physical, social, economical, biological and electrical systems.

A Hyperchaotic System is usually characterized as a chaotic system with more than one positive Lyapunov exponent implying that the dynamics expand in more than one direction giving rise to thicker and more complex chaotic dynamics. The first Hyperchaotic System was discovered by Rossler (1979). Chaos is an interesting nonlinear phenomenon and has been extensively studied in the last two decades (Alligood *et al.*, 1997; Lorenz, 1963; Rossler, 1979; Pecora and Carroll, 1990; Fabiny and Wiesenfield, 1991; Niu *et al.*, 2002; Blasius *et al.*, 1999; Kocarev and Parlitz, 1995; Boccaletti *et al.*, 1997; Tao, 1999; Ott *et al.*, 1990; Ho and Hung, 2002; Huang *et al.*, 2004; Chen, 2005; Sundarapandian and Suresh, 2011; Sundarapandian and Karthikeyan, 2011; Lu *et al.*, 2004; Sundarapandian, 2011a-h; Zhao and Lu, 2008; Park and Kwon, 2003; Tan *et al.*, 2003; Vincent, 2008; Slotine and Sastry, 1983; Utkin, 1993; Sundarapandian and Sivaperumal, 2011; Pang and Liu, 2011; Wang and Liu, 2006; Hahn, 1967).

Synchronization of chaotic systems is a phenomenon which may occur when two or more chaotic oscillators are coupled or when a chaotic oscillator drives another chaotic oscillator. Because of the butterfly effect which causes the exponential divergence of the trajectories of two identical Chaotic Systems started with nearly the same initial conditions, synchronizing two Chaotic Systems is seemingly a very challenging problem.

Pecora and Carroll (1990) deployed control techniques to synchronize two identical Chaotic Systems and showed that it was possible for some chaotic systems to be completely synchronized. From then on, chaos synchronization has been widely explored in a variety of fields including physical systems (Fabiny and Wiesenfield, 1991), chemical systems (Niu *et al.*, 2002), ecological systems (Blasius *et al.*, 1999), secure communications (Kocarev and Parlitz, 1995; Boccaletti *et al.*, 1997; Tao, 1999), etc.

In most of the chaos synchronization approaches, the master-slave or drive-response formalism is used. If a particular chaotic system is called the Master or

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Drive System and another Chaotic System is called the Slave or Response System then the idea of the synchronization is to use the output of the Master System to control the Slave System so that the output of the Slave System tracks the output of the Master System asymptotically.

Since, the seminal research by Pecora and Carroll (1990), a variety of impressive approaches have been proposed for the synchronization of chaotic systems such as the OGY Method (Ott *et al*., 1990), Active Control Method (Ho and Hung, 2002; Huang *et al*., 2004; Chen, 2005; Sundarapandian and Suresh, 2011; Sundarapandian and Karthikeyan, 2011), Adaptive Control Method (Lu *et al*., 2004; Sundarapandian, 2011f, e, b, a, g) Sampled-Data Feedback Synchronization Method (Zhao and Lu, 2008), Time-Delay Feedback Method (Park and Kwon, 2003), Backstepping Method (Tan *et al*., 2003; Vincent, 2008), Sliding Mode Control Method (Slotine and Sastry, 1983; Utkin, 1993; Sundarapandian and Sivaperumal, 2011; Sundarapandian, 2011d, c, h), etc.

In this study, researchers investigate the global chaos synchronization of uncertain hyperchaotic systems, viz., identical hyperchaotic Pang System (Pang and Liu, 2011), identical hyperchaotic Wang Systems (Wang and Liu, 2006) and Non-identical hyperchaotic Pang and hyperchaotic Wang Systems. Researchers consider the general case when the parameters of the Hyperchaotic Systems are unknown.

SYSTEMS DESCRIPTION

The hyperchaotic Pang System (Pang and Liu, 2011) is described by the dynamics:

$$\begin{aligned} \dot{x}_1 &= a(x_2 - x_1) \\ \dot{x}_2 &= cx_2 - x_1x_3 + x_4 \\ \dot{x}_3 &= -bx_3 + x_1x_2 \\ \dot{x}_4 &= -d(x_1 + x_2) \end{aligned} \qquad (1)$$

Where:

x_1-x_4 = The state variables

a-d = Positive, constant parameters of the system

The 4D system Eq. 1 is hyperchaotic when the parameter values are taken as:

$$a = 36,\ b = 3,\ c = 20 \text{ and } d = 2$$

The state orbits of the hyperchaotic Pang Chaotic System Eq. 1 are shown in Fig. 1. The hyperchaotic Wang System (Wang and Liu, 2006) is described by:

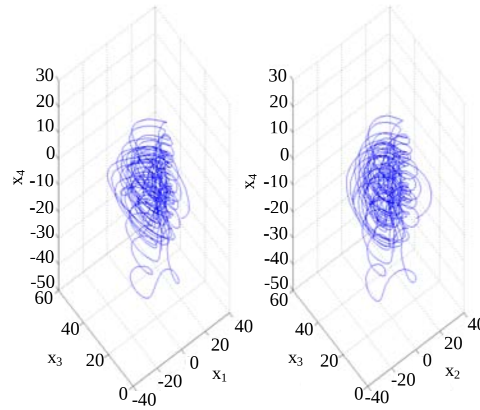

Fig. 1: State orbits of the hyperchaotic Pang Chaotic System

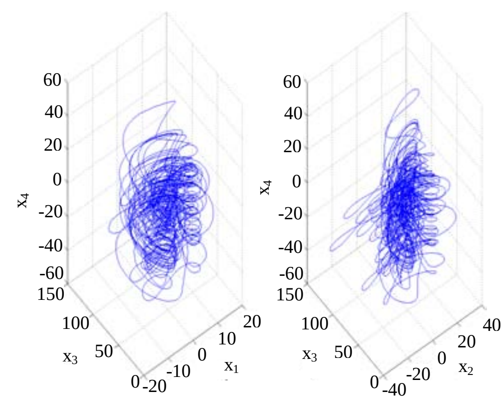

Fig. 2: State orbits of the Hyperchaotic Wang System

$$\begin{aligned} \dot{x}_1 &= \alpha(x_2 - x_1) \\ \dot{x}_2 &= \beta x_1 - x_1x_3 + x_4 \\ \dot{x}_3 &= -\gamma x_3 + \varepsilon x_1^2 \\ \dot{x}_4 &= -\delta x_1 \end{aligned} \qquad (2)$$

where, x_1-x_4 are the state variables and α, β, γ, δ and ε are positive constant parameters of the system. The 4D system Eq. 2 is hyperchaotic when the parameter values are taken as:

$$\alpha = 10,\ \beta = 40,\ \gamma = 2.5,\ \delta = 10.6 \text{ and } \varepsilon = 4$$

The state orbits of the hyperchaotic Wang Chaotic System Eq. 2 are shown in Fig. 2.

ADAPTIVE SYNCHRONIZATION OF IDENTICAL HYPERCHAOTIC PANG SYSTEMS

Theoretical results: In this study, researchers deploy adaptive control to achieve new results for the global chaos synchronization of identical hyperchaotic Pang

Systems (Pang and Liu, 2011) where the parameters of the Master and Slave Systems are unknown. As the Master System, we consider the hyperchaotic Pang dynamics described by:

$$\begin{aligned} \dot{x}_1 &= a(x_2 - x_1) \\ \dot{x}_2 &= cx_2 - x_1x_3 + x_4 \\ \dot{x}_3 &= -bx_3 + x_1x_2 \\ \dot{x}_4 &= -d(x_1 + x_2) \end{aligned} \tag{3}$$

Where:
x_1-x_4 = The state variables
a-d = Unknown, real and constant parameters of the system

As the Slave System, researchers consider the controlled hyperchaotic Pang dynamics described by:

$$\begin{aligned} \dot{y}_1 &= a(y_2 - y_1) + u_1 \\ \dot{y}_2 &= cy_2 - y_1y_3 + y_4 + u_2 \\ \dot{y}_3 &= -by_3 + y_1y_2 + u_3 \\ \dot{y}_4 &= -d(y_1 + y_2) + u_4 \end{aligned} \tag{4}$$

Where:
y_1-y_4 = The state variables
u_1-u_4 = The nonlinear controllers to be designed

The chaos synchronization error is defined by:

$$e_i = y_i - x_i, \ (i = 1\text{-}4) \tag{5}$$

The error dynamics is easily obtained as:

$$\begin{aligned} \dot{e}_1 &= a(e_2 - e_1) + u_1 \\ \dot{e}_2 &= ce_2 + e_4 - y_1y_3 + x_1x_3 + u_2 \\ \dot{e}_3 &= -be_3 + y_1y_2 - x_1x_2 + u_3 \\ \dot{e}_4 &= -d(e_1 + e_2) + u_4 \end{aligned} \tag{6}$$

Let us now define the adaptive control functions:

$$\begin{aligned} u_1(t) &= -\hat{a}(e_2 - e_1) - k_1e_1 \\ u_2(t) &= -\hat{c}e_2 - e_4 + y_1y_3 - x_1x_3 - k_2e_2 \\ u_3(t) &= \hat{b}e_3 - y_1y_2 + x_1x_2 - k_3e_3 \\ u_4(t) &= \hat{d}(e_1 + e_2) - k_4e_4 \end{aligned} \tag{7}$$

where, $\hat{a}-\hat{d}$ are estimates of a-d, respectively and k_i (i = 1-4) are positive constants. Substituting Eq. 7 into Eq. 6, the error dynamics simplifies to:

$$\begin{aligned} \dot{e}_1 &= (a - \hat{a})(e_2 - e_1) - k_1e_1 \\ \dot{e}_2 &= (c - \hat{c})e_2 - k_2e_2 \\ \dot{e}_3 &= -(b - \hat{b})e_3 - k_3e_3 \\ \dot{e}_4 &= -(d - \hat{d})(e_1 + e_2) - k_4e_4 \end{aligned} \tag{8}$$

Let us now define the parameter estimation errors as:

$$e_a = a - \hat{a}, \ e_b = b - \hat{b}, \ e_c = c - \hat{c} \text{ and } e_d = d - \hat{d} \tag{9}$$

Substituting Eq. 9 into Eq. 8, researchers obtain the error dynamics as:

$$\begin{aligned} \dot{e}_1 &= e_a(e_2 - e_1) - k_1e_1 \\ \dot{e}_2 &= e_ce_2 - k_2e_2 \\ \dot{e}_3 &= -e_be_3 - k_3e_3 \\ \dot{e}_4 &= -e_d(e_1 + e_2) - k_4e_4 \end{aligned} \tag{10}$$

For the derivation of the update law for adjusting the estimates of the parameters, the Lyapunov approach is used. We consider the quadratic Lyapunov function defined by:

$$\begin{aligned} &V(e_1, e_2, e_3, e_4, e_a, e_b, e_c, e_d) \\ &= \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_4^2 + e_a^2 + e_b^2 + e_c^2 + e_d^2\right) \end{aligned} \tag{11}$$

which is a positive definite function on R^8. We also note that:

$$\dot{e}_a = -\dot{\hat{a}}, \ \dot{e}_b = -\dot{\hat{b}}, \ \dot{e}_c = -\dot{\hat{c}} \text{ and } \dot{e}_d = -\dot{\hat{d}} \tag{12}$$

Differentiating Eq. 11 along the trajectories of Eq. 10 and using Eq. 12, we obtain:

$$\begin{aligned} \dot{V} = &-k_1e_1^2 - k_2e_2^2 - k_3e_3^2 - k_4e_4^2 + \\ &e_a\left[e_1(e_2 - e_1) - \dot{\hat{a}}\right] + e_b\left[-e_3^2 - \dot{\hat{b}}\right] + \\ &e_c\left[e_2^2 - \dot{\hat{c}}\right] + e_d\left[-e_4(e_1 + e_2) - \dot{\hat{d}}\right] \end{aligned} \tag{13}$$

In view of Eq. 13 the estimated parameters are updated by the following law:

$$\begin{aligned} \dot{\hat{a}} &= e_1(e_2 - e_1) + k_5e_a \\ \dot{\hat{b}} &= -e_3^2 + k_6e_b \\ \dot{\hat{c}} &= e_2^2 + k_7e_c \\ \dot{\hat{d}} &= -e_4(e_1 + e_2) + k_8e_d \end{aligned} \tag{14}$$

where, k_4-k_7 are positive constants. Substituting Eq. 14 into Eq. 13, we obtain:

$$\begin{aligned} \dot{V} = &-k_1e_1^2 - k_2e_2^2 - k_3e_3^2 - k_4e_4^2 - \\ &k_5e_a^2 - k_6e_b^2 - k_7e_c^2 - k_8e_d^2 \end{aligned} \tag{15}$$

which is a negative definite function on R^8. Thus, by Lyapunov Stability Theory (Hahn, 1967), it is immediate that the synchronization error e_i (i = 1-4) and the parameter estimation error e_a-e_d decay to zero exponentially with time. Hence, we have proved the following result.

Theorem 1: The identical hyperchaotic Pang Systems Eq. 3 and 4 with unknown parameters are globally and exponentially synchronized via the adaptive control law (Eq. 7) where the update law for the parameter estimates is given by Eq. 14 and k_i (i = 1, 2,..., 8) are positive constants.

Also, the parameter $\hat{a}(t)$, $\hat{b}(t)$, $\hat{c}(t)$ and $\hat{d}(t)$ estimates exponentially converge to the original values of the parameters a-d, respectively as $t\to\infty$.

Numerical results: For the numerical simulations, the fourth order Runge-Kutta Method with time-step $h = 10^{-6}$ is used to solve the hyperchaotic systems Eq. 3 and 4 with the adaptive control law Eq. 14 and the parameter update law Eq. 14 using MATLAB. Researchers take:

$$k_i = 4 \text{ for } i = 1, 2, \ldots\ldots, 8$$

For the hyperchaotic Pang Systems Eq. 3 and 4, the parameter values are taken as:

$$a = 36,\ b = 3,\ c = 20,\ d = 2$$

Suppose that the initial values of the parameter estimates are:

$$\hat{a}(0) = 12,\ \ \hat{b}(0) = 4,\ \ \hat{c}(0) = 2,\ \ \hat{d}(0) = 21$$

The initial values of the Master System Eq. 3 are taken as:

$$x_1(0) = 12,\ \ x_2(0) = 18,\ \ x_3(0) = 35,\ \ x_4(0) = 6$$

The initial values of the Slave System Eq. 4 are taken as:

$$y_1(0) = 20,\ \ y_2(0) = 5,\ \ y_3(0) = 16,\ \ y_4(0) = 22$$

Figure 3 shows the global chaos synchronization of the identical hyperchaotic Pang Systems Eq. 3 and 4. Figure 4 shows that the estimated values of the parameters, viz., $\hat{a}(t)$, $\hat{b}(t)$, $\hat{c}(t)$ and $\hat{d}(t)$ and converge exponentially to the system parameters:

$$a = 36,\ b = 3,\ c = 20 \text{ and } d = 2 \text{ as } t\to\infty$$

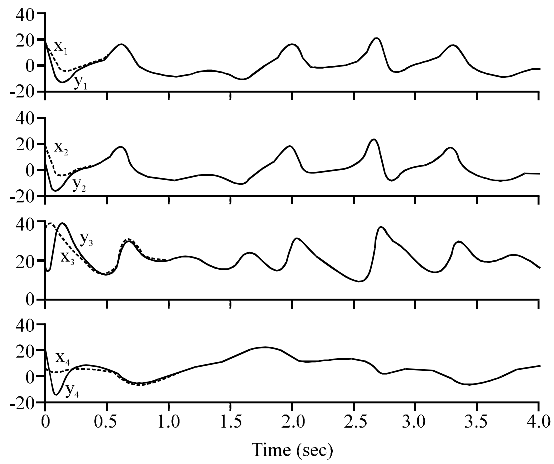

Fig. 3: Complete synchronization of Hyperchaotic Pang Systems

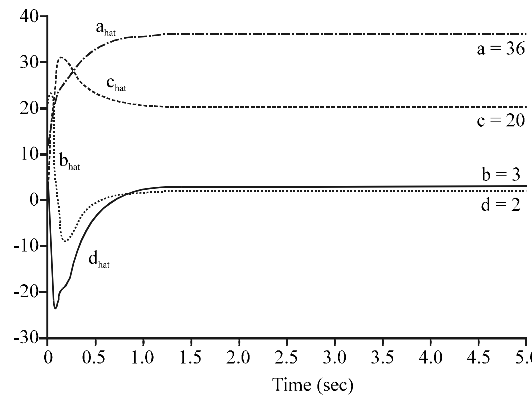

Fig. 4: Parameter estimates $\hat{a}(t)$, $\hat{b}(t)$, $\hat{c}(t)$ and $\hat{d}(t)$

ADAPTIVE SYNCHRONIZATION OF IDENTICAL HYPERCHAOTIC WANG SYSTEMS

Theoretical results: In this study, we deploy adaptive control to achieve new results for the global chaos synchronization of identical hyperchaotic Wang Systems (Wang and Liu, 2006) where the parameters of the Master and Slave Systems are unknown. As the Master System, we consider the hyperchaotic Wang dynamics described by:

$$\begin{aligned}\dot{x}_1 &= \alpha(x_2 - x_1)\\ \dot{x}_2 &= \beta x_1 - x_1x_3 + x_4\\ \dot{x}_3 &= -\gamma x_3 + \varepsilon x_1^2\\ \dot{x}_4 &= -\delta x_1\end{aligned} \tag{16}$$

where, x_1-x_4 are the state variables and α, β, γ, δ and ε are unknown, real and constant parameters of the system. As the Slave System, we consider the controlled hyperchaotic Wang dynamics described by:

$$\begin{aligned}\dot{y}_1 &= \alpha(y_2 - y_1) + u_1\\ \dot{y}_2 &= \beta y_1 - y_1 y_3 + y_4 + u_2\\ \dot{y}_3 &= -\gamma y_3 + \varepsilon y_1^2 + u_3\\ \dot{y}_4 &= -\delta y_1 + u_4\end{aligned} \quad (17)$$

Where:

y_1-y_4 = The state variables

u_1-u_4 = The nonlinear controllers to be designed

The chaos synchronization error is defined by:

$$\begin{aligned}e_1 &= y_1 - x_1\\ e_2 &= y_2 - x_2\\ e_3 &= y_3 - x_3\\ e_4 &= y_4 - x_4\end{aligned} \quad (18)$$

The error dynamics is easily obtained as:

$$\begin{aligned}\dot{e}_1 &= \alpha(e_2 - e_1) + u_1\\ \dot{e}_2 &= \beta e_1 + e_4 - y_1 y_3 + x_1 x_3 + u_2\\ \dot{e}_3 &= -\gamma e_3 + \varepsilon(y_1^2 - x_1^2) + u_3\\ \dot{e}_4 &= -\delta e_1 + u_4\end{aligned} \quad (19)$$

Let us now define the adaptive control functions:

$$\begin{aligned}u_1(t) &= -\hat{\alpha}(e_2 - e_1) - k_1 e_1\\ u_2(t) &= -\hat{\beta} e_1 - e_4 + y_1 y_3 - x_1 x_3 - k_2 e_2\\ u_3(t) &= \hat{\gamma} e_3 - \hat{\varepsilon}(y_1^2 - x_1^2) - k_3 e_3\\ u_4(t) &= \hat{\delta} e_1 - k_4 e_4\end{aligned} \quad (20)$$

where, $\hat{\alpha}, \hat{\beta}, \hat{\gamma}, \hat{\delta}$ and $\hat{\varepsilon}$ are estimates of α, β, γ, δ and ε, respectively and k_i (i = 1-4) are positive constants. Substituting Eq. 20 into Eq. 19, the error dynamics simplifies to:

$$\begin{aligned}\dot{e}_1 &= (\alpha - \hat{\alpha})(e_2 - e_1) - k_1 e_1\\ \dot{e}_2 &= (\beta - \hat{\beta}) e_1 - k_2 e_2\\ \dot{e}_3 &= -(\gamma - \hat{\gamma}) e_3 + (\varepsilon - \hat{\varepsilon})(y_1^2 - x_1^2) - k_3 e_3\\ \dot{e}_4 &= -(\delta - \hat{\delta}) e_1 - k_4 e_4\end{aligned} \quad (21)$$

Let us now define the parameter estimation errors as:

$$\begin{aligned}&e_\alpha = \alpha - \hat{\alpha},\ e_\beta = \beta - \hat{\beta},\ e_\gamma = \gamma - \hat{\gamma},\\ &e_\delta = \delta - \hat{\delta} \text{ and } e_\varepsilon = \varepsilon - \hat{\varepsilon}\end{aligned} \quad (22)$$

Substituting Eq. 22 into Eq. 21, we obtain the error dynamics as:

$$\begin{aligned}\dot{e}_1 &= e_\alpha(e_2 - e_1) - k_1 e_1\\ \dot{e}_2 &= e_\beta e_1 - k_2 e_2\\ \dot{e}_3 &= -e_\gamma e_3 + e_\varepsilon(y_1^2 - x_1^2) - k_3 e_3\\ \dot{e}_4 &= -e_\delta e_1 - k_4 e_4\end{aligned} \quad (23)$$

For the derivation of the update law for adjusting the estimates of the parameters, the Lyapunov approach is used. We consider the quadratic Lyapunov function defined by:

$$\begin{aligned}&V(e_1, e_2, e_3, e_4, e_\alpha, e_\beta, e_\gamma, e_\delta, e_\varepsilon)\\ &= \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_4^2 + e_\alpha^2 + e_\beta^2 + e_\gamma^2 + e_\delta^2 + e_\varepsilon^2\right)\end{aligned} \quad (24)$$

which is a positive definite function on R^9. We also note that:

$$\dot{e}_\alpha = -\dot{\hat{\alpha}},\ \dot{e}_\beta = -\dot{\hat{\beta}},\ \dot{e}_\gamma = -\dot{\hat{\gamma}},\ \dot{e}_\delta = -\dot{\hat{\delta}} \text{ and } \dot{e}_\varepsilon = -\dot{\hat{\varepsilon}} \quad (25)$$

Differentiating Eq. 24 along the trajectories of Eq. 23 and using Eq. 25, we obtain:

$$\begin{aligned}\dot{V} = &-k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 - k_4 e_4^2 + e_\alpha\left[e_1(e_2 - e_1) - \dot{\hat{\alpha}}\right] +\\ &e_\beta\left[e_1 e_2 - \dot{\hat{\beta}}\right] + e_\gamma\left[-e_3^2 - \dot{\hat{\gamma}}\right] + e_\delta\left[-e_1 e_4 - \dot{\hat{\delta}}\right] +\\ &e_\varepsilon\left[e_3(y_1^2 - x_1^2) - \dot{\hat{\varepsilon}}\right]\end{aligned} \quad (26)$$

In view of Eq. 26 the estimated parameters are updated by the following law:

$$\begin{aligned}\dot{\hat{\alpha}} &= e_1(e_2 - e_1) + k_5 e_\alpha\\ \dot{\hat{\beta}} &= e_1 e_2 + k_6 e_\beta\\ \dot{\hat{\gamma}} &= -e_3^2 + k_7 e_\gamma\\ \dot{\hat{\delta}} &= -e_1 e_4 + k_8 e_\delta\\ \dot{\hat{\varepsilon}} &= e_3(y_1^2 - x_1^2) + k_9 e_\varepsilon\end{aligned} \quad (27)$$

where, k_i (i = 5,...., 9) are positive constants. Substituting Eq. 27 into Eq. 26, we obtain:

$$\begin{aligned}\dot{V} = &-k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 - k_4 e_4^2 - k_5 e_\alpha^2 -\\ &k_6 e_\beta^2 - k_7 e_\gamma^2 - k_8 e_\delta^2 - k_9 e_\varepsilon^2\end{aligned} \quad (28)$$

which is a negative definite function on R^9. Thus, by Lyapunov Stability Theory (Hahn, 1967), it is immediate that the synchronization error e_i (i = 1-4) and the parameter estimation error e_α, e_β, e_γ, e_δ and e_ε decay to zero exponentially with time. Hence, we have proved the following result.

Theorem 2: The identical hyperchaotic Wang Systems Eq. 16 and 17 with unknown parameters are globally and exponentially synchronized via the adaptive control law Eq. 20 where the update law for the parameter estimates is given by Eq. 27 and k_i (i = 1, 2, ..., 9) are positive constants. Also, the parameter estimates and $\hat{\alpha}(t)$, $\hat{\beta}(t)$, $\hat{\gamma}(t)$, $\hat{\delta}(t)$ and $\hat{\varepsilon}(t)$ exponentially converge to the original values of the parameters α, β, γ, δ and ε, respectively as $t\to\infty$.

Numerical results: For the numerical simulations, the fourth order Runge-Kutta Method with time-step $h = 10^{-6}$ is used to solve the hyperchaotic systems Eq. 16 and 17 with the adaptive control law Eq. 20 and the parameter update law Eq. 27 using MATLAB. We take:

$$k_i = 4 \text{ for } i = 1, 2, ..., 9$$

For the hyperchaotic Wang Systems Eq. 16 and 17, the parameter values are taken as:

$$\alpha = 10, \beta = 40, \gamma = 2.5, \delta = 10.6, \varepsilon = 4$$

Suppose that the initial values of the parameter estimates are:

$$\hat{\alpha}(0) = 5,\quad \hat{\beta}(0) = 10,\quad \hat{\gamma}(0) = 7,\quad \hat{\delta}(0) = 14,\quad \hat{\varepsilon}(0) = 9$$

The initial values of the Master System Eq. 16 are taken as:

$$x_1(0) = 21, x_2(0) = 7, x_3(0) = 16, x_4(0) = 18$$

The initial values of the Slave System Eq. 17 are taken as:

$$y_1(0) = 4, y_2(0) = 25, y_3(0) = 30, y_4(0) = 11$$

Figure 5 shows the global chaos synchronization of the identical hyperchaotic Wang Systems Eq. 16 and 17. Figure 6 shows that the estimated values of the parameters, $\hat{\alpha}(t), \hat{\beta}(t), \hat{\gamma}(t), \hat{\delta}(t)$ and $\hat{\varepsilon}(t)$ viz., converge exponentially to the system parameters:

$$\alpha = 10, \beta = 40, \gamma = 2.5, \delta = 10.6 \text{ and } \varepsilon = 4 \text{ as } t\to\infty$$

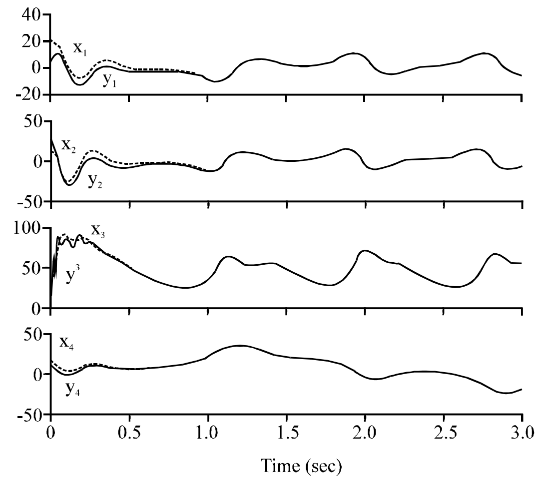

Fig. 5: Complete synchronization of Hyperchaotic Wang Systems

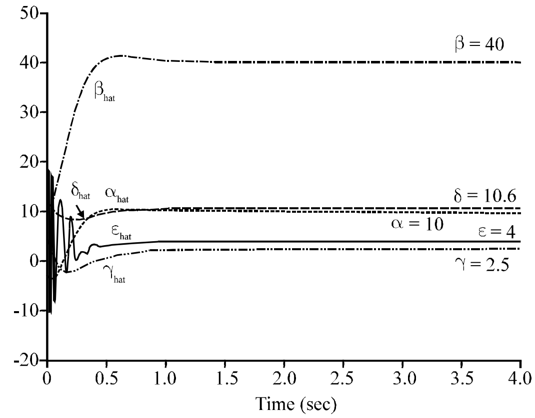

Fig. 6: Parameter estimates $\hat{\alpha}(t), \hat{\beta}(t), \hat{\gamma}(t), \hat{\delta}(t)$ and $\hat{\varepsilon}(t)$

ADAPTIVE SYNCHRONIZATION OF HYPERCHAOTIC PANG AND HYPERCHAOTIC WANG SYSTEMS

Theoretical results: In this study, we discuss the global chaos synchronization of non-identical hyperchaotic Pang system (Pang and Liu, 2011) and hyperchaotic Wang system (Wang and Liu, 2006) where the parameters of the master and slave systems are unknown. As the master system, we consider the hyperchaotic Pang System described by:

$$\begin{aligned} \dot{x}_1 &= a(x_2 - x_1) \\ \dot{x}_2 &= cx_2 - x_1x_3 + x_4 \\ \dot{x}_3 &= -bx_3 + x_1x_2 \\ \dot{x}_4 &= -d(x_1 + x_2) \end{aligned} \tag{29}$$

Where:

x_1-x_4 = The state variables
a-d = Unknown, real and constant parameters of the system

As the Slave System, we consider the controlled hyperchaotic Wang dynamics described by:

$$\begin{aligned} \dot{y}_1 &= \alpha(y_2 - y_1) + u_1 \\ \dot{y}_2 &= \beta y_1 - y_1 y_3 + y_4 + u_2 \\ \dot{y}_3 &= -\gamma y_3 + \varepsilon y_1^2 + u_3 \\ \dot{y}_4 &= -\delta y_1 + u_4 \end{aligned} \tag{30}$$

where, y_1-y_4 are the state variables, α, β, γ, δ and ε are unknown, real and constant parameters of the system and u_1-u_4 are the nonlinear controllers to be designed. The synchronization error is defined by:

$$\begin{aligned} e_1 &= y_1 - x_1 \\ e_2 &= y_2 - x_2 \\ e_3 &= y_3 - x_3 \\ e_4 &= y_4 - x_4 \end{aligned} \tag{31}$$

The error dynamics is easily obtained as:

$$\begin{aligned} \dot{e}_1 &= \alpha(y_2 - y_1) - a(x_2 - x_1) + u_1 \\ \dot{e}_2 &= \beta y_1 - c x_2 + e_4 - y_1 y_3 + x_1 x_3 + u_2 \\ \dot{e}_3 &= -\gamma y_3 + b x_3 + \varepsilon y_1^2 - x_1 x_2 + u_3 \\ \dot{e}_4 &= -\delta y_1 + d(x_1 + x_2) + u_4 \end{aligned} \tag{32}$$

Let us now define the adaptive control functions:

$$\begin{aligned} u_1(t) &= -\hat{\alpha}(y_2 - y_1) + \hat{a}(x_2 - x_1) - k_1 e_1 \\ u_2(t) &= -\hat{\beta} y_1 + \hat{c} x_2 - e_4 + y_1 y_3 - x_1 x_3 - k_2 e_2 \\ u_3(t) &= \hat{\gamma} y_3 - \hat{b} x_3 - \hat{\varepsilon} y_1^2 + x_1 x_2 - k_3 e_3 \\ u_4(t) &= \hat{\delta} y_1 - \hat{d}(x_1 + x_2) - k_4 e_4 \end{aligned} \tag{33}$$

where, $\hat{a}, \hat{b}, \hat{c}, \hat{d}, \hat{\alpha}, \hat{\beta}, \hat{\gamma}, \hat{\delta}$ and $\hat{\varepsilon}$ are estimates of a, b, c, d, α, β, γ, δ and ε, respectively and k_i (i = 1-4) are positive constants. Substituting Eq. 33 into Eq. 32, the error dynamics simplifies to:

$$\begin{aligned} \dot{e}_1 &= (\alpha - \hat{\alpha})(y_2 - y_1) - (a - \hat{a})(x_2 - x_1) - k_1 e_1 \\ \dot{e}_2 &= (\beta - \hat{\beta}) y_1 - (c - \hat{c}) x_2 - k_2 e_2 \\ \dot{e}_3 &= -(\gamma - \hat{\gamma}) y_3 + (b - \hat{b}) x_3 + (\varepsilon - \hat{\varepsilon}) y_1^2 - k_3 e_3 \\ \dot{e}_4 &= -(\delta - \hat{\delta}) y_1 + (d - \hat{d})(x_1 + x_2) - k_4 e_4 \end{aligned} \tag{34}$$

Let us now define the parameter estimation errors as:

$$\begin{aligned} & e_a = a - \hat{a}, \;\; e_b = b - \hat{b}, \;\; e_c = c - \hat{c}, \;\; e_d = d - \hat{d} \\ & e_\alpha = \alpha - \hat{\alpha}, \;\; e_\beta = \beta - \hat{\beta}, \;\; e_\gamma = \gamma - \hat{\gamma}, \\ & e_\delta = \delta - \hat{\delta}, \;\; e_\varepsilon = \varepsilon - \hat{\varepsilon} \end{aligned} \tag{35}$$

Substituting Eq. 35 into Eq. 34, we obtain the error dynamics as:

$$\begin{aligned} \dot{e}_1 &= e_\alpha(y_2 - y_1) - e_a(x_2 - x_1) - k_1 e_1 \\ \dot{e}_2 &= e_\beta y_1 - e_c x_2 - k_2 e_2 \\ \dot{e}_3 &= -e_\gamma y_3 + e_b x_3 + e_\varepsilon y_1^2 - k_3 e_3 \\ \dot{e}_4 &= -e_\delta y_1 + e_d(x_1 + x_2) - k_4 e_4 \end{aligned} \tag{36}$$

We consider the quadratic Lyapunov function defined by:

$$V = \frac{1}{2}\begin{pmatrix} e_1^2 + e_2^2 + e_3^2 + e_4^2 + e_a^2 + e_b^2 + e_c^2 + \\ e_d^2 + e_\alpha^2 + e_\beta^2 + e_\gamma^2 + e_\delta^2 + e_\varepsilon^2 \end{pmatrix} \tag{37}$$

which is a positive definite function on R^{13}. We also note that:

$$\begin{aligned} & \dot{e}_a = -\dot{\hat{a}}, \;\; \dot{e}_b = -\dot{\hat{b}}, \;\; \dot{e}_c = -\dot{\hat{c}}, \;\; \dot{e}_d = -\dot{\hat{d}} \\ & \dot{e}_\alpha = -\dot{\hat{\alpha}}, \; \dot{e}_\beta = -\dot{\hat{\beta}}, \; \dot{e}_\gamma = -\dot{\hat{\gamma}}, \; \dot{e}_\delta = -\dot{\hat{\delta}}, \; \dot{e}_\varepsilon = -\dot{\hat{\varepsilon}} \end{aligned} \tag{38}$$

Differentiating Eq. 37 along the trajectories of Eq. 36 and using Eq. 38, we obtain:

$$\begin{aligned} \dot{V} = & -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 - k_4 e_4^2 + e_a\left[-e_1(x_2 - x_1) - \dot{\hat{a}}\right] + \\ & e_b\left[e_3 x_3 - \dot{\hat{b}}\right] + e_c\left[-e_2 x_2 - \dot{\hat{c}}\right] + e_d\left[e_4(x_1 + x_2) - \dot{\hat{d}}\right] + \\ & e_\alpha\left[e_1(y_2 - y_1) - \dot{\hat{\alpha}}\right] + e_\beta\left[e_2 y_1 - \dot{\hat{\beta}}\right] + e_\gamma\left[-e_3 y_3 - \dot{\hat{\gamma}}\right] + \\ & e_\delta\left[-e_4 y_1 - \dot{\hat{\delta}}\right] + e_\varepsilon\left[e_3 y_1^2 - \dot{\hat{\varepsilon}}\right] \end{aligned} \tag{39}$$

In view of Eq. 39 the estimated parameters are updated by the following law:

$$\begin{aligned} & \dot{\hat{a}} = -e_1(x_2 - x_1) + k_5 e_a, \;\; \dot{\hat{\alpha}} = e_1(y_2 - y_1) + k_9 e_\alpha \\ & \dot{\hat{b}} = e_3 x_3 + k_6 e_b, \qquad \dot{\hat{\beta}} = e_2 y_1 + k_{10} e_\beta \\ & \dot{\hat{c}} = -e_2 x_2 + k_7 e_c, \qquad \dot{\hat{\gamma}} = -e_3 y_3 + k_{11} e_\gamma \\ & \dot{\hat{d}} = e_4(x_1 + x_2) + k_8 e_d, \;\; \dot{\hat{\delta}} = -e_4 y_1 + k_{12} e_\delta \\ & \qquad \dot{\hat{\varepsilon}} = e_3 y_1^2 + k_{13} e_\varepsilon \end{aligned} \tag{40}$$

where, k_i (i = 5, ... , 13) are positive constants. Substituting Eq. 40 into Eq. 39, we obtain:

$$\begin{aligned} \dot{V} = & -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 - k_4 e_4^2 - k_5 e_a^2 - k_6 e_b^2 - \\ & k_7 e_c^2 - k_8 e_d^2 - k_9 e_\alpha^2 - k_{10} e_\beta^2 - k_{11} e_\gamma^2 - k_{12} e_\delta^2 - k_{13} e_\varepsilon^2 \end{aligned} \tag{41}$$

which is a negative definite function on R^{13}. Thus, by Lyapunov Stability Theory (Hahn, 1967), it is immediate that the synchronization error e_i (i = 1, 2, 3, 4) and all the parameter estimation errors decay to zero exponentially with time. Hence, we have proved the following result.

Theorem 3: The non-identical hyperchaotic Pang System Eq. 29 and hyperchaotic Wang System Eq. 30 with unknown parameters are globally and exponentially synchronized via the adaptive control law Eq. 33 where the update law for the parameter estimates is given by Eq. 40 and k_i (i = 1, 2, ..., 13) are positive constants. Also, the parameter $\hat{a}(t), \hat{b}(t), \hat{c}(t), \hat{d}(t), \hat{\alpha}(t), \hat{\beta}(t), \hat{\gamma}(t), \hat{\delta}(t)$ and $\hat{\varepsilon}(t)$ estimates exponentially converge to the original values of the parameters a, b, c, d, α, β, γ, δ and ε, respectively as $t\to\infty$.

Numerical results: For the numerical simulations, the fourth order Runge-Kutta Method with time-step $h = 10^{-6}$ is used to solve the hyperchaotic systems Eq. 29 and 30 with the adaptive control law Eq. 33 and the parameter update law Eq. 40 using MATLAB. We take $k_i = 4$ for i = 1, 2, 3, ..., 13.

For the hyperchaotic Pang and hyperchaotic Wang Systems, the parameters of the systems are chosen so that the systems are hyperchaotic. Suppose that the initial values of the parameter estimates are:

$$\hat{a}(0) = 2,\ \hat{b}(0) = 5,\ \hat{c}(0) = 10,\ \hat{d}(0) = 12$$
$$\hat{\alpha}(0) = 7,\ \hat{\beta}(0) = 9,\ \hat{\gamma}(0) = 15,\ \hat{\delta}(0) = 22,\ \hat{\varepsilon}(0) = 25$$

The initial values of the Master System Eq. 29 are taken as:

$$x_1(0) = 27,\ x_2(0) = 11,\ x_3(0) = 28,\ x_4(0) = 6$$

The initial values of the slave system Eq. 30 are taken as:

$$y_1(0) = 10,\ y_2(0) = 26,\ y_3(0) = 9,\ y_4(0) = 30$$

Figure 7 shows the global chaos synchronization of hyperchaotic Pang and hyperchaotic Wang systems. Figure 8 shows that the estimated values of the parameters, viz., $\hat{a}(t), \hat{b}(t), \hat{c}(t), \hat{d}(t), \hat{\alpha}(t), \hat{\beta}(t), \hat{\gamma}(t), \hat{\delta}(t)$ and $\hat{\varepsilon}(t)$ converge exponentially to the system parameters a = 36, b = 3, c = 20, d= 2, α = 10, β = 40, γ = 2.5, δ = 10.6 and ε = 4, respectively as $t\to\infty$.

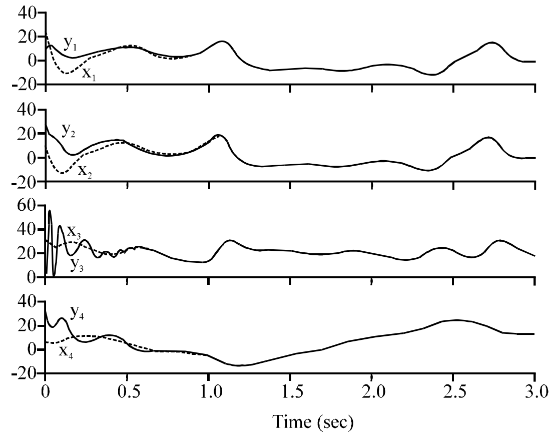

Fig. 7: Complete synchronization of hyperchaotic Pang and Wang Systems

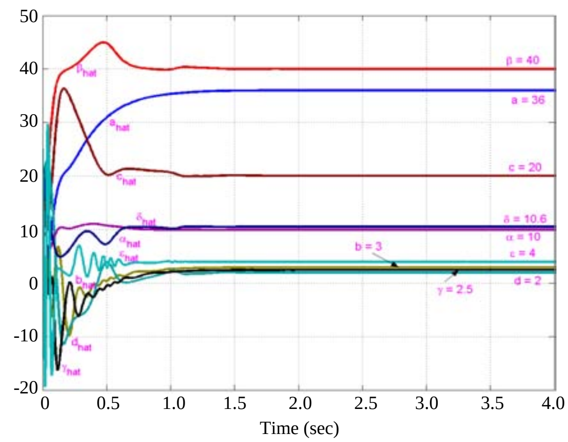

Fig. 8: Parameter estimates $\hat{a}(t), \hat{b}(t), \hat{c}(t), \hat{d}(t), \hat{\alpha}(t), \hat{\beta}(t)$ and $\hat{\gamma}(t)$

CONCLUSION

In this study, we have derived new results for the adaptive synchronization of identical hyperchaotic Pang Systems in 2011, identical hyperchaotic Wang Systems in 2006 and Non-identical hyperchaotic Pang and hyperchaotic Wang Systems with unknown parameters. The adaptive synchronization results derived in this study are established using Lyapunov Stability Theory. Since, the Lyapunov exponents are not required for these calculations, the Adaptive Control Method is a very effective and convenient for achieving global chaos synchronization for the uncertain Hyperchaotic Systems discussed in this study.

Numerical simulations are given to illustrate the effectiveness of the adaptive synchronization schemes derived in this study for the global chaos synchronization of identical and non-identical uncertain hyperchaotic Pang and hyperchaotic Wang Systems.

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