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On Filtration of Multidimensional Signals with Linearly Coordinated Components

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Abstract: The given study considers the problem of coordinated selection (filtration) component parts of observed set sequences in the assumption of observed set sequences in the assumption that there is a set of nonzero constant coefficients allowing to receive zero sums of their (parts) values on all interval of registration (model of linear coherence) is considered. As important circumstance is the component equality that serves not a salience of one of them as linearly depending on others. It is possible to specify procedures of hindrances autocompensation when processing signals using additional channels of their registration as an important example. Possibility of principle of empirical values orthogonal design use in these conditions on hyperplane determined by set of the analyzed "true" sizes is investigated. Main computing ratios for estimation linearly coordinated component at known parameters of coherence are received. Besides, procedure of linear coherence model parameter estimation on the basis of empirical data is offered.

Key words: Multidimensional processes estimation, coherence (interrelation) linear models, parameter estimation, orthogonal design

INTRODUCTION

Within this research coherence (interrelation) linear model of some set sequences of Eq. 1 (Woodrow, 1989) is called:

$$Z_i = \{z_{ki}\}, i = 1,..,M; k = 1,2,..,$$
$$\sum_{i=1}^{M} a_i z_{ki} \equiv 0, k = 1,2,.. \quad (1)$$

where, a_i, i = 1,...,M material parameters (coefficients) of coherence. Without breaking similarity, it is supposed that M are defined by the smallest amount of sequences for which the identity is carried out (Eq. 1).

This model adequately reflects situation when observed set of variables is generated by one source, or evolves under its strong influence. It is possible to specify manifestations of hindrances source, general for various radio channels as real corresponding situations (Woodrow, 1989) or coordinated behavior of stock prices of various companies, the monetary turn determined by volume in stock market. In study the generalized model form Eq. 1 is considered:

$$\sum_{i=1}^{M} a_{i?} z_{ki} = 0, k = 1,2,.. \quad (2)$$

where, $r \in \{1,...,M\}$ and one of equalities of normalization is carried out:

$$a_{rr} = 1 \quad (3)$$

Thus, we mark out the fact that the variable in Eq. 2 with the corresponding index for any reasons is allocated as dependent on other variables. For example, other variables can represent the impacts on system which are specially organized at identification (Gantmakher, 1967; Anderson, 1975).

MAIN POINTS

We will present a full possible set of rated coefficients in the matrix form with a single diagonal:

$$A = \{a_{ir}\} = (\vec{a}_1 \vec{a}_2 .. \vec{a}_M) \quad (4)$$

and as it is easy to show, it can be formed using only one column vector:

$$A = \vec{a}_r \times \vec{c}_r^T \quad (5)$$

where the symbol T means transposing and:

$$\vec{c}_r^T = (a_{1r}^{-1}, .., a_{Mr}^{-1}) \quad (6)$$

From here follows ratios for any two columns of matrix Eq. 4 and squares of their Euclidean norms:

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$$\vec{a}_k = \vec{a}_r / a_{kr},\ \ k,r \in \{1,..,M\};$$
$$\|\vec{a}_k\|^2 = \|\vec{a}_r\|^2 / a_{kr}^2,\ \ k,r \in \{1,..,M\} \tag{7}$$

Components of the observed (registered) multidimensional sequence x_{ki} in type of various reasons will differ from coordinated that it is possible to express as additive ratios:

$$x_{ki} = z_{ki} + u_{ki},\ i = 1,..,M;\ k = 1,2.... \tag{8}$$

In this regard, there is the first main objective: how to estimate their linearly coordinated parts meeting condition (Eq. 2) in fixed matrix (Eq. 4) on registered values of sequence component (Eq. 8).

Second task is connected with estimation of linear coherence model parameters when their choice on the basis of aprioristic reasons is impossible.

We will note that the existing practice of model approximation is based on use of the Smallest Squares (SSM) Method (Marple, 1990; Rektoris, 1985) (principle) when as a measure of linear dependence functionalities approximation quality is:

$$F_r = \vec{a}_r^T R \vec{a}_r \tag{9}$$

So the set of parameters which is the variation task solution is accepted by the most acceptable:

$$F_r = \hat{\vec{a}}_r^T R \hat{\vec{a}}_r = \min \vec{d}^T R \vec{d}, \forall \vec{d} =$$
$$(d_{1r},..,d_M)^T \in R^M; \hat{a}_{rr} = d_r = 1 \tag{10}$$

Where $\hat{\vec{a}}_r = (\hat{a}_{1r}..,\hat{a}_{Mr})^T$:

$$R = X^T X = \{R_{ik}\},\ i,k = 1,..M;$$
$$X = (\vec{x}_1,..,\vec{x}_M)^T, \vec{x}_r = (x_{ir},..,x_{Nr})^T \tag{11}$$

Here and further N-duration is processed selection. Let D is the dimension matrix (M-1)×(M-1) received from R matrix at deletion of r of line and column and vector $\vec{p}_r$ represent its r column with deleted r-element.

Then as it is easy to show (Bolshakov and Karimov, 2007), the necessary condition of performance (Eq. 10) is the equation for a vector of estimates of parameters $\hat{\vec{b}}_r = (\hat{a}_{1r},..,\hat{a}_{r-1,r},\hat{a}_{r+1,r},..,\hat{a}_{Mr})^T$:

$$D_r \hat{\vec{b}}_r = -\vec{p}_r \tag{12}$$

So if the matrix in the left part is nonexceptional, required vector is defined from a ratio (a sufficient condition of a minimum):

$$\hat{\vec{b}}_r = -D_r^{-1} \vec{p}_r \tag{13}$$

and the minimum value of functionality (Eq. 10) turns out at substitution of this representation in its left part:

$$F_r = \|\vec{x}_r\|^2 - \vec{p}_r^T D_r^{-1} \vec{p}_r \tag{14}$$

It is obvious that for other index similar representations have to take place:

$$\hat{\vec{b}}_k = -D_k^{-1} \vec{p}_k \tag{15}$$

$$F_k = \|\vec{x}_k\|^2 - \vec{p}_k^T D_k^{-1} \vec{p}_k \tag{16}$$

It is easy to understand that for vectors (Eq. 13 and 15) and ratios of type (Eq. 7) will be carried out only if from the coordinated sequences are equal in the right parts (Eq. 8) of deviation identically zero. Thus equalities will also be carried out:

$$F_k = 0,\ k \in \{1,..,M\} \tag{17}$$

that is the minimum values of all functionalities of type (Eq. 9) are identical and equal to zero.

It is clear that generally the specified properties of the received estimates of parameters will be absent. In this sense such estimates aren't adequate that is the consequence of non-invariancy of functionality (Eq. 9) to transformations of type (Eq. 7) at any sequences (Eq. 8). Therefore, it is advisable to construct instead of (Eq. 7 and 9) measure of approximation quality, invariant to transformations of type, linearly to coordinated parts of sequence component (Eq. 8) and also procedure of approximation parameters estimation which results to these transformations satisfy.

COMPUTING RATIOS

We will note that generally separate elements of the right parts in ratios (Eq. 8) are unknown, including probabilistic properties of deviations from coherence (second composed in the right parts). Therefore, it is expedient to apply again some variation principle allowing to receive computing ratios for estimates of hypothetical coordinated parts at vectors of coherence model parameters (Eq. 1) chosen from these or those reasons it is obvious that in the latter case it is about filtration option. In necessary (Anderson, 1975; Hui *et al*., 2008) cases as a stage of justification of parameter values, it is expedient to carry out their estimation directly according to empirical data for what it is also natural to use the adequate variation principle. We will assume that the vector is:

$$\vec{a}_r = (a_{1r},..,a_{Mr})^T$$

fixed taking into account condition (Eq. 3) and we will enter vectors:

$$\vec{v}_{kr} = (v_{k1},..,v_{kM})^T \quad (18)$$

where it is carried out equalities:

$$v_{ki} = z_{ki}, i \neq r; v_{rr} = -\sum_{\substack{m=1 \\ m\neq r}}^{M} a_{mr} z_{km} \quad (19)$$

Within this work design of vectors is carried out:

$$\vec{w}_k = (x_{k1},..,x_{kM})^T \quad (20)$$

on vectors of type (Eq. 18) with use of the variation principle:

$$G_{kr} = \| \vec{w}_k - \vec{v}_{kr}\} \|^2 = \min \| \vec{w}_k - \vec{d} \|^2, \quad \forall \vec{d} = (d_1,..,d_M)^T \in R^M, d_r = 1 \quad (21)$$

We will remind that the symbol ||•|| means Euclidean norm (square root component from the sum of squares). That the vector of type (Eq. 18) was a projection of vector (Eq. 20) also enough performance of equation system is necessary:

$$\partial G_{kr} / \partial z_{ki} = 0, i = 1,..,M; i \neq r \quad (22)$$

As a result of differentiation from (Eq. 21) we receive the equation for estimates of the coordinated component parts (Eq. 8):

$$B_r \vec{s}_{kr} = \vec{y}_{kr} - x_{kr} \vec{t}_r \quad (23)$$

Where:

$$\vec{y}_{kr} = (x_{k1},..,x_{k,r-1}, x_{k,r+1},..,x_{kM})^T;$$
$$\vec{t}_r = (a_{1r},..,a_{r-1,r}, a_{r+1,r},..,a_{Mr})^T;$$
$$\vec{s}_{kr} = (z_{k1},..,z_{k,r-1}, z_{k,r+1},..,z_{kM})^T$$

$$B_r = I + \vec{t}_r \vec{t}_r^T, I = diag(1,..,1) \quad (24)$$

It is easy to show correctness of ratio for the inverted to (Eq. 24) matrixes:

$$B_r^{-1} = I - h \vec{t}_r \vec{t}_r^T \quad (25)$$

where (taking into account definitions in Eq. 23):

$$h = 1/(1 + \| \vec{t}_r \|^2) = 1/ \| \vec{a}_r \|^2 \quad (26)$$

Therefore, the solution of Eq. 23 has an appearance:

$$\vec{s}_{kr} = \vec{y}_{kr} - (\vec{w}_k., \vec{a}_r)) \vec{t}_r / \| \vec{a}_r \|^2 \quad (27)$$

where the symbol (.,) means a scalar product in Euclidean space of vectors. In turn for z_{kr} serves as an assessment:

$$\hat{z}_{kr} = -(\vec{t}_r, \vec{s}_{kr}) \quad (28)$$

Having substituted representation (Eq. 27) here, taking into account (Eq. 26) and definitions from (Eq. 23) it is easy to receive the following ratio

$$\hat{z}_{kr} = x_{kr} - (\vec{w}_k, \vec{a}_r) / \| \vec{a}_r \|^2 \quad (29)$$

Uniting Eq. 27 and 29 taking into account designations in Eq. 23 and conditions Eq. 3, it is easy to receive the general ratio for coordinated part estimates vector:

$$\vec{\hat{z}}_k = (\hat{z}_{1k},..,\hat{z}_{Mk})^T = \vec{w}_k - (\vec{w}_k, \vec{a}_r) \vec{a}_r / \| \vec{a}_r \|^2 \quad (30)$$

It is easy to understand that substitution here according to transformation (Eq. 7) of representation $\vec{a}_r = \vec{a}_m \times a_{mr}$ yields the same result of estimation (Zhang *et al*., 2005; Xiaozhe *et al*., 2006). In other words, the ratio is invariant to the specified transformations that allows to speak about its adequacy in this sense.

It is also easy to receive the following ratios for functionals (Eq. 21) which defines squares of vector estimates norms of observed size deviations from the coordinated parts (Eq. 8):

$$G_{kr} = (\vec{a}_r, \vec{w}_k)^2 / \| \vec{a}_r \|^2 \quad (31)$$

and squares of estimate vectors norms (Eq. 30) of these parts:

$$\| \vec{\hat{z}}_k \|^2 = \| \vec{w}_k \|^2 - (\vec{w}_k, \vec{a}_r)^2 / \| \vec{a}_r \|^2 \quad (32)$$

Summarizing (Eq. 31 and 32), we have equality:

$$G_{kr} + \| \vec{\hat{z}}_k \|^2 = \| \vec{w}_{Л} \|^2 \quad (33)$$

which answers the principle of orthogonal projection. It is easy to show that its value also is invariant to transformations of type (Eq. 7).

Meaning representation: Summarizing these functionals on all selection, taking into account definition (Eq. 11), we receive a ratio:

$$G_r = \sum_{k=1}^{N} G_{kr} = \vec{a}_r^T R \vec{a}_r / \| \vec{a}_r \|^2 \quad (34)$$

defining cooperative measure of deviation of observed sizes from their estimates linearly of the coordinated parts at a known vector of coefficients of coherence.

It should be noted that (Eq. 7) pre-education $\vec{a}_r = \vec{a}_m \times a_{mr}$ determined by ratio does not change values of functionals (Eq. 31 and 33) that it is also possible to consider property of adequacy.

Meaning representations (Eq. 8), from ratio (Eq. 30) it is easy to receive equality for difference of the received assessment and "true" of vectors:

$$\vec{\Delta}_k = \vec{\bar{z}}_k - \vec{z}_k = -(\vec{z}_k, \vec{a}_k)\vec{a}_k / \| \vec{a}_k \|^2 + \vec{u}_k - (\vec{u}_k, \vec{a}_k)\vec{a}_k / \| \vec{a}_k \|^2$$

Where:

$$\vec{u} = (u_{k1}, .., u_{kM})^T$$

Let's assume that for vector of noise conditions are satisfied:

$$E[\vec{u}_k] = 0$$

$$E[u_{kr} u_{km}] = \sigma_r^2 \delta_{rm}$$

Where:

E = The symbol means expectation

δ_{rm} = A Kronecker delta

Then shift of the received assessment equally:

$$E[\vec{\Delta}_k] = (\vec{z}_k, \vec{a}_k)\vec{a}_k / \| \vec{a}_k \|^2$$

From this it follows that when performing a condition of coherence (Eq. 2) assessment (Eq. 30) turns out not displaced and for it similar equality is also carried out:

$$(\vec{\Delta}_k, \vec{a}_k) = (\vec{\bar{z}}_k, \vec{a}_k) = 0$$

In assumption of noise components dispersion uniformity (So *et al*., 2005) (Eq. 10) we easily receive a ratio for expectation of norm of vector of a difference of an initial vector and its assessment:

$$E[\| \vec{\Delta}_k \|^2] = (M-1)\sigma^2$$

Let's consider the second ask formulated above: parameter estimation of models of the linear coordination on empirical data. Meaning (Eq. 12 and 23) designations used in ratios (Eq. 33), it is easy to transform a ratio to type:

$$G_r = (R_{rr} + 2(\vec{p}_r, \vec{t}_r) + \vec{t}_r^T D_r \vec{t}_r) / \| 1 + \vec{t}_r \|^2 \quad (34)$$

Let columns of a matrix $Q_r = (\vec{q}_1^r, .., \vec{q}_{M-1}^r)$ be latent vectors of matrix D_r so the ratio is carried out:

$$D_r Q_r = Q_r L_r \quad (35)$$

where, L = diag (λ_1,..,λ_{M-1}) and for determinancy latent vectors are ordered on decrease of corresponding eigenvalues which are positive:

$$\lambda_k^r \geq \lambda_{k+1}^r > 0, k = 1, .., M-2 \quad (36)$$

In a type of matrix symmetry D_r the basis from its latent vectors is orthogonal and complete in Euclidean space of corresponding dimension (Gantmakher, 1967). Therefore, it is possible to put:

$$\vec{p}_r = Q_r \vec{h}_r; \quad \vec{t}_r = Q_r \vec{v}_r \quad (37)$$

In turn substitution of these representations in (Eq. 34) gives somewhat more prime ratio:

$$G_r = (R_{rr} + 2(\vec{h}_r, \vec{v}_r) + \vec{v}_r^T L_r \vec{v}_r) / (1 + \| \vec{v}_r \|^2) \quad (38)$$

For minimization of this functional it is expedient to use method of gradient descent (Gholami *et al*., 2003).

SUMMARY

It should be noted that such rationing does not assume selection of the variable corresponding to simple coefficient as dependent on others.

The way of component estimation is defined by a ratio (Eq. 30) from which in particular follows that the result does not depend on option of the specified normalization.

The parameter estimation of the linear coherence model can be performed on the basis of functional minimization (Eq. 34). Ratios for its gradient that allows to construct procedure of minimization with use of gradient descent are received. For a case of two variables specific ratios allowing to calculate required coefficients of linear coherence model are received.

CONCLUSION

Inverse problem of coordinated (dependent) parts of many-dimensional sequence components set linear estimation is considered when coefficients of coherence model are normalized so that one of them (any) was equal to unit.

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