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Image Restoration

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Abstract: The problem of compensation is considered arising from the registration of hardware distortion images, the model of which is presented by two-dimensional Fredholm integral equations of the first kind. Under this model, an idea of an additive image component is obtained accessible for the recovery and the method of adaptive regularization for its calculations is developed on the basis of empirical data within the conditions of an a priori uncertainty about the properties of the additive noises.

Key words: Images, recovery, deconvolution, regularization, additive noises

INTRODUCTION

The ratio (Bates and McDonnell, 1989; Gonzalez *et al*., 2004; Pratt, 1978) is the model used often for an image registration:

$$v_{nm} = u_{nm} + \varepsilon_{nm},\ n = 1, ..., N;\ m = 1, ..., M \quad (1)$$

where, ε_{nm} is an unknown error during the registration of a signal part of the type:

$$u_{nm} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} r(x_n - x, y_m - y)\, f(x, y)dxdy \quad (2)$$

Where:

r(x, y) = Hardware function of the registration system (the kernel of an integral equation) which is assumed to be known
f(x, y) = An unknown input impact of the registration system which makes the main interest

The objective is to restore an unknown impact (Eq. 2) f(x, y) according to the known elements of the left part in Eq. 1 and the kernel r(x, y). The restoring operator Φ(x, y, v, r) used at that is also natural to seek in the class of linear ones, meaning the recovery procedure:

$$\hat{f}(x, y) = \Phi(x, y, V, r) \quad (3)$$

Where:

$\hat{f}(x, y)$ = Input impact assessment (restored image)
V = A set of recorded data

$$V = \{v_{nm}\},\ n = 1, ..., N;\ m = 1, ..., M \quad (4)$$

The complexity and ambiguity of restoring operator development is conditioned by the mathematical incorrectness of an inverse problem concerning the solution of an integral (Eq. 2) which requires the use of regularization methods when empirical data are applied (Eq. 4). The most well-known method of regularization is the application of a restoring operator of the following form (Gonzalez *et al*., 2004):

$$\hat{f}_F(x, y) = \int_{-\pi}^{\pi}\int_{-\pi}^{\pi} \frac{V(z, t)R^*(z, t)}{(|R(z, t)|^2} + \frac{\alpha D(z, t))\exp(j(zx + ty))dzdt}{(4\pi^2)} \quad (5)$$

where, the asterisk denotes a complex conjugation:

$$V(z, t) = \sum_{n=1}^{N}\sum_{m=1}^{M} v_{nm} \exp(-jz(n-1))\exp(-jt(m-1)) \quad (6)$$

$$R(z, t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} r(x, y)\exp(-jxz)\exp(-jyt)dxdy \quad (7)$$

Where:

D(z, t) = The stabilizer which is a positive function of two variables, slightly decreasing in the field of integration
α = A small positive number which is called the regularization parameter

The limits of integration in the operator (Eq. 5) are determined by the property of Fourier function transformant periodicity of the frequency functions sampled with a constant pitch (equidistant) of an argument.

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At the basis of an operator development (Eq. 4) the theorem on Fourier transformant of the convolution is used (Eq. 2). The second term in the denominator of an integrand expression in Eq. 5 plays the role of a compensator concerning a too rapid decrease of the kernel Fourier transformant module as compared to the decrease of the Fourier transformant module in respect of empirical data (Eq. 6). The stabilizer of the following form is chosen rather frequently:

$$D(z, t) = z^2 + t^2 \tag{8}$$

This form corresponds to the variational principle of Euclidean norm squares minimization concerning the first private derivative estimates of input impacts (for restored images).

The selection of regularization parameter is carried out heuristically, although it is recommended to use the so-called residual principle within the conditions of a priori information about the properties of the second term in Eq. 1.

This method was developed by Sizikov, Tikhonov and Arsenin and Leonov. Its application allows you to receive the recovery results resistant to the registration error impacts with an appropriate choice of the regularization parameter. However, you may consider as a drawback some artificial introduction in the restoring stabilizer operator of (generally speaking) a heuristically chosen form and the need for a priori knowledge concerning the registration accuracy level for a signal part of the empirical data at an adequate choice of the regularization parameter.

The aim of this research is the development of an image reconstruction method, based on the use of linear representations via the basic functions generated by an equation kernel (Eq. 2). This allows to get a greater adequacy of regularization procedure including an adaptive estimation of error level concerning the registration of the empirical data signal part.

MAIN PART

Construction of recovery process images: Let's assume that the kernel of Eq. 2 (instrumental function) may be represented as the following product:

$$r(x, y) = r_1(x)\, r_2(y) \tag{9}$$

This form adequately reflects the properties of the hardware features for the most recording systems for example in radiolocation (distance-azimuth) as well as at the use of optical sensors for which an additional condition of a circular symmetry is performed:

$$r(x, y) = r_1(x)\, r_1(y) \tag{10}$$

at that the Gaussian Model serves as the basic model:

$$r_1(t) = K \exp(-t^2/2\sigma^2) \tag{11}$$

where, K is some positive ratio. Keeping in mind that all considered functions are the models of physically implemented processes let's assume their square integrability (the belonging to two-dimensional space L_2) and in particular the limitations of the Euclidean standards:

$$\| r \|^2 = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} r_1^2(x)\, r_2^2(y)\, dxdy < \infty \tag{12}$$

$$\| f \|^2 = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f^2(x, y)\, dxdy < \infty \tag{13}$$

Let's introduce a lineal:

$$f_1(x, y) = \sum_{n=1}^{N}\sum_{m=1}^{M} a_{nm}\, r_1(x_n - x)\, r_2(y_m - y) \tag{14}$$

where, the matrix elements $A = \{a_{nm}\}$, n = 1, ..., N; m = 1, ..., M are real numbers. Then, Rektoris any function from L_2 may be uniquely represented in the following form:

$$f(x, y) = f_1(x, y) + f_2(x, y) \tag{15}$$

where, the following equations are performed for the function f_2 (x, y):

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_2(x, y)\, r_1(x_n - x)\, r_2(y_m - y)\, dxdy = 0 \tag{16}$$

$$n = 1, ..., N;\ m = 1, ..., M$$

Therefore, after the substitution of the representation (Eq. 15) in the equation system (Eq. 1) taking into account (Eq. 2 and 16) we may easily obtain the matrix equation:

$$V = BAC + E \tag{17}$$

where, $E = \{\varepsilon_{nm}\}$, n = 1, ..., N; m = 1, ..., M and the elements of square matrices $B = \{b_{ik}\}$, i, k = 1, ..., N and $C = \{c_{nm}\}$, n, m = 1, ..., M are determined from the following relations:

$$b_{ik} = \int_{-\infty}^{\infty} r_1(x_i - x)\, r_1(x_k - x) dx \quad (18)$$

$$c_{nm} = \int_{-\infty}^{\infty} r_2(y_n - y)\, r_2(y_m - y) dy \quad (19)$$

Let's note that the second component of the right-hand side of the representation (Eq. 15) concerning the desired solution does not influence the result of an image registration and therefore its restoration is impossible. Thus, a natural and an adequate representation of the required assessment concerning an input impact is a linear form Eq. 14, the coefficient matrix which must satisfy (Eq. 17). Thus, the task of input exposure restoration is reduced to the problem of linear coefficient matrix estimation (Eq. 14) according to responce registration results. Obviously, the matrices with the elements (Eq. 18 and 19) are symmetric and positively definite. Therefore, there are such orthogonal matrices of eigenvectors and the diagonal matrices of eigenvalues for which the following conditions are performed (stroke denotes the transposition of a matrix) (Lancaster, 1969):

$$\begin{aligned} &BQ = QL, \\ &CG = GP, \\ &Q'Q = QQ' = diag(1, ...1), \\ &G'G = GG' = diag(1, ...1), \\ &L = diag(\lambda_1, ..., \lambda_n), \\ &P = diag(p_1, ..., p_M), \\ &\lambda_k \geq \lambda_{k+1} \geq 0,\ k = 1, ..., N\text{-}1, \\ &p_k \geq p_{k+1} \geq 0,\ k = 1, ..., M\text{-}1 \end{aligned} \quad (20)$$

Thus Eq. 17 may be transformed to the following form:

$$W = LTP + O \quad (21)$$

Where:

$$W = Q'VG \quad (22)$$

$$O = Q'EG \quad (23)$$

$$T = Q'AG \quad (24)$$

The matrix elements:

$$Z = LVP = \{z_{ik}\},\ i = 1, ..., N;\ k = 1, ..., M \quad (25)$$

are determined by the following correlations:

$$z_{ik} = \lambda_i p_k t_{ik},\ i = 1, ..., N;\ k = 1, ..., M \quad (26)$$

Therefore, if the following conditions are performed:

$$\lambda_i = 0,\ i > J_L;\ p_k = 0,\ k > J_P \quad (27)$$

Then in:

$$z_{ik} = 0,\ i = J_L + 1, ..., N;\ k = J_P + 1, ..., M \quad (28)$$

Thus, it is natural to require the performance of the equalities:

$$t_{ik} = 0,\ i = J_L + 1, ..., N;\ k = J_P + 1, ..., M \quad (29)$$

The remaining elements of the required matrix may be reduced to the following condition:

$$\| W_{11} - L_1 T_{11} P_1 \|^2 = d^2 \quad (30)$$

where, the symbol $\|\|^2$ denotes the square of the Euclidean matrix standard:

$$\begin{aligned} &L_1 = diag(\lambda_1, ..., \lambda_{J_L});\ P_1 = diag(p_1, ..., p_{J_P}); \\ &T_{11} = Q_1'AG_1 \end{aligned} \quad (31)$$

and the matrices Q_1 and G_1 are rectangular ones and include a corresponding number of eigenvectors (the number of columns is equal to the number of non-zero eigenvalues).

The right part in Eq. 30 is determined from the following considerations. Since, the failure of similar (Eq. 28) equations to zero for the elements of the matrix W is conditioned by the presence of image registration errors then the relation:

$$s^2 = \sum_{n = J_L + 1}^{N} \sum_{m = J_P + 1}^{M} w_{nm}^2 / (N - J_L)(M - J_P) \quad (32)$$

may serve as an estimate of their mean square. Then, to estimate the sum of squared errors for other components of the matrix W it is natural to use the following ratio:

$$d^2 = s^2 NM / (N - J_L)(M - J_P) \quad (33)$$

It is clear that for the calculation of the matrix T_{11} one (Eq. 30) is not enough as it has an infinite number of solutions. In order to select one of them it is offered to use the variation principle of a standard square minimization of a restored component (Eq. 14) that is $\|f_2\|$.

The substitution of Eq. 14 in the definition (Eq. 13) after the conversion taking into account (Eq. 20) and definitions (Eq. 24) allows to obtain the following representation for the required standard:

$$\| f_1 \|^2 = \sum_{n=1}^{J_L} \sum_{m=1}^{L_p} \lambda_n p_m t_{nm}^2 \quad (34)$$

Let's present in the following form Lagrange function of the proposed variation minimization problem (Eq. 33) at the performance of the condition (Eq. 30):

$$FL(\mu, T_{11}) = \mu \sum_{n=1}^{J_L} \sum_{m=1}^{L_p} \lambda_n p_m t_{nm}^2 + \sum_{i=1}^{J_L} \sum_{k=1}^{J_p} (w_{ik} - \lambda_i p_k t_{ik})^2 - d^2 \quad (35)$$

The equation to zero the first partial derivatives according to the required matrix elements provides the representations for the components of a variation problem solution (regularized solution):

$$\hat{t}_{ik}(\mu) = w_{ik}/(\mu+\lambda_i p_k),\ i = 1, ..., J_L;\ k = 1, ..., J_p \quad (36)$$

The substitution of this representation to the condition (Eq. 30) provides a nonlinear equation for the indefinite Lagrange parameter:

$$\mu^2 \sum_{i=1}^{J_L} \sum_{k=1}^{J_p} w_{ik}^2/(\mu+\lambda_i p_k)^2 = d^2 \quad (37)$$

Not difficult make sure at equity next.

Statement 1: Equation 30 has a positive root only when the the following inequality is performed:

$$\sum_{i=1}^{J_L} \sum_{k=1}^{J_p} w_{ik}^2 \geq d^2 \quad (38)$$

Since, the left part of Eq. 36 is increased monotonically with the increase of $\mu \geq 0$ (positive derivative), then this root is the only one and it is equal to zero only if the following condition is performed:

$$d^2 = 0 \quad (39)$$

In the latter case the estimates (Eq. 37) take the following form:

$$\hat{t}_{ik}(0) = w_{ik}/\lambda_i p_k,\ i = 1, ..., J_L;\ k = 1, ..., J_p \quad (40)$$

i.e., the components are restored in an asymptotically correct way (Eq. 14). Equation 36 may be easily converted to a form suitable for the application of Simple Iteration Method.

$$\mu = d/[\sum_{i=1}^{J_L} \sum_{k=1}^{J_p} w_{ik}^2/(\mu+\lambda_i p_k)^2]^{1/2} \quad (41)$$

At that it is reasonable to use zero (initial) approximation (if Eq. 38) is satisfied). Then the next parameter value will be the most possible and so on that is the approach to the root will be a two-way one. To calculate the presentation coefficient matrix (Eq. 14) let's use the following ratio:

$$\hat{A} = Q_1 \hat{T}_{11} G_1' \quad (42)$$

Thus, all relations which allow to calculate a reconstructed image on the basis of the presentation (Eq. 14) are obtained.

Examples and illustrations: At first let's consider the model (Eq. 10 and 11). At that (Eq. 18) provides the following:

$$b_{nm} = c_{nm} = 2K\sigma(\pi)^{1/5} \exp(-(x_n - x_m)^2/4\sigma^2) \quad (43)$$

Assuming that the spatial sampling is performed uniformly (equidistant) with an increment Δ, let's present (Eq. 40) in a standard way ($h = \Delta/\sigma$):

$$b_{nm} = c_{nm} = 2\Delta K(\pi)^{1/5} h \exp(-(n-m)^2/4h^2) \quad (44)$$

where $h = \sigma/\Delta$. A special interest is presented by the dependence of non-zero eigenvalues number J_L of the matrices (Eq. 43) on the value of parameter h. At that it is reasonable to have a simple relationship which allows to perform the assessment of this characteristics. It is natural to use the comparison of the cast eigenvalues as a guideline with the maximum use of inequality, for example:

$$\lambda_k \leq 0.001\lambda_1 \quad (45)$$

at the performance of which an own number with such an index is assumed to be zero. Then, the minimum one from such a set of indexes is taken for J_L. Table 1 provides the estimations of the values J_L for the following case $N = M = 1000$.

Table 1 data and computational experiments at other values of dimensions show that when the inequality $M \geq 100$ is performed a good approximation is obtained at the use of the following correlation:

$$J_L \approx [0.85 X_{max}/\varpi] \quad (46)$$

Table 1: Estimation of values J_L

h	J_L
0.1	1000
0.5	1000
1.0	851
1.5	569
2.0	423
2.5	322
3.0	282
4.0	212
10.0	85

where, X_{max} is the spatial size of a registered image domain and the square brackets denote the integer part of a number. Thus, the rank of the matrix (Eq. 42) is determined by the ratio of the determination domain size to the size of the Gaussian parameter σ. Let's note also that the ratio 0.85 matches the probability integral value at an argument equal to one. It is also known that this parameter tends to zero if delta function will be a Gaussian limit.

CONCLUSION

This study proposes a new method for an inverse problem of image restoration solution. It is based on the use of function basis that is determined by the kernel of an integral equation. Also the method of an Adaptive Regularizing algorithm is developed for the calculation of approximate solutions concerning integral equations according to empirical data.

The proposed method of image reconstruction takes into account the information on the input image adequately and allows to take into account the distortions arising from the registration of the empirical data adaptively.

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REFERENCES

Bates, R.H. and M.J. McDonnell, 1989. Image Restoration and Reconstruction. Clarendon Press, Oxford, UK., Pages: 283.

Gonzalez, R.C., R.E. Woods and S.L. Eddins, 2004. Digital Image Processing Using MATLAB. Pearson Prentice Hall, New Jersey, USA., ISBN-13: 978-0130085191, Pages: 609.

Lancaster, P., 1969. Theory of Matrices. Academic Press, New YorK, England, Pages: 277.

Pratt, W.K., 1978. Digital Image Processing. Jons Willey & Sons Inc., New Jersey, USA., Pages: 735.