International Journal of Soft Computing 8 (3): 163-170, 2013
ISSN: 1816-9503

A New Engineering Optimization Method: African Wild Dog Algorithm

[1]C. Subramanian, [2]A.S.S. Sekar and [3]K. Subramanian
[1]Department of Civil Engineering, Thanthai Periyar Government Institute of Technology, Vellore, India
[2]Department of Civil Engineering, A.C. College of Engineering and Technology, Karaikudi, India
[3]Department of Civil Engineering, Coimbatore Institute of Technology, Coimbatore, India

Abstract: This study introduces a new parameter free meta-heuristic optimization algorithm, African Wild Dog Algorithm (AWDA) to solve engineering optimization problems. Meta-heuristic algorithms imitate natural phenomena, e.g., physical annealing in simulated annealing, human memory in a tabu search and evolution in evolutionary algorithms. AWDA mimics the communal hunting behavior of African wild dogs. As the currently available metaheuristic optimization algorithms require a set of algorithmic parameters to be tuned to yield optimal performance, AWDA does not require any parameter except pack size and termination criterion. The AWDA, code was tested in several benchmark engineering optimization problems taken from literature. The optimization results indicate that AWDA may yield better solutions than other Meta-heuristic algorithms.

Key words: Meta-heuristic optimization, African wild dog algorithm, engineering optimization, memory, parameters

INTRODUCTION

In last few decades, many new meta-heuristic algorithms have been developed and used in engineering optimization. This meta-heuristic algorithms do not have limitations in using resources, (e.g., music inspired harmony search (Geem *et al*., 2001). However, nature is the principal source of inspiration. Biologically-inspired algorithms are one of the main categories of the nature-inspired meta-heuristic algorithms. More specifically, these algorithms are based on the selection of the fittest in biological systems which have evolved by natural selection over millions of years (Yang, 2011). Various bio-inspired optimization algorithms have been presented in literature. The most popular methods are Genetic Algorithm (GA) (Goldberg, 1989), Particle Swarm Optimization (PSO) (Eberhart and Kennedy, 1995), Ant Colony Optimization (ACO) (Dorigo *et al*., 1996) and Tabu Search (TS) (Glover, 1977). Moreover, Bat Algorithm (BA) (Yang, 2010), Cuckoo Search (CS) (Gandomi *et al*., 2011), Firefly Algorithm (FA) (Yang, 2009), Glowworm Swarm Optimization (GSO) (Krishnanand and Ghose, 2006) and Hunting Search (HuS) (Oftadeh *et al*., 2010) are some of the new bio-inspired algorithms. Besides bio-inspired algorithms, there are the nature-inspired algorithms that mimic physical phenomena such as Simulated Annealing (SA) (Kirkpatrick, 1984), Big Bang-Big Crunch (BB-BC) (Erol and Eksin, 2006) and Charged System Search (CSS) (Kaveh and Talatahari, 2010).

All the mentioned meta-heuristic optimization algorithms require a set of algorithmic parameters to be tuned to yield optimal performance. The number of parameters has a major impact on the practical applicability of optimization algorithms. A good algorithm would consist of a small number of problem-specific parameters. In this study, a new parameter free meta-heuristic optimization algorithm, African Wild Dog Algorithm (AWDA) is introduced to solve engineering optimization problems. The AWD algorithm is based on the communal hunting behaviour of African wild dogs. Preliminary studies indicate that AWDA may be superior over GA, HS and other mathematical optimization methods.

AFRICAN WILD DOG META-HEURISTIC ALGORITHM

Meta-heuristic algorithms imitate natural phenomena, e.g., physical annealing in simulated annealing, human memory in a tabu search and evolution in evolutionary algorithms. A new wild dog meta-heuristic algorithm is conceptualized using the communal hunting behavior of

Corresponding Author: C. Subramanian, Department of Civil Engineering, Thanthai Periyar Government Institute of Technology, Vellore, India

African wild dogs. African wild dogs live in packs of upto 20 adults and their dependent young. Communal hunting is one of the most conspicuous aspects of the behavior of social carnivores. The studies of carnivore ecology suggested that communal hunting might favour sociality, either by increasing the size of prey that could be killed or by improving hunting success (Creel and Creel, 1995). Coordination between the members of an African wild dog pack is seen throughout a hunt. At several stages, the effectiveness appears to depend on the number of cooperating hunters. This communal hunting behavior is similar to the optimization process which results in finding a global solution as determined by an objective function. The location of each dog compared to the prey determines its chance of catching the prey. Similarly, the objective function value is determined by the set of values assigned to each decision variable. The new wild dog meta-heuristic algorithm is developed based on a model of cooperative hunting of animals when searching for food.

In continuous optimization problems, the evaluation of a solution is carried out by putting values of decision variables into the objective function or fitness function. This calculates the function value which includes cost, efficiency and/or error.

Compared to communal hunting in a continuous optimization problem each wild dog is replaced with a solution of the problem. It should be noted that communal hunting of animals and the meta-heuristic algorithm have a primary difference. In communal hunting of wild dogs, dogs can see the prey or sense the smell of the prey and determine its location. In contrast, in optimization problems researchers have no indication of the optimum solution/point. In communal hunting of animals however, the solution (prey) is dynamic and the dogs (based on the current location of the prey) must coordinate their position. In optimization problems instead, the optimum solution is static and does not change its position during the search process. In fact, both real and artificial group hunting have their own difficulties. To resemble this dynamics of the hunting process in the algorithm, each dog move towards other dog based their favorable position (fitness). The procedure of the AWD algorithm consists of the following steps:

- Step 1: Specify the optimization problem and parameters of the algorithm
- Step 2: Randomly initialize the wild dog pack
- Step 3: Evaluate the fitness of all wild dogs
- Step 4: Coordinated movement of wild dog pack
- Step 5: Repeat steps 3 and 4 until the termination criterion is satisfied

Step 1: Initialize the optimization problem and algorithm parameters. First, the optimization problem is specified as follows:

Minimize:

$$f(x) \text{ s.t. } x_i \in X_i,\ i = 1, 2, \ldots, N \quad (1)$$

Where:

f (x) = The objective function
x = The set of each design variable (x_i)
X_i = The set of the possible range of values for each design variable (continuous design variables) that is ${}_L x_i \le X_i \le {}_U x_i$
N = The number of design variables

The number of wild dogs and termination criterion (maximum number of iterations) are also specified in this step. No other parameters are required to solve the optimization problem in wild dog algorithm.

Step 2: Randomly initialize the wild dog pack. Based on the number of wild dogs, the hunting pack matrix is filled with randomly generated solution vectors.

Step 3: Evaluate the fitness of all dogs. The values of objective function are computed and the fitness values of wild dogs are evaluated. The wild dogs are ranked according to their fitness values.

Step 4: Coordinated movement of wild dogs. In step 4, each dog i will move to a new position $x_{i,new}$ towards another dog j having higher fitness value based on probability:

$$x_{i,\,new} = x_i + rand \times (x_i - x_j) \times c \times (a/b) \quad (2)$$

Where:

rand = Vector of size N having random values varying from 0 to 1
c = Step reduction coefficient = 1-(Iteration number/max iterations)
a = Mean euclidian distance of all dogs
b = Euclidian distance between dog i and j

Step 5: Repeat steps 3 and 4 until the termination criterion is satisfied. In step 5, the computations are terminated when the termination criterion is satisfied. If not, steps 3 and 4 are then repeated. The termination criterion has been defined as the maximum number of iterations.

To further elaborate on the wild dog meta-heuristic algorithm, consider the following unconstrained minimization problem with two design variables (Goldstein and Price, 1971):

$$f(x) = \{1+(x_1+x_2+1)^2\ (19-14x_1+3x_1^2-14x_2+6x_1x_2+3x_2^2)\} \times \{30+(2x_1-3x_2)^2(18-32x_1+12x_1^2+48x_2-36x_1x_2+27x_2^2)\} \quad (3)$$

This function is an eighth-order polynomial in two variables that is well behaved near each minimum value. The function has four local minima, one of which is global as follows: f(1.2, 0.8) = 840.0, f(1.8, 0.2) = 84.0, f(0.6, 0.4) = 30.0 and $f^*(0, 1.0) = 3.0$ (global minimum) (Goldstein and Price, 1971).

The AWD was initially structured with randomly generated solution vectors within the bounds prescribed for this example (i.e., -5.0 to 5.0) as shown in Fig. 1 and objective function values are evaluated. As the problem is a minimization problem, the fitness values are calculated as the inverse of objective function value so that the wild dog with least objective function value will get higher fitness value. These were sorted according to the fitness value (step 3).

In step 4, each dog i will move to a newposition $x_{i,new}$ towards another dog j having higher fitness value (favorable position to catch the prey) based on probability. For example a wild dog having rank 4 will move towards any of the wild dog having rank <4 (i.e, 1, 2 or 3). Therefore, a wild dog having rank 1 in a particular iteration will not move in next iteration by this way the best solution is preserved to next iterations. The probability of dog i moving towards dog j is given by:

$$Pj = \frac{Fj}{\Sigma F} \quad (4)$$

Where:

Fj = Fitness of dog j

ΣF = Sum of fitness of all dogs having fitness value higher than fitness of dog i

For example the probability of movement of wild dog ranked 4 towards wild dog ranked 1 is more than the probability of movement towards wild dog ranked 2. This ensures the balance between exploration and exploitation behaviour of algorithm.

The probability of finding the global optimum, $x^* = (0, 1.0)$ increased with number of iterations as shown in Table 1. Finally after 100 iterations, the AWD algorithm found a near optimal solution, $x^* = (0.0000004603, -0.9999988730)$ that had a function value of -3.0000000005.

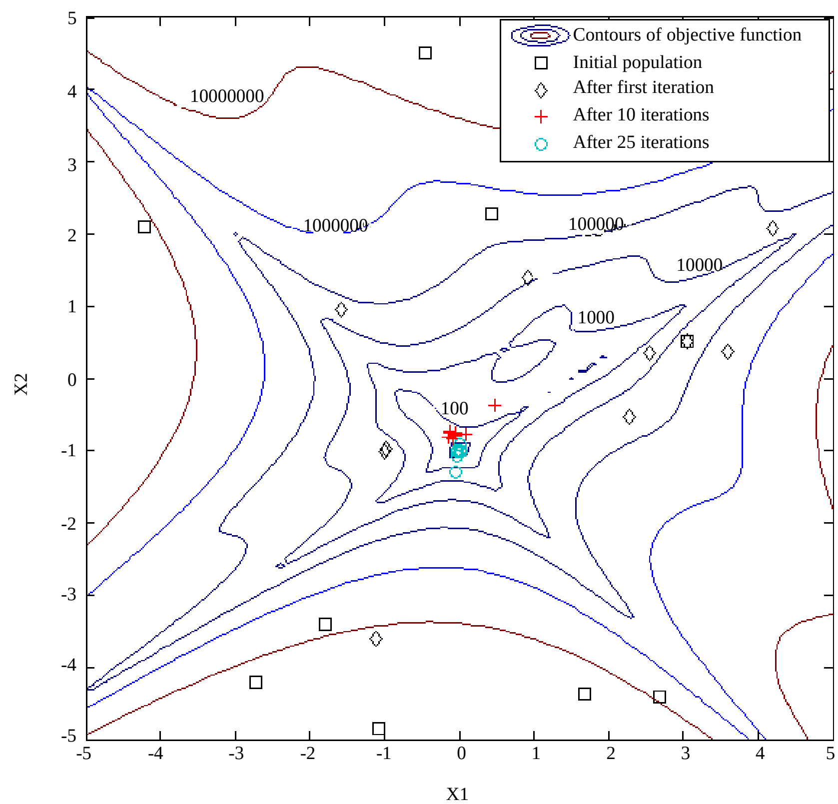

Fig. 1: Goldstein and Price function

Table 1: Optimal results for the Goldstein and Price function using AWD algorithm

	Initial population			Subsequent population		
Ranks	x_1	x_2	f (x)	x_1	x_2	f (x)
1	3.0457	0.5117	27424.96	-0.9855	-0.9787	1924.18
2	0.4412	2.2868	352995.32	-1.0067	-1.0126	2198.14
3	-1.7890	-3.3990	5858768.27	2.5523	0.3587	3781.08
4	2.6751	-4.4039	7465082.34	0.9223	1.4050	13398.81
5	-4.2176	2.0928	15602843.16	3.0457	0.5117	27424.96
6	-2.7150	-4.2160	28313944.61	2.2807	-0.5376	37900.20
7	1.6706	-4.3750	35417169.16	-1.5778	0.9445	39134.69
8	2.0144	4.0687	52665654.96	4.1948	2.0811	47990.78
9	-0.4570	4.5173	54783069.01	3.5865	0.3660	309396.15
10	-1.0777	-4.8557	219459536.88	-1.1139	-3.6076	15947610.43
Ranks	After 10 iterations			After 25 iterations		
1	-0.0861	-0.8249	18.36	0.0026	-1.0028	3.00691
2	-0.1471	-0.8130	19.45	0.0061	-1.0096	3.06296
3	-0.0579	-0.7868	28.53	0.0191	-1.0022	3.10579
4	-0.0572	-0.7782	31.19	-0.0277	-1.0116	3.18607
5	-0.1191	-0.7551	32.68	-0.0205	-0.9860	3.24184
6	-0.1326	-0.7410	35.31	0.0281	-1.0106	3.32859
7	-0.0507	-0.7668	35.49	0.0333	-1.0139	3.49577
8	-0.0435	-0.7580	39.36	-0.0210	-1.0648	4.99130
9	0.0844	-0.7692	40.70	0.0035	-0.9200	5.83866
10	0.4776	-0.3689	461.59	-0.0436	-1.2898	206.52392
Ranks	After 50 iterations			After 100 iterations		
1	-0.000019914	-0.99997699	3.000000428	0.0000004603	-0.9999988730	3.0000000005
2	0.000608971	-0.99997886	3.000090882	0.0000011637	-0.9999980683	3.0000000015
3	-0.000548237	-0.99983554	3.000106826	0.0000007009	-1.0000024265	3.0000000030
4	0.000453312	-0.99874510	3.000608473	0.0000047518	-1.0000013500	3.0000000079
5	0.000748549	-0.99871193	3.000648939	-0.0000055388	-0.9999973037	3.0000000141
6	-0.000402966	-1.00155969	3.000957749	0.0000021460	-1.0000080373	3.0000000328
7	0.001972044	-0.99997555	3.000970620	-0.0000167632	-0.9999724029	3.0000004997
8	-0.001595440	-0.99937854	3.001019959	-0.0000754692	-1.0000005937	3.0000014257
9	-0.005752862	-1.02202721	3.198261225	-0.0007916424	-1.0025625947	3.0025640002
10	0.002818328	-0.97390961	3.276559392	0.0005466802	-0.9951328958	3.0096822616

EXAMPLES

The computational procedures described above have been implemented in a MATLAB computer program on a pentium 42.4 GHz computer. In this study, three standard engineering optimization benchmark examples from the literature are presented to demonstrate the efficiency and robustness of the proposed AWD meta-heuristic algorithm.

Pressure vessel design: The cylindrical pressure vessel capped at both ends by hemispherical heads (Fig. 2) must be designed for minimum cost (Sandgren, 1990). The compressed air tank has a working pressure of 3000 psi and a minimum volume of 750 ft^3 and must be designed according to the ASME code on boilers and pressure vessels. The total cost results from a combination of welding, material and forming costs. The thickness of the cylindrical skin (Ts), the thickness of the spherical head (Th), the inner Radius (R) and the length of the cylindrical segment of the vessel (L) were included as optimization variables. Thicknesses

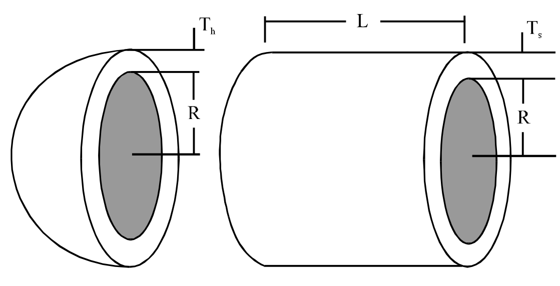

Fig. 2: Schematic of pressure vessel

can only take discrete values which are integer multiples of 0.0625 inch. and R and L have continuous values of $40 \le R \le 80$ and $20 \le L \le 60$ inch., respectively. The optimization problem can be stated as follows:

Minimize:

$$f(Ts, Th, R, L) = 0.6224T_sRL + 1.7781T_hR^2 + 3.1661T_s^2 L + 19.84T_h^2 R \quad (5)$$

Constraints are set in accordance with the ASME design codes; g_3 represents the constraint on the minimum volume of 750 ft^3. The constraints are stated as follows:

$$g_1 = -T_s + 0.0193R \le 0 \quad (6)$$

$$g_2 = -T_h + 0.00954R \le 0 \quad (7)$$

$$g_3 = -\pi R^2 L - \frac{4}{3}\pi R^3 + 750 \times 1278 \le 0 \quad (8)$$

$$g_4 = L - 240 \le 0 \quad (9)$$

$$g_5 = 1.1 - T_s \le 0 \quad (10)$$

$$g_6 = 0.60 - T_h \le 0 \quad (11)$$

This problem was earlier analyzed using GA (Wu and Chow, 1995) and HS (Lee and Geem, 2005). Optimization results are shown in Table 2. With 25 wild dogs, AWDA found the global optimum of 7198.008 within 25,000 function evaluations (i.e., 1000 optimization iterations). Table 2 compares the results obtained by AWDA with those reported in the literature.

Sandgren achieved the optimal values of $7980.894 using the Branch and Bound Method. Wu and Chow obtained the minimum cost of $7207.494 using the GA-based approach. The best solution obtained by HS algorithm was $7207.494. The AWD algorithm achieves a design with a best solution vector of (1.125, 0.625, 58.2901 and 43.6928) and a minimum cost of $7198.008 without violating any constraint. The results obtained using the AWD algorithms were better optimized than earlier solutions.

Minimization of weight of spring: The problem consists of minimizing the weight f(x) of a tension/compression spring subject to constraints on shear stress, surge frequency and minimum deflection as shown in Fig. 3.

The design variables are the mean coil diameter D (= x_1); the wire diameter d (= x_2) and the number of active coils N (= x_3). The problem can be stated as:

Minimize:

$$f(x) = (x_3 + 2) x_2 x_1^2 \quad (12)$$

Subject to:

$$g_1 = 1 - \frac{x_2^3 x_3}{71785 x_1^4} \le 0 \quad (13)$$

$$g_2 = \frac{4x_2^2 - x_1 x_2}{12566(x_2 x_1^3 - x_1^4)} + \frac{1}{5108 x_1^2} - 1 \le 0 \quad (14)$$

$$g_3 = 1 - \frac{140.45 x_1}{x_2^3 x_3} \le 0 \quad (15)$$

$$g_4 = \frac{x_2 + x_1}{1.5} - 1 \le 0 \quad (16)$$

Belegundu solved this problem using eight different mathematical optimization techniques (Belegundu, 1982). Arora also solved this problem using a numerical optimization technique called constraint correction at constant cost (Arora, 1989). This problem was also analyzed using GA-Based Method (Coello, 2000a) and improved harmony search algorithm (Mahdavi *et al*., 2007). After 30,000 function evaluations the best solution is obtained at x = (0.0516558; 0:3559185; 11.336039)

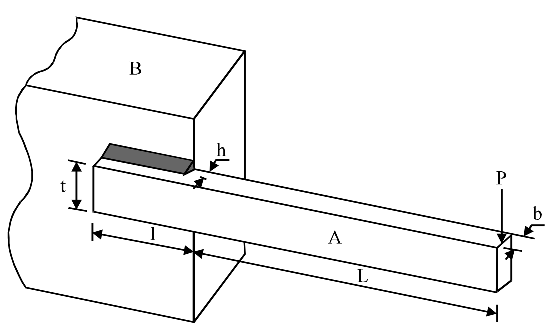

Fig. 3: Welded beam structure

Table 2: Optimal results for the pressure vessel design

Optimal design variables (x) and objective function value	Branch and Bound Method (Sandgren, 1990)	GA (Wu and Chow,1995)	HS (Lee and Geem, 2005)	Present study
T_s	1.125	1.1250	1.1250	1.1250
T_h	0.625	0.6250	0.6250	0.6250
R	48.970	58.1978	58.2789	5193.5186
L	106.720	44.2930	43.7549	180.1496
Cost	7980.894	7207.4940	7198.4330	7198.0080

Table 3: Optimal results for the minimization of weight of spring

Optimal design variables (x) and objective function value (f (x))	Numerical optimization technique (Arora, 1989)	Mathematical optimization techniques (Belegundu, 1982)	GA-Based Method (Coello, 2000a)	Improved harmony search algorithm (Mahadavi *et al.*, 2007)	Present study
x_1	0.053396	0.050000	0.051989	0.05115438	0.05165583
x_2	0.399180	0.315900	0.363965	0.34987116	0.35591847
x_3	9.185400	14.250000	10.890522	12.07643210	11.33603852
f(x)	0.012730	0.012833	0.012681	0.01267060	0.01266531

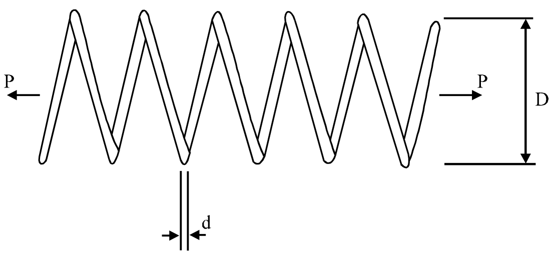

Fig. 4: Tension/compression spring

with corresponding function value equal to f(x) = 0.0126653. No constraints are active for this solution. Table 3 shows the best solution of this problem obtained using the AWD algorithm and compares the AWDA results with solutions reported by other researchers. It is apparent from the Table 2 that the result obtained using AWDA algorithm is better than those reported earlier in the literature.

Welded beam design problem: The welded beam structure shown in Fig. 4 is a practical design problem that has been often used as a benchmark for testing different optimization methods. This includes mathematical optimization algorithms (Ragsdell and Phillips, 1976) such as APPROX (Griffith and Stewart's successive linear approximation), DAVID (Davidon-Fletcher-Powell with a penalty function), SIMPLEX (Simplex Method with a penalty function) and RANDOM (Richardson's Random Method) algorithms. GA-based methods (Deb, 1991, 2000; Coello, 2000a, b), Harmony Search Method (Lee and Geem, 2005) and improved harmony search algorithm (Mahdavi *et al.*, 2007) were other methods used to solve this problem.

The objective is to find the minimum fabricating cost of the welded beam subject to constraints on shear stress τ, bending stress σ, buckling load P_c, end deflection δ and side constraint. There are four design variables: h (= x_1), l (= x_2), t (= x_3) and b (= x_4). The mathematical formulation of the objective function f (x) which is the total fabricating cost mainly comprised of the set-up, welding labor and material costs is as follows:

Minimize:

$$f(x) = 1.10471x_1^2x_2 + 0.04811x_3x_4(14.0+x_2) \tag{17}$$

Subject to:

$$g_1(x) = \tau(x) - \tau_{max} \le 0 \tag{18}$$

$$g_2(x) = \sigma(x) - \sigma_{max} \le 0 \tag{19}$$

$$g_3(x) = x_1 - x_4 \le 0 \tag{20}$$

$$g_4(x) = \delta(x_1) - \delta_{max} \le 0 \tag{21}$$

$$g_5(x) = P - P_c(x) \le 0 \tag{22}$$

$$0.125 \le x_1 \tag{23}$$

$$0.1 \le x_2, x_3 \le 10 \tag{24}$$

$$0.20.1 \le x_4 \le 5 \tag{25}$$

Where:

$$\tau(x) = \sqrt{(\tau')^2 + 2\tau'\tau''\frac{x_2}{2R} + (\tau'')^2} \tag{26}$$

$$\tau' = \frac{P}{\sqrt{2x_1x_2}} \tag{27}$$

$$\tau'' = \frac{MR}{J} \tag{28}$$

$$M = P\left(L + \frac{x_2}{x}\right) \tag{29}$$

$$R = \sqrt{\frac{x_2^2}{4} + \left(\frac{x_1+x_3}{2}\right)^2} \tag{30}$$

$$J = \left\{2\sqrt{2}x_1x_2\left[\frac{x_2^2}{4} + \left(\frac{x_1+x_2}{2}\right)^2\right]\right\} \tag{31}$$

$$\sigma(x) = \frac{6PL}{x_4x_3^2} \tag{32}$$

$$\delta(x) = \frac{6PL}{x_3^2x_4} \tag{33}$$

Table 4: Optimal results for the welded beam design

Methods	Optimal design variables (x)				Objective function value f (x)
	h	l	t	b	
GA-Based Method (Coello, 2000a)	N/A	N/A	N/A	N/A	1.8245
GA-Based Method (Coello, 2000b)	0.2088	3.4205	8.9975	0.21	1.7483
GA-Based Method (Deb, 2000)	N/A	N/A	N/A	N/A	2.38
GA-Based Methods (Deb, 1991)	0.2489	6.173	8.1789	0.2533	2.4328
APPROX (Ragstell and Phillips, 1976)	0.2444	6.2189	8.2915	0.2444	2.3815
DAVID (Ragstell and Phillips, 1976)	0.2434	6.2552	8.2915	0.2444	2.3841
RANDOM (Ragstell and Phillips, 1976)	0.4575	4.7313	5.0853	0.66	4.1185
SIMPLEX (Ragstell and Phillips, 1976)	0.2792	5.6256	7.7512	0.2796	2.5307
Harmony Search Method (Lee and Geem, 2005)	0.2442	6.2231	8.2915	0.2443	2.3807
Improved harmony search algorithm (Mahdavi *et al*., 2007)	0.20573	3.47049	9.03662	0.20573	1.72485
Present study	0.2057296	3.4704889	9.0366244	0.2057296	1.7248522

$$P_c(x) = \frac{4.013E\sqrt{x_3^2 x_4^6/36}}{L^2}\left(1 - \frac{x_3}{2L}\sqrt{\frac{E}{4G}}\right) \quad (34)$$

Where:

P = 6000 lb
L = 14 inch.
δ_{max} = 0.25 inch.
E = 30×10^6 psi
G = 12×10^6 psi
τ_{max} = 13,600 psi
σ_{max} = 30,000 psi

The AWDA obtained optimum result after 150,000 function evaluations. The results are tabulated in Table 4. AWDA is better than the reported results except improved harmony search algorithm.

CONCLUSION

This study introduced a new meta-heuristic optimization algorithm, AWDA to solve engineering optimization problems. The African wild dog algorithm mimics the communal hunting behavior of African wild dogs. The AWDA, code was tested in several benchmark optimization problems taken from literature. The optimization results indicate that AWDA is efficient than other metaheuristic algorithms such as GA and HS.

REFERENCES

Arora, J.S., 1989. Introduction to Optimum Design. McGraw-Hill International, New York.

Belegundu, A.D., 1982. A study of mathematical programming methods for structural optimization. Ph.D. Thesis, Department of Civil and Environmental Engineering, University of Iowa, Iowa

Coello, C.A.C., 2000a. Constraint-handling using an evolutionary multiobjective optimization technique. Civ. Eng. Envir. Syst., 17: 319-346.

Coello, C.A., 2000b. Use of a self-adaptive penalty approach for engineering optimization problems. Comput. Ind., 41: 113-127.

Creel, S. and N.M. Creel, 1995. Communal hunting and pack size in African wild dogs, *Lycaon pictus*. Anim. Behav., 50: 1325-1339.

Deb, K., 1991. Optimal design of a welded beam via genetic algorithms. AIAA J., 29: 2013-2015.

Deb. K., 2000. An efficient constraint handling method for genetic algorithms. Comput. Methods Applied Mech. Eng., 186: 311-338.

Dorigo, M., V. Maniezzo and A. Colorni, 1996. Ant system: Optimization by a colony of cooperating agents. IEEE Trans. Syst. Cybern. Part B: Cybern., 26: 29-41.

Eberhart, R.C. and J. Kennedy, 1995. A new optimizer using particle swarm theory. Proceedings of the 6th International Symposium on Micro Machine and Human Science, October 4-6, 1995, Nagoya, Japan, pp: 39-43.

Erol, O.K. and I. Eksin, 2006. A new optimization method: Big Bang-Big Crunch. Adv. Eng. Software, 37: 106-111.

Gandomi, A.H., X.S. Yang and A.H. Alavi, 2011. Cuckoo search algorithm: A metaheuristic approach to solve structural optimization problems. Eng. Comput. 10.1007/s00366-011-0241-y.

Geem, Z.W., J.H. Kim and G.V. Loganathan, 2001. A new heuristic optimization algorithm: Harmony search. Simulation, 76: 60-68.

Glover, F., 1977. Heuristics for integer programming using surrogate constraints. Decis. Sci., 8: 156-166.

Goldberg, D.E., 1989. Genetic Algorithms in Search Optimization and Machine Learning. Addison-Wesley, USA.

Goldstein, A.A. and J.F. Price, 1971. On descent from local minima. Math. Comput., 25: 569-574.

Kaveh, A. and S. Talatahari, 2010. A novel heuristic optimization method: charged system search. Acta Mech., 213: 267-289.

Kirkpatrick, S., 1984. Optimization by simulated annealing: Quantitative studies. J. Stat. Phys., 34: 975-986.

Krishnanand, K.N. and D. Ghose, 2006. Glowworm swarm based optimization algorithm for multimodal functions with collective robotics applications. Multiagent Grid Syst., 2: 209-222.

Lee, K. and Z. Geem, 2005. A new meta-heuristic algorithm for continuous engineering optimization: harmony search theory and practice. Comput. Methods Applied Mechanics Eng., 194: 3902-3933.

Mahdavi, M., M. Fesanghari and E. Damangir, 2007. An improved harmony search algorithm for solving optimization problems. Applied Math. Comput., 188: 1567-1579.

Oftadeh, R., M.J. Mahjoob and M. Shariatpanahi, 2010. A novel meta-heuristic optimization algorithm inspired by group hunting of animals: Hunting search. Comput. Math. Appl., 60: 2087-2098.

Ragsdell, K.M. and D.T. Phillips, 1976. Optimal design of a class of welded structures using geometric programming. J. Eng. Ind., 98: 1021-1025.

Sandgren, E., 1990. Nonlinear integer and discrete programming in mechanical design optimization. J. Mech. Des., 112: 223-229.

Wu, S.J. and P.T. Chow, 1995. Genetic algorithms for nonlinear mixed discrete-integer optimization problems via meta-genetic parameter optimization. Eng. Optim., 24: 137-159.

Yang, X.S., 2009. Firefly Algorithms for multimodal optimization. Lecture Notes Comput. Sci., 5792: 169-178.

Yang, X.S., 2010. A new metaheuristic bat-inspired algorithm. In: In: Nature Inspired Cooperative Strategies for Optimization Studies in Computational Intelligence J.R. Gonzalez (Eds.)., Springer Berlin, 284 Springer pp: 65-74.

Yang, X.S., 2011. Nature-Inspired Metaheuristic Algorithms. 2nd Edn., Luniver Press, UK., ISBN: 9781905986286, Pages: 160.