

Hardware Complexity Reduction of Parallel FIR Filter Structures Based on Fast FIR Algorithm

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Abstract: The main objective of the study is to reduce the hardware cost considerably. Parallel Finite Impulse Response (FIR) filters can be implemented with less hardware cost at different level of parallelism using Fast Symmetric Convolution algorithm. Multiplications are major part in FIR filter implementation. The number of required multipliers is reduced by effectively cascading the short length FIR filter using symmetric coefficients in the sub-filter section. The reduction of multipliers and adders are more advantages in terms of silicon area. For example, the proposed structure of a four-parallel 144-tap FIR filter saves 33 multipliers and 7 adders whereas for a eight-parallel 576-tap FIR filter saves 351 multipliers and 11 adders. Finally, when the length of filter is large, the proposed parallel FIR filter structure saves the hardware cost in terms of multiplications in both odd and even length.

Key words: Hardware complexity, symmetric coefficients, Finite Impulse Response (FIR), sub-filter, odd

INTRODUCTION

Finite impulse response filters are the most important blocks in digital signal processing systems. These filters can be used for wide range of applications. In generally, parallel FIR digital filters can be used for either low power or high speed applications. Researchers have proposed various structures to improve the performance of FIR filter in terms of area and complexity. In the proposed structure symmetric convolution plays an important role in fast FIR filtering. The primary object is to increase symmetrical coefficient based sub filters used in fast fir filtering.

The major research topic in the last few decades is to reduce the complexity of the parallel FIR filtering. The polyphase decomposition is a technique used to design parallel FIR filters (Chung and Parhi, 2002). Using this technique, Fast Filtering algorithms for smaller block sizes are developed. Researchers have designed a large block sizes by cascading small sized parallel FIR filtering blocks and then effectively utilize the sub-filters to reduce the number of multiplications used in parallel FIR filtering (Parker and Parhi, 1997; Chung and Parhi, 2002; Parhi, 1999; Mou and Duhamel, 1991; Acha, 1989; Cheng and Parhi, 2004, 2005, 2007; Lin and Mitra, 1996; Synopsys Inc., 2008; Blahut, 1985; Nussbaumer, 1981; Lim and Liu, 1988; Kwan and Tsim, 1987; Lim and Liu, 1988). Fast FIR algorithm of L-parallel filter and (2L-1) sub-filter blocks each of length N/L was introduced by

Parker and Parhi (1997), Chung and Parhi (2002) and Parhi (1999) when the block size L increases the hardware implementation cost also increases along with that.

In FFA required number of multipliers ($L \times N$) has been reduced to $(2N-N/L)$ (Cheng and Parhi, 2005). An Iterated Short Convolution algorithm saves hardware cost when the length of the FIR filter is large (Cheng and Parhi, 2004). Here, researchers have designed a new parallel FIR filter structure of symmetric convolution based even and odd length can further reduce the amounts of multipliers required in the sub filter blocks (Tsao and Choi, 2010, 2012).

FAST FIR FILTER ALGORITHM

Generally, the N-tap FIR filter which can be expressed as:

$$y(n) = \sum_{i=0}^{N-1} h(i)x(n-i) \quad (1)$$
$$n = 0, 1, 2, \dots, \infty$$

where, $\{h(n)\}$ having FIR filter coefficients of length N and $\{x(n)\}$ is an infinite length input sequence. The polyphase decomposition can be derived from L-parallel FIR filters by decomposing $X(Z)$, $Y(Z)$ and $H(Z)$ into L subsequences as:

$$Y(Z) = X(Z)H(Z)$$

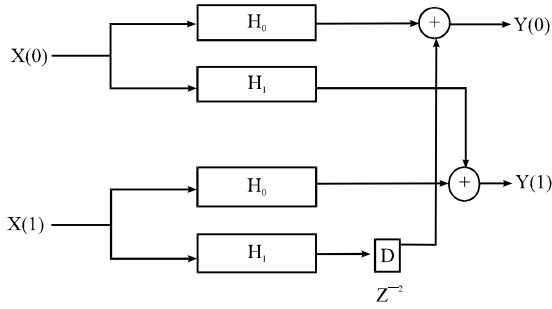


Fig. 1: Traditional two parallel FIR filter structure

$$\sum_{k=0}^{L-1} Y_k(Z^L)Z^{-k} = \sum_{i=0}^{L-1} X_i(Z^L)Z^{-i} \sum_{j=0}^{L-1} H_j(Z^L)Z^{-j} \quad (2)$$

For i, j and $k = 0, 1, 2, \dots, L-1$

From this equation, L-parallel FIR filter needs L^2 sub-filter operations each of its length N/L and requires N/L multiply and addition operations. So, the L-parallel FIR filter needs LN multiply and addition operations which is always linear with block size (Blahut, 1985):

$$Y_0 + Z^{-1}Y_1 = (X_0 + Z^{-1}X_1)(H_0 + Z^{-1}H_1) \quad (3)$$

$$= H_0X_0 + Z^{-2}H_1X_1 + Z^{-1}(H_0X_1 + H_1X_0)$$

So that:

$$Y_0 = H_0X_0 + Z^{-2}H_1X_1 \quad (4)$$

$$Y_1 = H_0X_1 + H_1X_0$$

The traditional two parallel FIR filter implementation was obtained from polyphase decomposition as shown in Fig. 1. This filter structure requires $2N$ multiplications and $2N-2$ additions. Equation 4 can be rewritten as:

$$Y_0 = H_0X_0 + Z^{-2}H_1X_1 \quad (5)$$

$$Y_1 = (H_0 + H_1)(X_0 + X_1) - H_0X_0 - H_1X_1$$

which is shown in Fig. 2. The existing two parallel Fast FIR filter structure as shown in Fig. 3 was obtained from Eq. 6:

$$Y_0 = \left\{ \frac{1}{2}[(H_0 + H_1)(X_0 + X_1) + (H_0 - H_1)(X_0 - X_1)] - H_1X_1 \right\} + Z^{-2}H_1X_1$$

$$Y_1 = \frac{1}{2}[(H_0 + H_1)(X_0 + X_1) - (H_0 - H_1)(X_0 - X_1)] \quad (6)$$

Similarly, traditional and existing three parallel fast FIR filter implementation was obtained from Eq. 7 and 8, respectively:

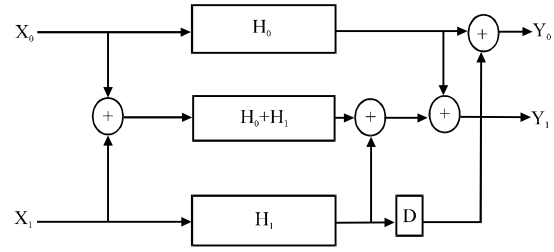


Fig. 2: Two parallel fast FIR filter structure

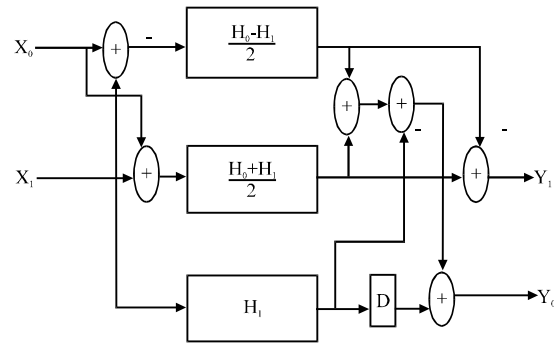


Fig. 3: Existing two parallel fast FIR filter structure

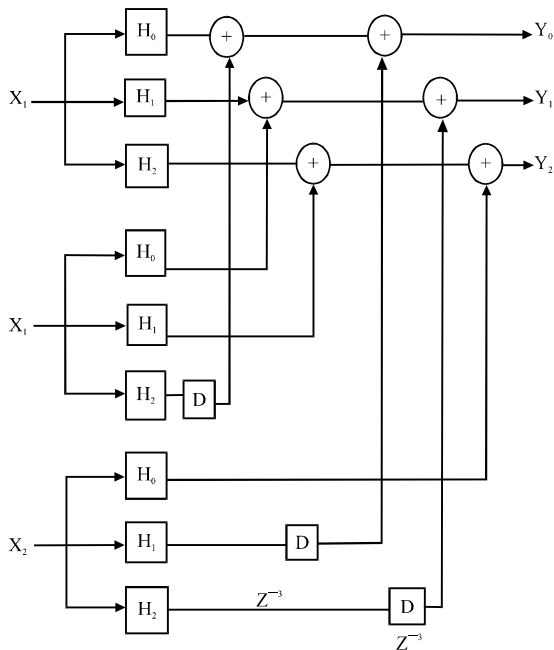


Fig. 4: Traditional three parallel fast FIR filter structure

$$Y_0 = H_0X_0 + Z^{-3}H_2X_1 + Z^{-3}H_1X_2$$

$$Y_1 = H_1X_0 + H_0X_1 + Z^{-3}H_2X_2 \quad (7)$$

$$Y_2 = H_2X_0 + H_1X_1 + H_0X_2$$

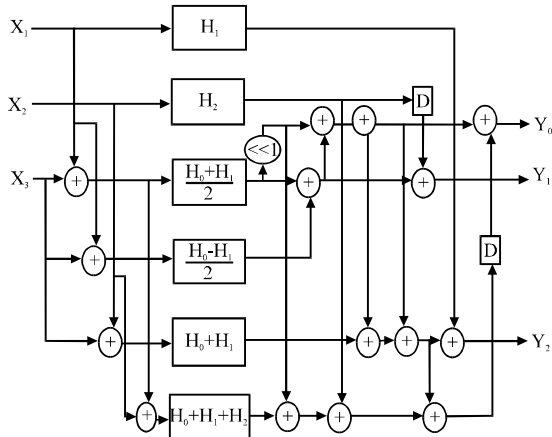


Fig. 5: Existing three parallel fast FIR filter structure

This implementation as shown in Fig. 4 and 5:

$$\begin{aligned}
 Y_0 &= (H_0+H_1)(X_0+X_1) - \frac{1}{2}[(H_0+H_1)(X_0+X_1) - \\
 &\quad (H_0-H_1)(X_0-X_1)] - H_1X_1 + Z^{-3}\{(H_0+H_1+H_2) \\
 &\quad (X_0+X_1+X_2) - (H_0+H_1)(X_0+X_1) - [(H_0+H_2) \\
 &\quad (X_0+X_2) - \{(H_0+H_1)(X_0+X_1) - \frac{1}{2}[(H_0+H_1) \\
 &\quad (X_0+X_1) - (H_0-H_1)(X_0-X_1)] - H_1X_1\} - H_2X_2\} - H_2X_2\} \\
 Y_1 &= \frac{1}{2}[(H_0+H_1)(X_0+X_1) - (H_0-H_1)(X_0-X_1)] + Z^{-3}H_2X_2 \\
 Y_2 &= H_1X_1 + \{(H_0+H_2)(X_0+X_2) - \{(H_0+H_1)(X_0+X_1) - \\
 &\quad \frac{1}{2}[(H_0+H_1)(X_0+X_1) - (H_0-H_1)(X_0-X_1)] - H_1X_1\} - H_2X_2\}
 \end{aligned} \tag{8}$$

IMPROVED FAST PARALLEL FIR FILTER STRUCTURES

From the Eq. 6, a two parallel FIR filter can be rewritten as:

$$\begin{aligned}
 Y_0 &= H_0X_0 + Z^{-2} \left\{ \frac{1}{2}[(H_0+H_1)(X_0+X_1) + \right. \\
 &\quad \left. (H_0-H_1)(X_0-X_1)] - H_0X_0 \right\} \\
 Y_1 &= \frac{1}{2}[(H_0+H_1)(X_0+X_1) - (H_0-H_1)(X_0-X_1)]
 \end{aligned} \tag{9}$$

is shown in Fig. 6. This two parallel FFA can be cascaded with an another two parallel FFA to produce four parallel FFA as shown in study. Similarly, six and eight parallel FFA structures can be obtained. From Eq. 8, the three parallel FIR filter can be rewritten as:

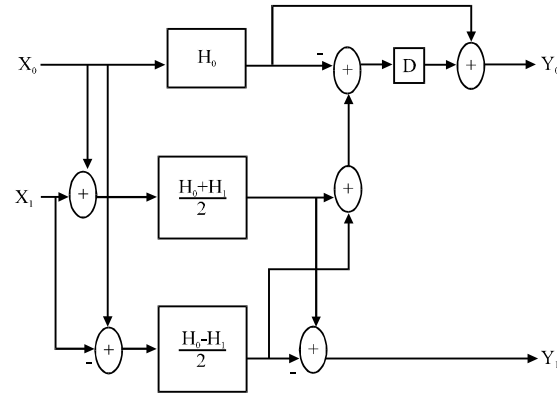


Fig. 6: Proposed two parallel fast FIR filter structure

$$\begin{aligned}
 Y_0 &= \frac{1}{2}[(H_0+H_1)(X_0+X_1) + (H_0-H_1)(X_0-X_1) - H_1X_1] - \\
 &\quad Z^{-3} \left\{ \frac{1}{2}[(H_0+H_1)(X_0+X_1) - (H_0-H_1)(X_0-X_1)] + H_1X_1 + \right. \\
 &\quad \left. (H_0+H_2)(X_0+X_2) - (H_0+H_1+H_2)(X_0+X_1+X_2) \right\} \\
 Y_1 &= \frac{1}{2}[(H_0+H_1)(X_0+X_1) - (H_0-H_1)(X_0-X_1)] - \\
 &\quad Z^{-3} \left\{ \frac{1}{2}[(H_0+H_1)(X_0+X_1) + (H_0-H_1)(X_0-X_1)] - \right. \\
 &\quad \left. \frac{1}{2}[(H_0+H_2)(X_0+X_2) + (H_0-H_2)(X_0-X_2)] - H_1X_1 \right\} \\
 Y_2 &= \frac{1}{2}[(H_0+H_2)(X_0+X_2) - (H_0-H_2)(X_0-X_2)] + H_1X_1
 \end{aligned} \tag{10}$$

The implementation as shown in Fig. 7. The proposed four parallel fast FIR filter is shown in Fig. 8. All the six sub-filter blocks contain symmetric coefficients in this proposed structure. Hence, the number of multiplications used in this structure have been reduced significantly.

Example 1: Consider a 21-tap FIR filter with a set of symmetric coefficients can be applied to the proposed 3 parallel FIR filter:

$$\{h(0), h(1), h(2), \dots, h(20)\}$$

Where:

$$\begin{aligned}
 h(0) &= h(20) \\
 h(1) &= h(19) \\
 h(2) &= h(18) \dots h(8) = h(11) \\
 h(9) &= h(10)
 \end{aligned}$$

from that researchers can obtain two more sub-filter blocks with symmetric coefficients as:

$$H_0 \pm H_2 = \{h(0) \pm h(2), h(1) \pm h(5), \dots, h(18) \pm h(20)\}$$

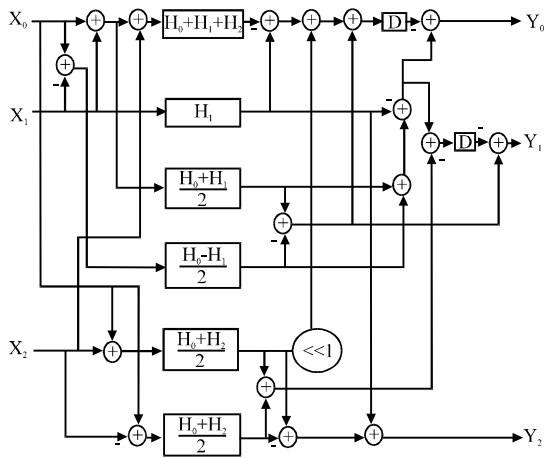


Fig. 7: Proposed three parallel fast FIR filter structure

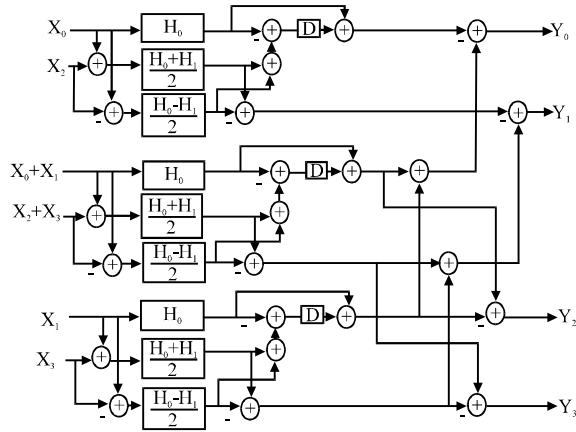


Fig. 8: Proposed four parallel fast FIR filter structure

Where as:

$$h(0) \pm h(2) = \pm h(18) \pm h(20)$$

$$h(3) \pm h(5) = \pm h(15) \pm h(17)$$

$$h(6) \pm h(8) = \pm h(12) \pm h(14)$$

The symmetric sub-filter blocks are $\{1/2(H_0+H_2), 1/2(H_0+H_1), H_0+H_1+H_2, H_1\}$.

Example 2: When researchers consider a 17-tap FIR filter the symmetric coefficients of sub-filter blocks are $1/2\{(H_0+H_1), (H_0+H_1)\}$.

HARDWARE COMPLEXITY COMPUTATION

Parallel FIR filters with long block sizes which can be designed by cascading smaller length fast parallel filters. For example, a P-Parallel FFA can be cascaded with a Q-Parallel FFA to produce an (P×Q) Parallel filtering structure for implementation. The number of required multiplications for an L-Parallel filter with $L = L_1L_2$. L_p is given by:

$$M = \frac{N}{\prod_{i=1}^p L_i} \prod_{i=1}^p M_i$$

Where:

p = The number of FFAs used L_i is the block size of the FFA at step p

M_i = The number of sub filter blocks that result from the i th FFA

N = The length of the filter

Based on the value of $\frac{N}{\prod_{i=1}^p L_i}$ is separated into two cases:

Case 1:

When:

$$\left\lceil \frac{N}{\prod_{i=1}^p L_i} \right\rceil$$

is even then:

$$M = \left\lceil \frac{N}{\prod_{i=1}^p L_i} \right\rceil \left(\prod_{i=1}^p M_i - \frac{S}{2} \right)$$

Case 2:

When:

$$\left\lceil \frac{N}{\prod_{i=1}^p L_i} \right\rceil$$

is odd then:

$$M = \left\lceil \frac{N}{\prod_{i=1}^p L_i} \right\rceil \prod_{i=1}^p M_i - \frac{S}{2} (Y)$$

$$Y = \left\lceil \frac{N}{\prod_{i=1}^p L_i} \right\rceil - 1$$

Where:

S = The number of sub-filter blocks containing symmetric coefficients

Y = The odd number which has to multiply with $S/2$

The number of the required adders in the sub-filters section can be calculated as follows:

$$A_{(sub-filter)} = \prod_{i=1}^p M_i \left(\left\lceil \frac{N}{\prod_{i=1}^p L_i} \right\rceil - 1 \right)$$

where, A is adder in the sub-filter block.

Table 1: Comparison of Proposed (P) short length FFA structures with Existing (E) works-number of required Multiplications/Additions (Mul./Add.)

Lengths	N-tap	Sub-filter		Saved		Mul.	Saved Mul.	
		Add.	Structure	Add.	Add.			
2	24	33	E.FFA	4	-2	30	6	
			P.FFA	6		24		
	72	105		E.FFA	4	-2	90	18
				P.FFA	6		72	
	144	213		E.FFA	4	-2	180	36
				P.FFA	6		144	
576	861		E.FFA	4	-2	720	144	
			P.FFA	6		576		
3	27	48		E.FFA	10	-7	46	8
				P.FFA	17		38	
	81	156		E.FFA	10	-7	136	26
				P.FFA	17		110	
	147	288		E.FFA	10	-7	246	48
				P.FFA	17		198	
591	1176		E.FFA	10	-7	986	196	
			P.FFA	17		790		

Table 2: Comparison of Proposed (P) cascading FFA structures with Existing (E) works-number of required Multiplications/Additions (Mul./Add.)

Lengths	N-tap	Sub-filter		Saved		Mul.	Saved Mul.	
		Add.	Structure	Add.	Add.			
6	27	72		E.FFA	53	-7	76	10
				P.FFA	60		66	
	81	234		E.FFA	53	-7	203	35
				P.FFA	60		168	
	147	432		E.FFA	53	-7	366	60
				P.FFA	60		306	
591	1764		E.FFA	53	-7	1439	245	
			P.FFA	60		1194		
4	24	45		E.FFA	31	7 (225%)	42	3
				P.FFA	24		39	
	72	153		E.FFA	31	7 (225%)	126	15
				P.FFA	24		111	
	144	315		E.FFA	31	7 (225%)	252	33
				P.FFA	24		219	
576	1287		E.FFA	31	7 (225%)	1008	141	
			P.FFA	24		867		
8	24	54		E.FFA	134	11 (8.2%)	73	10
				P.FFA	123		63	
	72	216		E.FFA	134	11 (8.2%)	211	40
				P.FFA	123		171	
	144	459		E.FFA	134	11 (8.2%)	414	81
				P.FFA	123		333	
576	1917		E.FFA	134	11 (8.2%)	1656	351	
			P.FFA	123		1305		

IMPLEMENTATION AND COMPLEXITY COMPARISON

The required number of multiplications and additions for a different N-tap filters at their different levels of parallelism are summarized. The number of saved multiplications are linearly increased with N-tap filters which used in FFA structure. Table 1 shows the comparison of proposed short length FFA structures against the existing one.

Table 2 shows the cascading structure results. This effective designing structure has reduced the hardware cost considerably. For example, four parallel filtering structure can save 2N/8 multiplications for

Table 3: Summary of proposed results for saving multiplications

N-TAP	L = 2 (%)	L = 4 (%)	L = 8 (%)	N-TAP	L = 3 (%)	L = 6 (%)
24	20	7.0	21.1	27	17.4	19.8
72	20	13.9	18.9	81	19.1	17.2
144	20	13.0	19.5	147	13.2	16.4
576	20	11.9	13.7	591	19.8	17.0

implementation. The required complexity of four parallel 72 and 144-tap FFA structures represent a hardware saving of nearly 13% when compared to the 3N/8 multiplications in the existing structure. The symmetric coefficients of proposed FFA structures are generated by MATLAB using Remez Exchange algorithm. Table 3 shows the summary of proposed structure for hardware saving in terms of percentage.

CONCLUSION

It is proposed a new area efficient fast parallel FIR filter structures which are beneficial to symmetric convolutions for both odd and even length filtering. This parallel FIR structure has been designed from traditional FIR filtering, linear and symmetric convolution based fast FIR algorithm. Parallel FIR filters with large block sizes have been obtained from cascading small length parallel filters. The number of multiplication and additions both can be considerably reduced by designing an efficient cascading structure for implementing the fast FIR filter. This structure contains many symmetric sub filters which reduces the overall hardware cost of the filter. So that, it is comparatively better performance than existing FFA structure. Finally, it has been proved that the hardware cost of parallel FIR filters can be reduced significantly.

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