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A Novel Rate-Distortion Optimized Tree Structured Hybrid Algorithm for Coding of Digital Images

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Abstract: This study proposes a novel segmentation based tree structured algorithm for efficient compression of digital images. The presented research combines the recent segmentation based coding approach, namely the binary space partition scheme and the popular Geometric Wavelet Coding Method to capture the curve singularities in a more effective way and to provide the sparse representation of the image. For partitioning the image domain in the BSP scheme, polar co-ordinate representation of straight line is used which improved the choice of bisecting lines available for partitioning thereby enhancing the probability of reducing the error functional. A rate-distortion optimization process is performed prior to encoding where a New Pruning algorithm is tried to prune the BSP tree and achieve the desired bit rate. The relative practical competency of this hybrid technique is investigated and the results are compared with state-of-the-art wavelet coders, recent segmentation based algorithms as well as the Original Geometric Wavelet Coding algorithm. This technique provides outstanding results in terms of rate-distortion compression by taking advantage of the curve singularities in the image. The algorithm is applied individually on tiled regions of the image rather than on the entire image. The results report a gain of 2.46 dB over the EZW algorithm and 1.74 dB over the SPIHT algorithm at the bit-rate 0.03125 bpp. Researchers also show that the presented algorithm reports a gain of 1.2 dB over the Original GW Method at the compression ratio of 256 for the Lena test image. The technique provides remarkable results in terms of rate-distortion compression by taking advantage of the edge singularities in the image. The improved GW algorithm was simulated using the 2010 version of MATLAB on still images of Lena and cameraman to validate its performance. The algorithm is highly complex in computation and requires enormous time for execution.

Key words: Geometric wavelets, binary space partition, rate-distortion optimization, image coding, pruning

INTRODUCTION

In the present scenario, the use of digital images has become an important part of the daily life. As the dependence on the digital media continues to grow, finding proficient ways of storing and passing on these large amounts of data has become a major concern. Since, the amount of space required to hold unadulterated images can be extremely large in terms of cost as well as of the huge bandwidth required to transmit them, researchers are seeking methods for efficient representations of these digital pictures to simplify their transmission and save disk space. At this point, the technique of image compression has become very essential and highly applicable. To date, substantial advancements in the field of image compression have been made, ranging from the traditional predictive coding approaches, classical and popular transform coding techniques and vector quantization to the more latest second generation coding schemes. Starting at 1 with the first digital picture in the early 1960s, the compression ratio has reached a saturation level of around 300:1 recently (Netravali and Haskell, 1988). Even then, the reconstructed image quality still remains as an important issue to be investigated. As the bit rate (the number of bits used to represent a pixel) decreases, the quality of the resulting image degrades. So, a tradeoff between the compression ratio and the tolerance in the visual quality degradation need to be considered during compression. Lately, the Discrete Cosine Transform (DCT) has been the most popular technique for image compression because of its optimal performance and ability to be implemented at a reasonable cost. Quite a lot of commercially Successful Compression algorithms, including the JPEG standard for still images and the MPEG standard for moving images are based on DCT. Wavelet-based image coding techniques are the latest development in the field of image compression offering multiresolution capability

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resulting in superior energy compaction and high quality reconstructed images at low bit rates (Antonini *et al.*, 1992). The discrete wavelet transform has come up as a cutting edge technology within the field of digital image compression. The wavelet transforms based coding approaches have taken over other classical methods particularly the cosine transform due to its capability to solve the problem of blocking artefacts which is a common phenomenon in DCT based compression (Islam and Pearlman, 1998). However, the EZW, the SPIHT, the SPECK, the EBCOT algorithms and the current JPEG 2000 standard are based on the Discrete Wavelet Transform (DWT) (Skodras *et al.*, 2001). The DWT based techniques also reduces the correlation between the neighbouring pixels and gives multi scale sparse representation of the image.

In spite of providing outstanding results in terms of rate-distortion compression, the Transform-Based Coding Methods do not take an advantage of the geometry of the edge singularities in an image. This led to the design of 'Second Generation' or the segmentation based image coding techniques that make use of the underlying geometry of edge singularities of an image (DeVore, 1998). To this day, almost all of the proposed 'Second Generation' algorithms (Kunt *et al.*, 1985) are not competitive with state of the art (dyadic) wavelet coding (Daubechies, 1990). In this regard, inspired by a recent progress in multivariate piecewise polynomial approximation, researchers put together the advantages of the Classical Method of coding using wavelets (Daubechies, 1992) and the segmentation based coding schemes to what can be described as a geometric wavelet approach.

This research focuses on a recent development in the field of piecewise polynomial approximation for image coding using geometric wavelets. This scheme efficiently captures curve singularities and provides a sparse representation of the image and thereby achieves better quality reconstructed images with higher compression ratios. Stress is given on the shared approach of image compression using geometric wavelets and the binary space partition scheme. The current study is envisaged to enhance the GW image coding method and its improved version. Researchers use the polar co-ordinate form of the straight line in the Binary Space Partition scheme (BSP). Here, the number of quantized bisecting lines is increased and hence probability of minimizing the cost functional and finding the optimal cut of the domain is improved a rate-distortion optimization process is performed prior to encoding where a New Pruning algorithm is tried to prune the BSP tree and achieve the desired bit rate. A new "geometric" context modeling scheme combined with arithmetic coding is designed to boost the performance of the algorithm.

LITERATURE REVIEW

Several segmentation algorithms have been proposed for image coding till date each claiming to be different or superior in some way. The first Segmentation-Based Coding Methods were suggested in the early 1980s (Kocher and Kunt, 1982). These algorithms partition the image into complex geometric regions using a Contour-Texture Coding Method (Leonardi and Kunt, 1986) over which it is approximated using low-order polynomials. The one of the most popular segmentation based coding schemes investigated by researchers in the early days were the quadtree-based image compression (Sullivan and Baker, 1991) which recursively divides the image signal into simpler geometric regions. Many variations of the 'Second Generation' coding schemes have since been announced that exploit the geometry of curve singularities of an image (Kunt *et al.*, 1987). In one of the outstanding 'Second Generation' Methods, Froment and Mallat (1992) constructed multi-scale wavelet-like edge detectors and showed how a function from the responses of a sparse collection of these detectors can be reconstructed. They reported good coding results at low bit-rates. Candes and Donoho (2001) constructed, a bivariate transform called curvelets intended to capture local multi-scale directional information. Cohen and Matei (2001) also presented a discrete construction of an edge-adapted transform which is closely related to nonlinear Lifting (Claypoole *et al.*, 2003). In a later research, the researchers enhance classical wavelet coding by detecting and coding the strong edges separately and then using wavelets to code a residual image (Cohen and Matei, 2001). Do and Vetterli (2005)'s construction of contourlets is similar but is a purely discrete construction. Coding algorithms that are geometric enhancements of existing wavelet transform based methods, where wavelet coefficients are coded using geometric context modelling also exist. But, all of these constructions are redundant, i.e., the output of the discrete transform implementations produces more coefficients than the original input data (Demaret *et al.*, 2006). Research on the possibility of using these new transforms to outperform wavelet based coding is still on-going.

The Binary Space Partition (BSP) scheme, a simple and efficient method for hidden-surface removal and solid modelling was introduced in 1990 (Dekel and Leviatan, 2005). The BSP technique was applied to the concept of image compression in 1996 and is adopted in the first stage of this study (Radha *et al.*, 1996). Later, binary partition trees were used as an efficient representation for image processing, segmentation and information retrieval (Salembier and Garrido, 2000). Recently, many second generation image compression algorithms such as the Bandelets (Le Pennec and Mallat, 2005), the Prune tree (Shukla *et al*., 2005), the Prune-Join tree, the GW Image Coding Method (Alani *et al*., 2007) and the like based on the sparse geometric representation have been introduced. Le Pennec and Mallat (2005) lately applied their 'Bandelets' algorithm to image coding where a warped-wavelet transform is computed to align with the geometric flow in the image and the edge singularities are coded using one-dimensional wavelet type approximations. The concept of combining the binary space partition scheme and geometric wavelets for compression of digital images were put forward by Dekel and Leviatan (2005) and Alani *et al*. (2007). Here, the bisecting lines of the BSP scheme are quantized using the normal form of straight line. This method successfully competes with state of the art wavelet methods such as the EZW (Shapiro, 1993), SPIHT (Said and Pearlman, 1996) and EBCOT algorithms (Taubman, 2000) and also beats the Recent Segmentation Based Methods. But, the algorithm turned out to be computationally intensive. An improvement was made to this research in 2011 by Chopra and Pal (2011). They used the slope intercept form of a straight line instead of the normal representation. This improved the possibility of minimizing the cost functional by increasing the choice of bisecting lines available for partitioning. This technique further increased the complexity of the algorithm.

The approach deviates from the context of multi-scale geometric processing even from the more general framework of harmonic analysis which is the theoretical basis for Transform Based Methods and also from the popular wavelet based studies and is based on the GW and Binary Space Partition Method. The main difference between the GW algorithm and recent research is that researchers use the polar coordinate representation of straight line for partitioning the domain thereby further improving the availability of partitioning lines and intern further minimizing the cost functional at each step of BSP scheme (Rehna and JeyaKumar, 2013). A new Pruning algorithm is tried to prune the BSP tree in the rate-distortion optimization process which is performed prior to encoding to achieve the desired bit rate. A new "geometric" context modeling scheme combined with arithmetic coding is designed to boost the performance of the algorithm. Other previous research that are found to be relatively close to ours are the study by Shukla *et al.* (2005) and Dekel and Leviatan (2005).

THE GW ALGORITHM

Geometric wavelets are multi-scale dictionary elements which are constructed directly from the data and have guarantees on the computational cost, the number of elements in the dictionary and the sparsity of the representation. Geometric Wavelets (GW) have been considered in context of image compression in 2007. It is a new multi-scale data representation technique which is useful for a variety of applications such as data compression, interpretation and anomaly detection. The GW is defined as:

$$\Psi_{\Omega 0}(f) \triangleq 1_{\Omega 0}\left(Q_{\Omega 0} - Q_{\Omega}\right) \quad (1)$$

Ω_0 here means one of the children of mother, Ω. It is possible to reconstruct the function f using:

$$f = \Sigma_{\Omega'} \Psi_{\Omega'}(f) \quad (2)$$

The basic concepts of the Geometric Wavelet Method are described in the study.

The BSP tree generation: The BSP technique can be described as follows. Given an image f, the algorithm divides convex polygonal domain Ω into two subsets Ω_0 and Ω_1 using a bisecting line. The subdivision is performed to minimize a given cost functional (Eq. 3). This partitioning process then operates recursively in a hierarchical manner on the subdomains until some exit condition is met. To be specific, researchers describe the algorithm of which is a BSP algorithm that identifies a compact geometric description of a target bivariate function. The goal in is to encode an optimal cut of the BSP tree to be precise, a sparse piecewise polynomial approximation of the original image based on the union of disjoint polygonal domains in the BSP tree (Paterson and Yao, 1990). Rate-distortion optimization strategies are used to meet a given bit rate.

For a given convex polygonal domain Ω, the algorithm finds two subdomains, Ω_0 and Ω_1 and two bivariate (linear) polynomials $Q_{\Omega 0}$ and $Q_{\Omega 1}$ that minimizes the given cost functional:

$$F(\Omega_0, \Omega_1) = \arg\min_{\Omega 0 \,\Omega 1} \left\|f - Q_{\Omega_0}\right\|^2_{\Omega_0} + \left\|f - Q\Omega_1\right\|^2_{\Omega_1} \quad (3)$$

where, Ω_0 and Ω_1 represent the subsets resulting from the subdivision of Ω (Ω_0 and Ω_1 should be considered as children for the mother Ω). The bivariate polynomial used is defined by:

$$Q_{\Omega i} = A_i x + B_i y + C_i \quad (4)$$

The polynomial interpolation is made using the Least Square Method, computing the difference between the image and the polynomial at a defined region Ω (Radha, 1990). The algorithm continues partitioning each region recursively until there are no enough pixels to subdivide or the approximation error is sufficiently small. The algorithm constructs a binary tree with the partitioning information. The algorithm needs to encode the information of the geometry, namely, the line that cut each sub-domain and the approximation function in each sub-domain represented by the polynomial coefficients. Figure 1 shows the steps involved in Binary Space Partitioning algorithm.

First, a line L divides the region Ω into two regions Ω_0 and Ω_1. The two regions Ω_0 and Ω_1 are further divided into Ω_{00}, Ω_{01} and Ω_{11}, Ω_{10}, respectively. These four regions are further divided into eight and so on until area of the sub-domain contains only a very few pixels. Then, it is represented in a tree structure as shown in Fig. 2.

The BSP Method is computationally very intensive. Therefore, the image is tiled first and then the BSP algorithm is applied independently on each tile, thereby creating a BSP forest (Rehna and Jeyakumar, 2014). The tile size is generally adopted is 128×128. The tiling of cameraman test image into tiles of size 128×128 is shown in Fig. 3.

The BSP scheme is applied on each tile of the image by using the polar coordinate form of the straight line. In polar coordinates on the Euclidean plane, a line is expressed as:

$$r = \frac{b}{\sin\theta - m\cos\theta} \tag{5}$$

Where:

m = The slope of the line
b = The y-intercept

Equation can be rewritten as:

$$r\sin\theta = mr\cos\theta + b \tag{6}$$

It is not possible to quantize the parameter m, as it is unbounded has value infinity for the straight lines which are parallel to y axis. This problem is solved by using the new parameter ϕ in place of min (Eq. 8) where ϕ is the angle between the line and the x-axis in the anticlockwise direction (Parameters θ and ϕ are shown in Figure 4). Subsequently, Eq. 4 reduces to:

$$r\sin\theta = \tan\phi . r\cos\theta + b \tag{7}$$

Here, the probability of minimizing the cost functional given in Eq. 1 is increased, compared to that when the normal form of straight line is used as in Eq. 5. The number of bisecting lines available for the partitioning of tile of dimension 128×128 in is 15740. In the improved GW approach, the number increased to 60775. But in the proposed algorithm this availability number further increased to 69780. Hence, this method provides a better choice of bisecting lines thereby giving more possibility to minimize the cost functional.

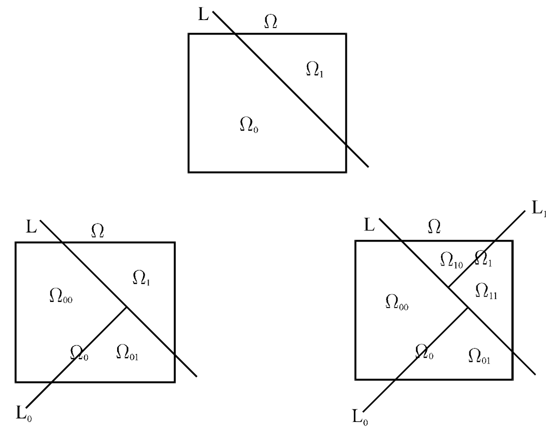

Fig. 1: Binary space partitioning of the domain Ω (two levels)

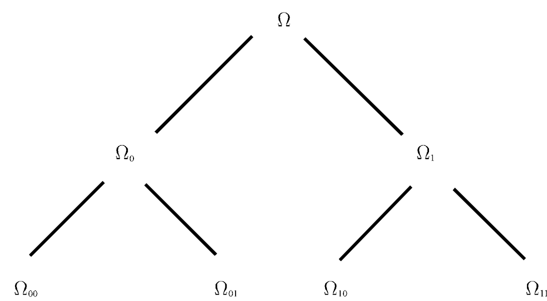

Fig. 2: BSP tree representation of the polygon in Fig. 1

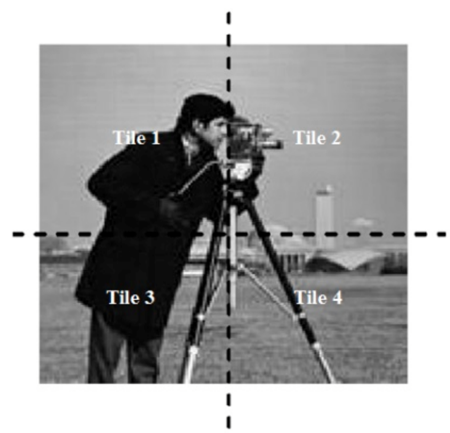

Fig. 3: Tiling of camera man image (Tile size 128×128)

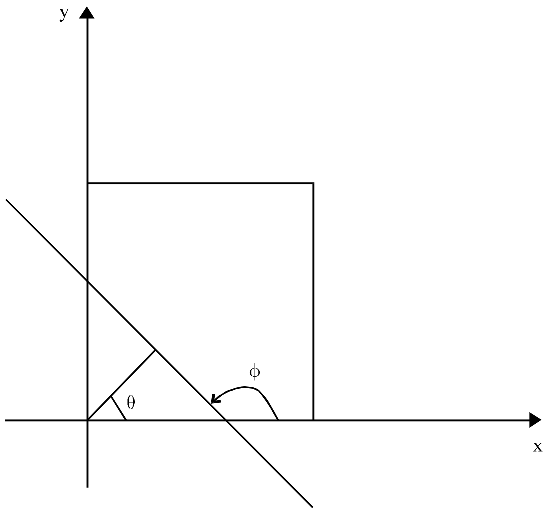

Fig. 4: Partition of the image domain into two subdomains-parameters θ and φ

The BSP tree is generated for each tile of the entire image according to the method discussed above, thus creating a BSP forest. The next step is to create the geometric wavelets for each node in the BSP tree.

Sparse geometric wavelet representation: Geometric wavelet, Ψ_Ω is a "local difference" component that belongs to the detail space between two levels in the BSP tree, a "low resolution" level associated with Ω and a "high resolution" level associated with Ω_0 as given in Eq. 1. Geometric wavelets also satisfy the vanishing moment property like isotropic wavelets, i.e., if f is locally a polynomial over Ω, then minimizing of Eq. 1 gives $Q_{\Omega 0} = Q_\Omega = 1$ and therefore $\Psi_{\Omega 0}(f) = 0$. Unlike classical wavelets, geometric wavelets do not satisfy the two scale relation and the biorthogonality property.

The BSP tree generated in the previous step consists of large number of nodes. Geometric wavelets are created for each node using Eq. 1. The GW Image Coding algorithm is based on the idea that among all the geometric wavelets only a "few" wavelets have large norm. Once all the geometric wavelets are created, they are arranged according to their L2 norm as shown in Eq. 8.

$$\left\|\Psi_{\Omega_{k1}}\right\|_2 \geq \left\|\Psi_{\Omega_{k2}}\right\|_2 \geq \left\|\Psi_{\Omega_{k3}}\right\|_2 \geq \ldots \quad (8)$$

Then, the sparse geometric representation is extracted using the greedy methodology of non-linear approximation (Temlyakov, 2003). Here, n wavelets are selected from the joint list of geometric wavelets over all tiles.

A rate-distortion optimization is performed prior to encoding where a new pruning algorithm is tried. The R-D curve for each node is generated by approximating the node by the quantized polynomial $\hat{p}(t)$ which is obtained by scalar quantizing the polynomial coefficients. In this Lagarangian cost based pruning, the R-D optimal pruning criterion for the given operating slope, λ is as follows: Prune the children if the sum of Lagarangian costs of the children is greater than or equal to the Lagarangian cost of the parent. Mathematically, this means that the children are pruned if $(D_{C1}+D_{C2})+\lambda(R_{C1}+R_{C2})\geq(D_P+\lambda R_P)$ where R_P and D_P are rate and distortions of the parent and R_{C1}, R_{C2}, D_{C1} and D_{C2} are the rate and distortions of the children, respectively.

Subsequently, function f is approximated using the n-term geometric wavelet sum given in Eq. 4 where n is the number of wavelets used in the sparse representation.

Encoding: To obtain a reasonable approximation of the image, it is essential that if a child is present in the sparse representation then the mother should also be there, i.e, the BSP tree should be connected. Therefore, instead of encoding an n-term tree approximation, researchers create an n+k geometric wavelet tree by considering more k nodes. The cost of imposing the condition of the connected tree structure is not very huge, since there is high probability that if a child is important all its ancestors are also important. The encoding of the geometry of the extracted connected tree structure saves bits as only optimal cut is to be encoded.

There are two sorts of data to be encoded, the geometry of the support of the wavelets participating in the sparse representation and the polynomial coefficients of the wavelet. Before encoding the extracted BSP forest, a small header is written to the compressed file. Header consists of the minimum and maximum values of the coefficients of the participating wavelet and the image graylevels. Out of header size of 26 bytes, 24 are used in the storage of the minimum and the maximum values of the coefficients while 2 bytes are utilized to store the extremal values of the image. "Root" geometric wavelets contribute most in the approximation, so each root wavelet is encoded. The encoding process is applied repeatedly for each of the geometric wavelet tree nodes in each tile.

Before the encoding step, in addition, a rate distortion optimization process is carried out in order to attain the desired bit rate. Pruning iterations are applied where at each iteration, the leaf node with minimal R-D slope is pruned until the desired rate is accomplished.

Encoding the geometry of the support of the wavelet: The following information is encoded for each of the participating node Ω:

- Number of children of Ω that participate in the sparse representation
- In case only one child is participating then whether it is the left or the right child
- If Ω is not a leaf node then the line that bisects Ω is encoded using the slope intercept form

Left child and right child are defined as the sets of the pixels satisfying the inequality $r\text{-}\tan\phi.r\sin\theta \leq b$ and $r\text{-}\tan\phi.r\sin\theta \geq b$, respectively. The leaf node is encoded by using the bit "1." Codes "00" and "01" are used for the one child symbol and the two children symbol, respectively. If only 1 child of Ω is participating in the sparse representation, then this event is encoded by using an additional bit. In case Ω is not a leaf node then the indices of the parameter ϕ and c of the bisecting line are encoded using the lossless variable length coding.

Encoding the wavelet coefficients: The coefficients of the wavelet polynomial, Q_Ω are quantized and encoded using an orthonormal representation of $\Pi_1(\Omega)$, where $\Pi_1(\Omega)$ is the set of all bivariate linear polynomials over Ω. A bit allocation scheme for the coefficients is applied using their distribution function (over all the domains) which is discussed in later sections. The "root" wavelet of each tile is always encoded.

Quantizing the wavelet coefficients: To ensure the stability of the quantization process of the geometric wavelet polynomial Q_Ω, researchers first need to find its representation in appropriate orthonormal basis. The orthonormal basis of $\Pi_1(\Omega)$ is found using the standard Graham-Schmidt procedure. Let $V_1(x, y) = 1$, $V_2(x, y) = x$, $V_3(x, y) = y$ and be the standard polynomial basis. Then, an orthonormal basis of $\Pi_1(\Omega)$ is given by:

$$\begin{aligned} U_1 &= \frac{V_1}{\|V_1\|} \\ U_2 &= \frac{V_2 - \{V_2, U_1\} U_1}{\|V_2 - \{V_2, U_1\} U_1\|} \\ U_3 &= \frac{V_3 - \{V_3, U_1\} U_1 - \{V_3, U_2\} U_2}{\|V_3 - \{V_3, U_1\} U_1 - \{V_3, U_2\} U_2\|} \end{aligned} \tag{9}$$

Where inner product and norm are associated with the space L2 (Ω). Let:

$$\Psi = \alpha U_1 + \beta U_2 + \gamma U_3 \tag{10}$$

be the representation of the geometric wavelet $\Psi \in \Pi_1(\Omega)$ in the orthonormal basis.

A bit allocation scheme is applied depending upon the distribution functions of the coefficients α, β and γ of the wavelets participating in the sparse representation. Researchers can infer from the graph that there is a very high probability for the coefficients α, β and γ to be small (the graph resembles a generalized-Gaussian function). Some large coefficients are also present due to root wavelets. Four bins are used to model the absolute value of the coefficients, bin limits are computed and passed to the decoder. In case all the three coefficients of the wavelet are small, i.e., they are present in the bin containing zero, then this event is encoded using single bit but if any one of them is not small then the bin number of each coefficient is encoded. After this quantized bits are written to the compressed file.

Figure 5 shows how the bit budget allocation of Lena at the bit-rate 0.03125 bits per pixel (bpp) is distributed among the GW algorithm components. Figure 6 shows the bit allocation distribution of Lena at the bit-rate 0.125 bits per pixel (bpp).

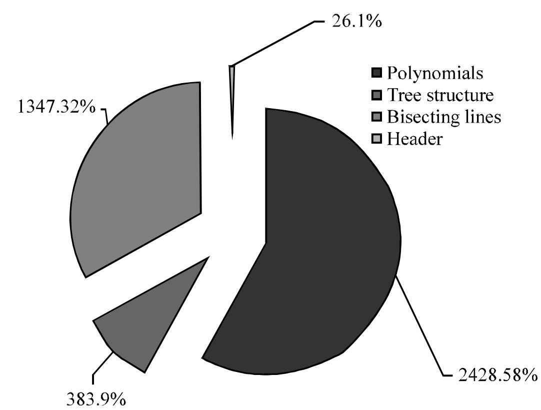

Fig. 5: Bit budget allocation for Lena at bit-rate, 0.03125 bpp. Output file size is 1 kbyte

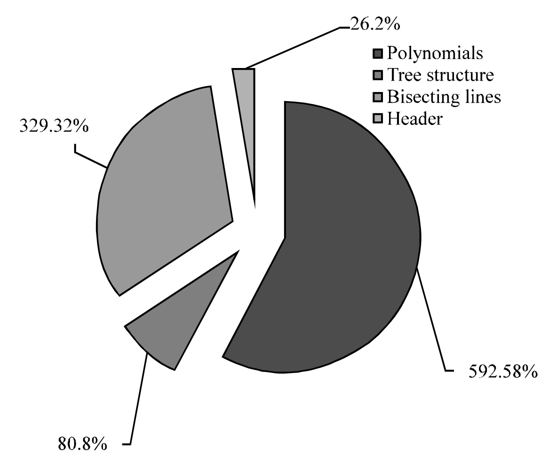

Fig. 6: Bit budget allocation for Lena at bit-rate, 0.125 bpp. Output file size is 4 kbyte

It can be inferred from the chart that at higher bit-rates, the bit budget for the polynomial coefficients relatively increases while the bit allocation for the bisecting lines decreases.

Decoding: In the decoding stage, the compressed bit stream is read to find whether the participating node is a root node has 1 child or 2 children or a leaf node. If one child is participating then by using bit stream identification, it is found whether it is left child or right child. If at least one of the children belongs to the sparse representation then the indexes of ϕ and b are decoded and using these index parameters ϕ and b of optimal cut are calculated. Thereafter, using this optimal cut, domain is partitioned into two subdomains and depending upon the situation vertex set of only one child or both children is found. This process is repeated until entire bit stream is read.

EXPERIMENTAL RESULTS

The proposed algorithm is tested on the still image of Lena of bit depth 8 and of size 512×512. The implementation is done using MATLAB. The Peak Signal to Noise Ratio (PSNR) based on Mean Square Error (MSE) is used as a measure of "quality". MSE and PSNR are given by the following relations:

$$MSE = \frac{1}{m \times n}\sum_{i=1}^{m}\sum_{j=1}^{n}\left(x_{i,j} - y_{i,j}\right)^2 \tag{11}$$

$$PSNR = 10\log_{10}\left[\frac{(255)^2}{MSE}\right] \tag{12}$$

Where:

- $m \times n$ = The image size
- $x_{i,j}$ = The original image
- $y_{i,j}$ = The reconstructed image
- MSE and PSNR = Inversely proportional to each other and higher value of the PSNR implies better quality reconstructed image

The performance of proposed method is compared against six algorithms. The PSNR values obtained by this method for the Lena image are compared with those obtained by the EZW, the SPIHT, the EBCOT and the Bandelets algorithms. Data presented in Table 1 shows that the proposed method outperforms the EZW, the SPIHT, the EBCOT and the Bandelets methods at low and medium bit rates.

The proposed method reports a gain of 1.35 dB over the SPIHT Method, 1.43 dB over the EBCOT Method and 2.19 dB over EZW algorithm at the compression ratio of 128:1 for the Lena test image. The presented algorithm

Table 1: Comparison of PSNR in dB with other state-of-the-art algorithms on test image, Lena

Compression ratio	Bite rate (bpp)	EZW	SPIHT	EBCOT	Bandlets	GW	Improved GW	Proposed method
256:1	0.03125	25.38	26.10	-	-	26.64	26.67	27.84
128:1	0.06250	27.54	28.38	28.30	-	28.72	28.78	29.73
64:1	0.12500	30.23	31.10	31.22	30.63	3.73	3.82	31.45

Fig. 7: Top left: original cameraman (256×256). Top center: reconstructed cameraman using the GW algorithm, 0.0625 bpp, PSNR = 22.93; top right: reconstructed cameraman using the proposed method, 0.0625 bpp, PSNR = 24.55. Bottom left: original Lena (512×512); bottom center: reconstructed Lena using the GW algorithm, 0.0625 bpp, PSNR = 28.72; bottom right: reconstructed Lena using the proposed method, 0.0625 bpp, PSNR = 29.73

Table 2: Comparison of PSNR in dB with other state-of-the-art algorithms on camera man test image

Compression ratio	Bite rate (bpp)	SPIHT	GW	Improved GW	Proposed method
128:1	0.0652	22.8	22.93	23.04	24.55
64:1	0.1250	25.0	25.07	25.29	26.31
32:1	0.2500	28.0	27.48	27.62	28.38

shows a gain of 1.01 dB over the original GW Method and 0.95 dB over the improved GW algorithm at a bit rate of 0.0625 bpp for the Lena image.

The PSNR comparison with other algorithms on the cmeraman test image is shown in Table 2. At the compression ratios of 128, 64 and 32, proposed method performs better than the SPIHT, GW and the improved GW. The proposed method reports a gain of 1.24 dB over the GW Method and 1.02 dB over the improved GW Method at the compression ratio of 64 for the cmeraman image. Figure 7 shows the reconstructed image of cmeraman and Lena using the algorithm, at the compression ratio of 128:1 and PSNR is 24.55 and 29.73, respectively.

CONCLUSION

In this research, researchers investigate the performance of a Hybrid algorithm for image compression using the geometric wavelets and the tree-structured binary space partition scheme. Researchers have improved the coding efficiency of the GW algorithm by using the polar coordinate form of straight line for best bisection in the line selection procedure. A new pruning algorithm was used to prune the BSP tree and to achieve the desired bit rate. The presented method produced PSNR values that are competitive with the state of art coders in literature. This method is applied to 8 bits gray scale images but it could be extended to color images also. The algorithm is found to be extremely complex in computation and has high execution time. This makes the technique of image coding less practically applicable. The algorithm works well with geometrically rich content images at low bit-rates. This study can be further taken up by researchers in view of reduction in computational complexity and time complexity of the algorithm without compromising on the signal to noise ratio.

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