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Solving Fuzzy Multi-Objective Non-Linear Illiteracy Problem Via Fuzzy Ranking Functions and Particle Swarm Optimization

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Abstract: Illiteracy Problem (IP) is an important problem in the entire world. The distance is an essential factor for the IP, so one of problems in IP is to select best locations to construct classrooms. In this study a multi-objective non-linear illiteracy problem is developed with uncertainty (Fuzzy) parameters (FMNIP). The fuzzy parameters of FMNIP are characterized as the triangular form. Fuzzy ranking functions have been used to transform the fuzzy parameter into crisp one. Finally, one of the Artificial Intelligent (AI) approaches called Particle Swarm Optimization technique (PSO) is developed to solve the proposed problem.

Key words: Fuzzy, fuzzy ranking, illiteracy problem, particle swarm optimization, FWNIP

INTRODUCTION

There are some real life problems related to the determination of suitable locations according to some criteria. Some studies discuss these problems like' researches related to land suitability analysis to achieve ideal usage of the available resources of land for example the selection of the suitable locations for the crop of rice according to the physic-chemical characteristics of different alluvial soils. There are other problems related to site location to determine the locations of emergency material warehouses (Garg and Sharma, 2012) for making waste management to determine appropriate site selection for disposal waste (Kai-Yuan, 1991) or to build hospitals and schools in a city for an appropriate sites with some criteria. These all problems may be exposed to it in real life and need to study each case and state the problem with its objectives and some constraints. These problems called multi-objective optimization problem (Karwowski and Mittal, 1986).

In this study discuss one of the site location problems to choose the best sites in a certain place. This problem is the Multi-objective Illiteracy Problem (MIP). MIP is a problem faced many countries. In some countries there are some cities facing illiteracy in abundance. These cities need an attention to facilitate the education of illiterate people. Some points of view will be studied to facilitate the education process. It must study and analysis the illiterate's view and how make people accept the education idea. Some illiterate people think of education as a wasting of time, effort and money. So, classrooms must be easy to reach and take the little of time from the housing places of people who want to learn. The cost is very important factor; it may be the transportation cost like money, time and distance. This cost must be reasonable (Verma *et al*., 2007; Zadeh, 1965).

In construction process, the budget is an important issue. The construction costs include the setup costs to establish classrooms. There are other important costs covers the teacher's compensations and the material costs called the running costs. To save the teacher's compensations, it must be choose the teachers from the local residents as much as possible (Bellman and Zadeh, 1970).

The main goal of the MIP is selecting the best sites to construct classrooms and enrolling a number of illiterate people in each classroom. The classroom will be at most 25 and at least 5 students (Garg and Sharma, 2011).

In this study MIP will be solved with fuzzy non-linear parameters. The fuzzy parameter assumed as triangle member ship function. The fuzzy ranking functions

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will be used to deal with the fuzzification then swarm optimization approach to solve the problem at hand.

MATERIALS AND METHODS

FMNIP definition: FMNIP is an important problem that faces the developing countries and also rich countries. This problem is considered as a decision problem concerning the location to choose the best sites to build illiteracy classrooms in suitable positions. It is needed to study the place that will be used in construction process. We have a certain place contains a group of centers and each center have a number of local units there are some Rural villages in each local unit, finally, it is needed to establish a number of classrooms in each rural village to educate illiterate people in this certain place. The chosen locations must be suitable to the circumstances for the residents. To solve this problem, it must be study the illiteracy from some points of views (Garg and Sharma, 2013).

People who need to learn (writing and reading) and persuade them with the idea of elimination of illiteracy, it must be choose the nearest locations to save costs of transportation whether are money, time and distance (Garg *et al*., 2012).

On the other hand, it is important to discuss the budget factor and make it minimum as much as possible. To construct a classroom in a certain site must need a setup cost for construction process, material costs for (chalks, blackboards, etc.) and teaching costs for teachers who teach for the illiterate students (Garg and Sharma, 2011).

Teaching costs must be reduced by selecting teachers from the same place where non learning population lives. The total cost for the establishing process and transportation cost are essential to consider and compute for all classrooms in all rural villages in all local units in all center's for the place (governorate) under consideration. All these mentioned costs must not exceed the planned budget (Zadeh, 1965).

Distance is playing an important role in this application. Distance here means (the distance of transportation from the residents student who want to learn and the location of the classroom). This study must take into account all distances for all students in all classrooms in all rural villages in all local units in all center's for the studying governorate. This parameter must be considered by calculating the shortest path. for this application, it is in need the GIS tool (such as ARCGIS, MAP-OBJECT, .., etc.) to deal with maps related to maps for the place (governorate) at hand and to compute distances especially the shortest distance between the suitable sites of the classrooms and the homes of the residents (Bellman and Zadeh, 1970). The main target of this research is to choose the best sites to establish illiteracy classrooms for illiteracy students and determine the number of enrolled students in each classroom (Karwowski and Mittal, 1986).

All these restrictions and circumstances must be kept in FMNIP model. In this model, the costs will be described as a fuzzy with triangular membership function. FMNIP; fuzzy mathematical model will be converted to the crisp model by Jimenez's Technique called fuzzy ranking functions (Garg and Sharma, 2011)

Problem parameters: The parameters that are needed for formulation are defined as:

- c; Represents the cth center, c = 1, 2, ..., c
- u; Represents the uth local unit in the center cth, u = 1, 2, …, u
- v; Represents a vth rural village for a uth local unit for a cth center, v =1, 2, …, v
- r; Represents a rth classroom for a vth rural village in a uth local unit in a cth center, r = 0,1,2,…,r
- Sc_{cuvr}; Represents the setup cost of a rth classroom in a vth rural village in a uth local unit in a cth center
- MSc; Represents the available maximum setup costs of the allocated classrooms in the whole governorate
- Rc_{cuvr}; Represents a running cost of a rth classroom in a vth rural village in a uth local unit in a cth center
- Mrc; Represents the available maximum running cost of the allocated classrooms in the whole governorate
- Tc_{cuvr}; Represents the sum of Sc_{cuvr} and Rc_{cuvr}
- Mtc; Represents the available maximum total cost of the allocated classrooms in the whole governorate
- Trc_{cuvr}; Represents the transportation cost of a rth classroom in a vth rural village in a u_{th} local unit in a c_{th} center
- MTRc; Represents the maximum transportation cost for all illiteracy registered students for all classrooms in the Governorate
- TRt_{cuvr}; The transportation time of of a rth classroom in a vth rural village in a uth local unit in a cth center
- MTRt; Represents maximum transportation time for all illiteracy registered students for all classrooms in the whole governorate
- Trd_{cuvr}; The transportation distance of a rth classroom in a vth rural village in a uth local unit in a cth center
- MTRd; Represents the maximum transportation distance for all illiteracy registered students for all classrooms in the whole Governorate

- MNr; Represents the maximum number of allocated building classrooms in the whole governorate
- MNi; Represents the maximum illiteracy inhabitants in the whole governorate

Decision variables: The decision variables of the model are as follows:

- $Clas_{cuvr}$: Decision variable takes the value 1 if a rth classroom located in a vth rural village in a uth local unit in a cth center and takes 0 otherwise
- Num_{cuvr}: Represents the Number of enrolled students for a rth classroom in a vth rural village in uth local unit in a cth center

FMNIP formulation: FMNIP will be described by 5 objectives functions from Eq. 1-5 and some constraints represented from Eq. 7-15. There exist some fuzzy parameters denoted by symbol. For the objectives functions Eq. 1 expresses the maximization of all classrooms r in all rural villages v in all local units u in all centers c in the whole governorate. In Eq. 2 represents the minimization of the total costs of the all classrooms (such that setup and running cost) in that whole place under studying (Zadeh, 1965).

On the other hand there exist three objectives belongs to the transportation costs in terms of (money, time and distances) for all classrooms R in all villages V in all local Units U in all in all Centers C. These equations formulated by Eq. 3-5. In addition, it must be observed the symbol. This notation concerns the fuzzy parameters (Garg *et al*., 2012).

It is found the three fuzzy parameters in Eq. 2-4 for total cost, transportation cost and transportation time, respectively. In Eq. 5 it is shown the transportation distance parameter (Tsd_{cuvr}) is stated by calculating of the shortest path (from the location of the homes of residents to the location of illiteracy classrooms) in all rural villages in all local units in all centers. This parameter needs Geographic Information Systems (GIS) tool to deal with the shortest path effectively. Afterwards, the objective functions will be formulated as follows (Garg *et al*., 2013):

$$\text{Max } Y_1 = \sum_{c=1}^{C}\sum_{U=1}^{U}\sum_{V=1}^{V}\sum_{r=0}^{R} Clas_{cuvr} \quad (1)$$

$$\text{Min } Y_2 = \sum_{c=1}^{C}\sum_{U=1}^{U}\sum_{V=1}^{V}\sum_{r=0}^{R} Tc_{cuvr} Clas_{cuvr} \quad (2)$$

$$\text{Min } Y_3 = \sum_{c=1}^{C}\sum_{U=1}^{U}\sum_{V=1}^{V}\sum_{r=0}^{R} TRC_{cuvr}\, Num_{cuvr}\, Clas_{cuvr} \quad (3)$$

$$\text{Min } Y_4 = \sum_{c=1}^{C}\sum_{U=1}^{U}\sum_{V=1}^{V}\sum_{r=0}^{R} TRt_{cuvr}\, Num_{cuvr}\, Clas_{cuvr} \quad (4)$$

$$\text{Min } Y_5 = \sum_{c=1}^{C}\sum_{U=1}^{U}\sum_{V=1}^{V}\sum_{r=0}^{R} TRd_{cuvr}\, Num_{cuvr}\, Clas_{cuvr} \quad (5)$$

All of the fuzzy parameters are used for the FMNIP models are defined as triangular fuzzy parameters and applying the membership functions in Eq. 6 (Garg and Sharma, 2013). If Ã is a fuzzy parameter as triangular, the membership function (Bellman and Zadeh, 1970) is formed by a three variables (e_1; e_2; e_3) as follows:

$$\mu_1(y) = \begin{cases} \dfrac{y-e_1}{e_2-e_1} & \text{if } e_1 \le y \le e_2 \\ \dfrac{y-e_2}{e_2-e_3} & \text{if } e_1 \le y \le e_2 \\ 0 & \text{Otherwise} \end{cases} \quad (6)$$

There exist some constraints according to the mentioned circumstances. These constraints will be formulated from Eq. 7-15. In this research there are four categories. The first category belongs to the classrooms represented by Eq. 7. Afterwards, the Eq. 8 and 9 described the second category related to illiteracy students:

$$\sum_{c=1}^{C}\sum_{u=1}^{U}\sum_{v=1}^{V}\sum_{r=0}^{R} Clas_{cuvr} \le MNr \quad (7)$$

$$\sum_{c=1}^{C}\sum_{u=1}^{U}\sum_{v=1}^{V}\sum_{r=0}^{R} Num_{cuvr}\, Clas_{cuvr} \le MNi \quad (8)$$

$$5 \le Num_{cuvr} \le 25 \quad (9)$$

On the other hand, the third category is very essential and concerns the total cost; Tc_{cuvr} for classroom r in rural village v in local unit u in center c. Tc includes two costs (setup cost Sc and running cost Rc). These costs will be computed for all classrooms R in all villages V in local Unit U in Centers C. it is observed some parameters related to the available budget for the maximum (total cost, setup cost and running cost) denoted by (MTc, MSc and MRc), respectively. For constraints in Eq. 10-12 the Tc_{cuvr} , Sc_{cuvr} and Rc_{cuvr} are triangular fuzzy parameters according to membership function defined in Eq. 6 must not exceed MTc, MSc and MRc, respectively:

$$\sum_{c=1}^{C}\sum_{u=1}^{U}\sum_{v=1}^{V}\sum_{r=0}^{R} TC_{cuvr}\, Clas_{cuvr} \le MTc \quad (10)$$

$$\sum_{c=1}^{C}\sum_{u=1}^{U}\sum_{v=1}^{V}\sum_{r=0}^{R} Sc_{cuvr}\, Clas_{cuvr} \le MSc \quad (11)$$

$$\sum_{c=1}^{C}\sum_{u=1}^{U}\sum_{v=1}^{V}\sum_{r=0}^{R} Rc_{cuvr}\, Clas_{cuvr} \le MRc \quad (12)$$

Finally, Eq. 13-15 represented the last category which include the constraints of transportation costs in terms of (money, time and distance) (Garg *et al.,* 2012). TSC_{cuvr}, Tst_{cuvr} and Tsd_{cuvr} described the transportation cost, time and distance, respectively, in all classrooms R in all rural Villages V in local Unit U in Centers C. the cost of transportation (money and time) are formed as fuzzy parameters with triangular form according to Eq. 6. The total transportation cost, time and distance also should be within (less than or equal) the planned total values MTST and MTRd, respectively (Garg and sharma 2011):

$$\sum_{c=1}^{C}\sum_{u=1}^{U}\sum_{v=1}^{V}\sum_{r=0}^{R} TRc_{cuvr}\, Num_{cuvr}\, Clas_{cuvr} \le MTRc \quad (13)$$

$$\sum_{c=1}^{C}\sum_{u=1}^{U}\sum_{v=1}^{V}\sum_{r=0}^{R} TRT_{cuvr}\, Num_{cuvr}\, Clas_{cuvr} \le MTRt \quad (14)$$

$$\sum_{c=1}^{C}\sum_{u=1}^{U}\sum_{v=1}^{V}\sum_{r=0}^{R} TRd_{cuvr}\, Num_{cuvr}\, Clas_{cuvr} \le MTRd \quad (15)$$

Ranking method for fuzzy numbers through expected intervals: Jimenez (Garg and Sharma 2013) has reported a ranking function for fuzzy parameters in view of the correlation of their expected intervals. This strategy is utilized for applying with the given FMNIP model with fuzzy parameters. The membership function of a fuzzy parameter $\tilde{N} = (n_1, n_2, n_3, n_4)$ can be written as Eq. 16 (Garg and Sharma, 2013):

$$\mu N(x) = \begin{cases} 0 & \forall x \in (-\infty, n_1], \\ h_N(x) & \forall x \in [n_1, n_2], \\ 1 & \forall x \in [n_2, n_3], \\ t_N(x) & \forall x \in [n_3, n_4], \\ 0 & \forall x \in (n_4, \infty] \end{cases} \quad (16)$$

So, as to warrant the presence and integrability of the inverse functions $h^{-1}_N(x)$ and $t^{-1}_N(x)$, it is supposed that $h_N(x)$ and $t_N(x)$ are continuous however $h_N(x)$ is increasing and $t_N(x)$ is decreasing. The Expected Interval (EXI) of a fuzzy variable is described as Garg and Sharma (2013):

$$EXI(\tilde{N}) = \left[G_i^A, G_2^A\right] = [\int_{n1}^{n2} x dh_N(x) \; \int_{n3}^{n4} x dt_N(x)] \quad (17)$$

Mathematically, the integration method will be applied (Integration by parts method) and will be changed the variable $\lambda = h_N(x)$, $\lambda = t_N(x)$:

$$EXI(\tilde{N}) = \left[G_1^A, G_2^A\right] = \left[\int_2^1 h_N^{(-1)}(()d)'', -\int_0^1 t_N^{(-1)}(()''d\right] \quad (18)$$

It is notably, the h_N and t_N are linear functions, however the expected interval will be defined differently as in Eq. 19 (Karwowski and Mittal, 1986) if the parameter Ñ is triangular or trapezoidal. if the number Ñ is a fuzzy number in a triangular form then $n_2=n_3=n$ and can be written as $\tilde{N} = (n_1, n, n_4)$:

$$EXI(\tilde{N}) = \left[\frac{1}{2}(n_1 + n_2), \frac{1}{2(n_3 + n_4)}\right] \quad (19)$$

If there are two fuzzy parameters Ñ and O the expected interval of Ñ-O is:

$$EXI(N - O) = \left[G^A{}_1 - G^O{}_2, G^A{}_2\right] = EXI(N)EXI(O) \quad (20)$$

Moreover, for any two of fuzzy parameters Ñ and O the degree for which Ñ is larger than O is showed as in Eq. 21. This equation was reported according to Jimenez method (Karwowski and Mittal, 1986):

$$\mu_m(N,O) = \begin{cases} 0 \text{ if } G_2^A - G_1^{\tilde{O}} < 0 \\ \dfrac{G_2^A - G_1^O}{(GN_2^n - G_1^{\tilde{O}}) - (G_1^N - G_2^{\tilde{O}})} \text{ if } 0 \in \left[\left(G_1^{\tilde{O}} - G_1^{\tilde{O}}\right), G_1^N - G_1^{\tilde{O}}\right] \\ 1 \text{ if } G_1^A - G_2^{\tilde{O}} > 0 \end{cases} \quad (21)$$

Equation 21 expressed the level of preference (the Ñ is preferred of O according to the μ_m (Ñ, õ). if μ_m (Ñ, õ) is 0.5 then the Ñ and õ are equal in the preference. This means the expected value of a fuzzy parameter is the half of its expected interval (Garg and Sharma 2013):

$$EXV(N) = \frac{G^{A}_{1} + G^{O}_{2}}{2} \quad (22)$$

If the fuzzy parameter Ñ is triangular or trapezoidal, it maps each fuzzy parameter by expected value as (Garg and Sharma, 2012):

$$EXV(\tilde{N}) = \frac{n_1 + n_2 + n_3 + n_4}{4} \quad (23)$$

Because of the μ_m (Ñ, O) is the level of preference of Ñ over O therefore assume μ_m (Ñ, O) ≥ λ this means that Ñ is greater than or equal to O at least in degree λ and it will be formed as Ñ≥O For two types of the constraints as the following (Karwowski and Mittal, 1986):

$$\tilde{N}_i x \geq \tilde{O}_i \;\; i = 1,\ldots, m \quad (24)$$

$$\tilde{N}_i x \leq \tilde{O}_i \, i = m + 1,\ldots, t \quad (25)$$

According to the Jimenez, Garg and Sharma (2013) a vector of decision $x \in R^n$ is feasible for λ degree if $\min_{i=1,\ldots,m} = \{\mu_m (Nx, \hat{o}\} = \lambda$. According to the membership function defined in Eq. 16 Ñ≤(ô I = 1, ..., m) is equivalent to:

$$\begin{aligned}&(G_2^{x_{N(i)}} - G_1^{xN_{(1)}}) / (G_2^{x_{N(i)}} - G_1^{xN_{(i)}}) - \\ &(G_2^{x_{N(i)}} - G_1^{N_{(1)}}) - (G_1^{N_i} - G_2^{Ni}) \geq i = 1\ldots, m\end{aligned} \quad (26)$$

And, Eq. 26 can be reformed as:

$$\begin{aligned}&\left[(1-"(")G_2^{N_{(i)}} + "("G_1^{N_{(i)}})\right] \\ &x \geq (G_2^{N_i} + (1-()G_1^{N_i} = 1,\ldots,m\end{aligned} \quad (27)$$

We can do the Eq. 28 for $N_i x \leq O_i$ so the equation equivalent to the following respectively (Garg and Sharma, 2012):

$$\left[(G_2^{N1} + (1-()G_1^{N_i}\right]x \quad (28)$$

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