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Intuitionistic Fuzzy Goal Geometric Programming Problem (IFG^2P^2) Based on Geometric Mean Method

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Abstract: In this study, a new approach of intuitionistic fuzzy goal programming is proposed. A non-linear intuitionistic fuzzy goal programming is solved here using geometric programming technique. Researchers have applied the proposed method on industrial waste water treatment design problem. Also, there is a comparison of results on industrial waste water treatment design problem with other methods.

Key words: Goal programming, geometric programming, non-linear optimization, industrial waste, fuzzy goal

INTRODUCTION

Intuitionistic Fuzzy Set (IFS) introduced by Atanassov (1986) is a growing field of research in different directions. Nowaday's goal programming in fuzzy environment is common whereas intuitionistic fuzzy goal programming is rare. In this study, researchers have worked on intuitionistic fuzzy goal programming problem where equations are non-linear. Geometric programming gives better result than non-linear programming (K-K-T conditions) which is already described by Ghosh and Roy (2012, 2013). Therefore, geometric programming is used here to solve non-linear goal programming problem. Nan and Li (2013), Ghosh and Roy (communicated) discussed arithmetic mean in intuitionistic fuzzy environment. In this study, researchers have used geometric mean in intuitionistic fuzzy goal programming. A numerical example and an application on industrial waste water treatment design problem is taken here as an illustration. Previously, Shih and Krishnan (1969), Evenson *et al*. (1969), Ecker and McNamara (1971) and Beightler and Philips (1976) have illustrated this design using dynamic programming, geometric programming. Later, Cao (2002) has discussed the same using fuzzy geometric programming. In this study, researchers have compared the results of industrial waste water treatment design problem with the results in another method.

FUZZY GOAL GEOMETRIC PROGRAMMING PROBLEM (FG^2P^2)

Find:
$$X=(x_1, x_2,..., x_q)^T \tag{1}$$

so as to $\widetilde{\text{Minimize}}\, f_{j0}(X)$ with target value C_{j0}, acceptance tolerance a_{j0}. subject to:

$$f_r(X) \le C_r,\ r=1,2,...,m,\ X=(x_1, x_2,..., x_q)^T > 0$$

Membership functions can be written as follows:

$$\mu_{f_{j0}}(X)=\begin{cases}1 & f_{j0}(X)\le C_{j0}\\ 1-\dfrac{f_{j0}(X)-C_{j0}}{a_{j0}} & C_{j0}\le f_{j0}(X)\le C_{j0}+a_{j0}\\ 0 & f_{j0}(X)\ge C_{j0}+a_{j0}\end{cases}$$

Hence, the crisp programming from fuzzy goal programming is:

$$\text{Maximize } \mu_{f_{j0}}(X) \tag{2}$$

Subject to:

$$0 \le \mu_{f_{j0}}(X) \le 1$$

$$f_r(x) \le C_r,\ r=1, 2,..., m,\ X=(x_1, x_2, ..., x_q)^T > 0$$

Equation 2 can be written as:

$$\begin{aligned}&\text{Maximize } \alpha \text{ subject to } \mu_{f_{j0}}(X)\ge\alpha\\ &f_r(X)\le C_r,\ r=1, 2, ..., m\\ &0\le\alpha\le 1,\ X=(x_1, x_2, ..., x_q)^T>0\end{aligned} \tag{3}$$

This is equivalent to the following geometric programming problem:

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$$\text{Maximize } \alpha^{-1} \text{ subject to } \frac{f_{j0}(X)}{a_{j0}(1-\alpha)+C_{j0}} \leq 1$$
$$\frac{f_r(X)}{C_r} \leq 1, \; r=1, 2,..., m, \; 0 \leq \alpha \leq 1, \quad (4)$$
$$X=(x_1, x_2,...,x_q)^T > 0$$

INTUITIONISTIC FUZZY GOAL GEOMETRIC PROGRAMMING PROBLEM (I FG²P2)

Find:

$$X=(x_1, x_2, ..., x_q)^T \quad (5)$$

so as to $\widetilde{\text{Minimize}}^i \, f_{j0}(X)$ with target value C_{j0}, acceptance tolerance a_{j0} rejection tolerance b_{j0}. Subject to:

$$f_r(X) \leq C_r, r = 1, 2,...,m, \; X = (x_1, x_2, ..., x_q)^T > 0$$

Membership and non-membership functions are (Fig. 1):

$$\mu_{f_{j0}}(X) = \begin{cases} 1 & f_{j0}(X) \leq C_{j0} \\ 1-\dfrac{f_{j0}(X)-C_{j0}}{a_{j0}} & C_{j0} \leq f_{j0}(X) \leq C_{j0}+a_{j0} \\ 0 & f_{j0}(X) \geq C_{j0}+a_{j0} \end{cases}$$

$$\vartheta_{f_{j0}}(X) = \begin{cases} 0 & f_{j0}(X) \leq C_{j0} \\ \dfrac{f_{j0}(X)-C_{j0}}{b_{j0}} & C_{j0} \leq f_{j0}(X) \leq C_{j0}+b_{j0} \\ 1 & f_{j0}(X) \geq C_{j0}+b_{j0} \end{cases}$$

Intuitionistic fuzzy goal programming can be transformed into crisp programming model using membership and non-membership function as:

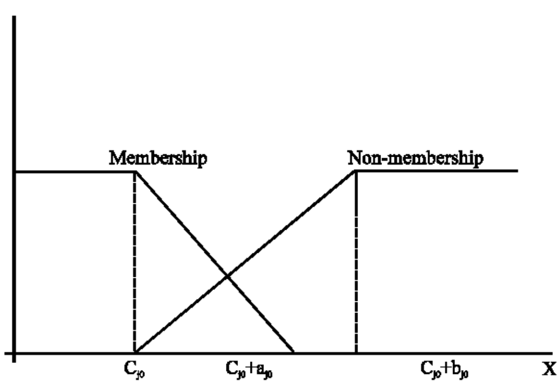

Fig. 1: Membership and non-membership function

$$\text{Maximum } \mu_{f_{j0}}(X), \text{ Minimize } \vartheta_{f_{j0}}(X)$$
$$\text{Subject to } 0 \leq \mu_{f_{j0}}(X) + \vartheta_{f_{j0}}(X) \leq 1$$
$$f_r(X) \leq C_r, \; r = 1, 2, ..., m \quad (6)$$
$$0 \leq \mu_{f_{j0}}(X), \vartheta_{f_{j0}}(X) \leq 1,$$
$$X=(x_1, x_2,...,x_q)^T > 0$$

This is equivalent to:

$$\text{Maximize } \alpha, \text{ Minimize } \beta$$
$$\text{Subject to } \mu_{f_{j0}}(X) \geq \alpha, \; \vartheta_{f_{j0}}(X) \leq \beta$$
$$f_r(X) \leq C_r, \; r = 1, 2,...,m \quad (7)$$
$$0 \leq \alpha+\beta \leq 1, \; 0 \leq \alpha, \; \beta \leq 1,$$
$$X = (x_1, x_2,..., x_q)^T > 0$$

It is easily seen that Max α is equivalent to Min $(1-\alpha)$ as $0 \leq \alpha \leq 1$. Taking geometric mean, the Eq. 7 can be written as:

$$\text{Minimize } \beta(1-\alpha)$$
$$\text{subject to } f_{j0}(X) \leq a_{j0} \times b_{j0} \, \beta(1-\alpha)+C_{j0}$$
$$f_r(X) \leq C_r, \; r = 1, 2, ..., m \quad (8)$$
$$0 \leq \alpha+\beta \leq 1, \; 0 \leq \alpha, \; \beta \leq 1,$$
$$X=(x_1, x_2, ..., x_q)^T > 0$$

Let us take $\beta(1-\alpha) = v>0$ then the above model becomes:

$$\text{Minimize } v$$
$$\text{subject to } \frac{f_{j0}(X)}{a_{j0} \times b_{j0} v + C_{j0}} \leq 1$$
$$\frac{f_r(X)}{C_r} \leq 1, \; r = 1, 2,..., m \quad (9)$$
$$0 \leq \alpha+\beta \leq 1, \; 0 \leq \alpha, \; \beta \leq 1,$$
$$v \in (0, 1), \; X=(x_1, x_2,..., x_q)^T > 0$$

The Eq. 9 is solved by geometric programming technique with v as parameter.

INDUSTRIAL WASTEWATER TREATMENT DESIGN

To optimize the treatment of industrial wastewater, a process flow from a paper and pulp industry has been considered. The treatment units indicate the removal of suspended solid and Biological Oxygen Demand (BOD) from the waste water. In this study, the design of treatment facilities is based on effluent containing BOD and essentially free from suspended solids. Wastewater treatment is consisted of primary clarification, secondary biological treatment (trickling filter followed by activated sludge or aerated lagoon), sludge disposal and tertiary treatment (coagulation, sedimentation, filtration for effluent of activated sludge and aerated lagoon; carbon adsorption for effluent of aerated lagoon). There are many combination of wastewater treatment process given in Table 1 (Beightler and Philips, 1976) to remove 5 days BOD (BOD_5).

In the study, researchers have taken the first design. There are consecutively four processes (primary clarifier, trickling filter, activated sludge and carbon adsorption):

Primary clarifier→Trickling filter↓
Carbon adsorption←Activated sludge

Let x_i be the percentage of remaining BOD_5 after each step. Then, after four processes the remaining percentage of BOD_5 will be x_1, x_2, x_3, x_4. The aim is to minimize the remaining percentage of BOD_5 with minimum annual cost as much as possible. The annual cost of BOD_5 removal by various treatments is shown in Table 2.

FUZZY GOAL GEOMETRIC PROGRAMMING PROBLEM (FG^2P^2)

Decision maker wants to remove about 98.5% $BODS_5$ and gives some relaxation 0.1 on this goal. Also, he sets another goal as annual cost should be about 300 (thousand \$) and gives flexibility 200 (thousand \$) on this goal. Then, the fuzzy goal programming problem is:

$$\widetilde{\text{Minimize}}\ f_1(x_1,x_2,x_3,x_4) = 19.4x_1^{-1.47} + 16.8x_2^{-1.66} + 91.5x_3^{-0.3} + 120x_4^{-0.33}$$

with target 300, acceptance tolerance 200.

$$\widetilde{\text{Minimize}}\ f_2(x_1,x_2,x_3,x_4) = x_1x_2x_3x_4$$

with target 0.015, acceptance tolerance 0.1, subject to x_1, x_2, x_3, x_4>0. Membership and non-membership functions are given:

$$\mu_{f_1}(x_1,x_2,x_3,x_4) = \begin{cases} 1 & f_1(x_1,x_2,x_3,x_4) \le 300 \\ 1-\dfrac{f_1(x_1,x_2,x_3,x_4)-300}{200} & 300 \le f_1(x_1,x_2,x_3,x_4) \le 500 \\ 0 & f_1(x_1,x_2,x_3,x_4) \ge 500 \end{cases}$$

$$\mu_{f_2}(x_1,x_2,x_3,x_4) = \begin{cases} 1 & f_2(x_1,x_2,x_3,x_4) \le 0.015 \\ 1-\dfrac{f_2(x_1,x_2,x_3,x_4)-0.015}{0.1} & 0.015 \le f_2(x_1,x_2,x_3,x_4) \le 0.115 \\ 0 & f_2(x_1,x_2,x_3,x_4 \ge 0.115 \end{cases}$$

Following Eq. 2-4, above equation can be written into Crisp programming problem as:

Table 1: Wastewater treatment design

Designs	Primary	Secondary	Tertiary
1	Primary clarifier	Trickling filter and activated sludge	Carbon adsorption
2	Primary clarifier	Trickling filter and aerated lagoon	Coagulation, sedimentation and filtration
3	Primary clarifier	Activated sludge	Carbon adsorption
4	Primary clarifier	Aerated lagoon	Coagulation, sedimentation and filtration
5	Primary clarifier	Trickling filter and activated sludge	Coagulation, sedimentation and filtration
6	Primary clarifier	Activated sludge	Coagulation, sedimentation and filtration
7	Primary clarifier	Activated sludge	None
8	Primary clarifier	Trickling filter and activated sludge	None
9	Primary clarifier	Aerated lagoon	None
10	Primary clarifier	Trickling filter	None

Table 2: List of annual costs in different treatments

Designs	Treatment	Annual cost
1	Primary clarifier	$19.4x_1^{-1.47}$
2	Trickling filter	$16.8x_2^{-1.66}$
3	Activated sludge	$91.5x_3^{-0.3}$
4	Carbon adsorption	$120x_4^{-0.33}$

Table 3: Optimal values of decision variables and objective functions of Eq. 9

Dual variables	Primal variables	Optimal objective functions	Membership and non-membership functions
$\delta_{01}^* = 1$	$x_1^* = 0.7059559$	$f_1^*(x_1, x_2, x_3, x_4) = 363.8048$	$\mu_{f_1} = (x_1, x_2, x_3, x_4) = 0.680976$
$\delta_{11}^* = 0.088967278$	$x_2^* = 0.7248393$	$f_2^*(x_1, x_2, x_3, x_4) = 0.0469024$	$\mu_{f_2} = (x_1, x_2, x_3, x_4) = 0.680976$
$\delta_{12}^* = 0.078784276$	$x_3^* = 0.1598653$		
$\delta_{13}^* = 0.435939662$	$x_4^* = 0.5733523$		
$\delta_{14}^* = 0.396308784$			
$\delta_{21}^* = 0.130781899$			

$$\begin{aligned}
&\text{Minimize } \alpha^{-1}\\
&\text{Subject to } 19.4x_1^{-1.47}+16.8x_2^{-1.66}+91.5x_3^{-0.3}+\\
&\qquad 120x_4^{-0.33} \le 200(1\text{-}\alpha)+300\\
&x_1x_2x_3x_4 \le 0.1(1-\alpha)+0.015,\\
&x_1, x_2, x_3, x_4 > 0,\ \alpha \in (0, 1)
\end{aligned} \tag{10}$$

Solving it using Cao's Geometric Programming Method taking α as a parameter where degree of difficulty is 5-(4+1) = 0, researchers have the results given in Table 3. The Table 3 shows that here 100−0.0469024×100 = 95.30976% BOD_5 removes with the cost of 363.8048 (thousand \$).

INTUITIONISTIC FUZZY GOAL GEOMETRIC PROGRAMMING PROBLEM (I FG^{2}P^2)

Let decision maker wants to remove about 98.5% BOD_5 and the tolerances of acceptance and rejection on this goal are 0.1 and 0.2, respectively. Also, he wants to remove the said amount of BOD_5 within 300 (thousand) \$ tolerances of acceptance and rejection on this goal are 200 and 300, respectively. Hence, the intuitionistic fuzzy goal programming problem is:

$$\widetilde{\text{Minimize}}^i\ f_1(x_1, x_2, x_3, x_4) = 19.4x_1^{-1.47}+16.8x_2^{-1.66}+91.5x_3^{-0.3}+120x_4^{-0.33}$$

with target 300, acceptance tolerance 200 and rejection tolerance 300:

$$\widetilde{\text{Minimize}}^i\ f_2(x_1, x_2, x_3, x_4) = x_1x_2x_3x_4$$

with target 0.015, acceptance tolerance 0.1 and rejection tolerance 0.2 subject to x_1, x_2, x_3, x_4>0. Membership and non-membership functions are given:

$$\mu_{f_1}(x_1,x_2,x_3,x_4) = \begin{cases} 1 & f_1(x_1,x_2,x_3,x_4) \le 300 \\ 1\text{-}\dfrac{f_1(x_1,x_2,x_3,x_4)\text{-}300}{200} & 300 \le f_1(x_1,x_2,x_3,x_4) \le 500 \\ 0 & f_1(x_1,x_2,x_3,x_4 \ge 500 \end{cases}$$

$$\vartheta_{f_1}(x_1,x_2,x_3,x_4) = \begin{cases} 0 & f_1(x_1,x_2,x_3,x_4) \le 300 \\ \dfrac{f_1(x_1,x_2,x_3,x_4)\text{-}300}{300} & 300 \le f_1(x_1,x_2,x_3,x_4) \le 600 \\ 1 & f_1(x_1,x_2,x_3,x_4 \ge 600 \end{cases}$$

$$\mu_{f_2}(x_1,x_2,x_3,x_4) = \begin{cases} 1 & f_2(x_1,x_2,x_3,x_4) \le 0.015 \\ 1\text{-}\dfrac{f_2(x_1,x_2,x_3,x_4)\text{-}0.015}{0.1} & 0.015 \le f_2(x_1,x_2,x_3,x_4) \le 0.115 \\ 0 & f_2(x_1,x_2,x_3,x_4 \ge 0.115 \end{cases}$$

Table 4: Optimal values of decision variables and objective functions of model (11)

Dual variables	Primal variables	Optimal objective functions	Membership and non-membership functions
$\delta_{01}^* = 1$	$x_1^* = 0.6380199$	$f_1^*(x_1, x_2, x_3, x_4) = 422.1483$	$\mu_{f_1} = (x_1, x_2, x_3, x_4) = 0.3892583$
$\delta_{11}^* = 0.088967278$	$x_2^* = 0.6627170$	$f_2^*(x_1, x_2, x_3, x_4) = 0.01504072$	$\vartheta_{f_1} = (x_1, x_2, x_3, x_4) = 0.4071612$
$\delta_{12}^* = 0.078784276$	$x_3^* = 0.09737155$		$\mu_{f_2} = (x_1, x_2, x_3, x_4) = 0.9995928$
$\delta_{13}^* = 0.435939662$	$x_4^* = 0.3653206$		$\vartheta_{f_2} = (x_1, x_2, x_3, x_4) = 0.00020358$
$\delta_{04}^* = 0.396308784$			
$\delta_{21}^* = 0.130781899$			

Table 5: Comparison of results using different methods

Methods	Total annual cost (thousand $)	Remaining BOD_5 in waste water	Removed BOD_5 (%)
FG^2P^2	363.8048	0.04690240	95.309760
IF G^2P^2 Arithmetic Mean (Ghosh and Roy, 2013)	359.2533	0.04919147	95.080853
IF G^2P^2 Geometric Mean	422.1483	0.01504072	98.495928

$$\vartheta_{f_2}(x_1,x_2,x_3,x_4) = \begin{cases} 0 & f_2(x_1,x_2,x_3,x_4) \le 0.015 \\ \dfrac{f_2(x_1,x_2,x_3,x_4)\text{-}0.015}{0.2} & 0.015 \le f_1(x_1,x_2,x_3,x_4) \le 0.215 \\ 1 & f_1(x_1,x_2,x_3,x_4 \ge 0.215 \end{cases}$$

Following Eq. 5-7, the Crisp programming problem is:

$$\begin{aligned} &\text{Minimize } v \\ &\text{subject to: } \frac{19.4x_1^{-1.47} + 16.8x_2^{-1.66} + 91.5x_3^{-0.3} + 120x_4^{-0.33}}{200\times 300v + 300} \le 1 \\ &\frac{x_1x_2x_3x_4}{0.1\times 0.2v + 0.015} \le 1,\ x_1,\ x_2,\ x_3,\ x_4 > 0,\ v \in (0, 1) \end{aligned} \tag{11}$$

Solving it using Cao's Geometric Programming Method having degree of difficulty 5-(4+1) = 0, researchers have the results given in Table 4.

Table 4 shows that membership and non-membership functions satisfy all the restrictions as in Eq. 6. The percentage of BOD_5 removed from the wastewater is $(100 - 0.01504072\times 100) = 98.495928\%$ which attains the set quota by the national standard and the annual total cost is 422.1483 (thousands $).

COMPARISON

Here is a comparison of results between other method and the proposed method (Table 5).

CONCLUSION

Researchers have applied fuzzy and intuitionistic fuzzy goal geometric programming on industrial waste water treatment design. Researchers have compared the results of various methods on industrial waste water treatment design. We have seen that in fuzzy goal geometric programming and intuitionistic fuzzy goal geometric programming with arithmetic mean, percentage of BOD_5 removal is almost same. But $4551.5 is saved in intuitionistic fuzzy goal geometric programming with arithmetic mean. In the proposed method, intuitionistic fuzzy goal geometric programming with geometric mean, 98.495928% BOD_5 is removed. Hence, for better purification intuitionistic fuzzy goal geometric programming with geometric mean is more appropriate.

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REFERENCES

Atanassov, K.T., 1986. Intuitionistic fuzzy sets. Fuzzy Sets Syst., 20: 87-96.

Beightler, C.S. and D.T. Philips, 1976. Applied Geometric Programming. John Wiley and Sons, New York, ISBN: 13-9780471063902, Pages: 590.

Cao, B.Y., 2002. Fuzzy Geometric Programming. Vol. 76, Kluwer Academic Publishers, London, ISBN: 13-9781461500094.

Ecker, J.G. and J. R. McNamara, 1971. Geometric programing and the preliminary design of industrial waste treatment plants. Water Resour. Res., 7: 18-22.

Evenson, O.E., G.T. Orlab and J.R. Monser, 1969. Preliminary selection of waste treatment systems. J. Water Poll. Control Fed., 41: 18-45.

Ghosh, P. and T.K. Roy, 2012. Goal geometric programming problem with product method. Proceedings of the 6th International Conference on Mathematical Sciences for Advancement of Science and Technology, December 21-23, 2012, Kolkata, India, pp: 95-107.

Ghosh, P. and T.K. Roy, 2013. Goal geometric programming problem (G^2P^2) with crisp and imprecise targets. J. Global Res. Comput. Sci. Vol. 4, No. 8.

Nan, J.X. and D.F. Li, 2013. Linear programming approach to matrix games with intuitionistic fuzzy goals. Int. J. Comput. Intell. Syst., 6: 186-197.

Shih, C. and P. Krishnan, 1969. Dynamic optimization for industrial waste treatment design. J. Water Poll. Control Fed., 41: 1787-1802.