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An Algorithm for Computing the Mittag-leffler Type Function in the Maple Symbolic Mathematics Package

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Abstract: The study proposes algorithms for the mittag-leffler function to be correctly calculated with a fractional parameter $1<\alpha<2$ in the MAPLE mathematical package.

Key words: Fractal parametric oscillator, Mittag-Leffler type function, Hankel contour, integral representation, Russia

INTRODUCTION

The mathematical modelling of fractal processes has gained widespread currency. Fractal processes are understood as various processes of natural, economic, and sociohistorical phenomena with respect to their fractal properties. A mathematical interpretation of such phenomena is provided by fractional calculus, the framework that has been well devised for over three centuries (Nakhushev, 2003).

A distinctive feature of mathematical constructions of fractal models is the extension of integer equations to real ones which brings us to power law solutions. Such solution is often a special entire function of the Mittag-Leffler type (Leffler, 1905).

Computation of this function by definition in systems such as MAPLE, often produces incorrect results. It cannot either be computed or yields the wrong answer due to error. Therefore, this study dwells on the algorithm for computing this function but first presents some of its properties.

The Mittag-Leffler type function and its properties: In study of Leffler (1905) Mittag-Leffler introduced a generalized exponential function for consideration:

$$E_\alpha(z)=\sum_{k=0}^{\infty}\frac{z^k}{\Gamma(1+\alpha k)}, z\in C, \alpha\in[0,\infty] \quad (1)$$

Here, $\Gamma(x)$ is the Euler gamma function. Ratio Eq. 1 shows that if $z = 0$, the Mittag-Leffler function becomes a constant $E_\alpha(0) = 1$ and in cases where $\alpha = 1$, it becomes an exponent $E_1(z) = \exp(z)$. For greater detail on the properties of the Mittag-Leffler function, the reader is referred to study of Leffler (1905). Study of Dzherbashyan (1966) examines the Mittag-Leffler type function:

$$E_\alpha(z)=\sum_{k=0}^{\infty}\frac{z^k}{\Gamma(1+\alpha k)}, z\in C, \alpha\in[0,\infty]$$

Function Eq. 2 is commonly referred to as the Mittag-Leffler type function. It provides a solution of many differential equations in fractional derivatives. For example, the fractal oscillator Eq. 4 including with the Caputo fractional derivative in this researchers study (Parovik, 2012):

$$\partial_{0t}^{\beta}u(\tau)+\omega^{\beta}u(t)=0, 1<\beta<2 \quad (3)$$

Has the following solution:

$$u(t)=C_1E_{\beta,1}\left(-(\omega t)^{\beta}\right)+C_2tE_{\beta,2}\left(-(\omega t)^{\beta}\right)$$

Where C_1 and C_2 are integration constants determined by initial conditions. Similarly, one can obtain functions of the Mittag-Leffler type in this researcher's studies Parovik (2013, 2014). Notably, there are some properties of the Mittag-Leffler type function arising from definition Eq. 2:

$$E_{\alpha,\beta}(0)=\frac{1}{\Gamma(\beta)}, E_{\alpha,1}(z)=E_\alpha(z)\Rightarrow E_{1,1}(z)=\exp(z);$$

$$E_{1,2}(z)=\frac{\exp(z)-1}{z}, E_{2,1}(z)=\mathrm{ch}\left(\sqrt{z}\right);$$

$$E_{2,2}(z)=\mathrm{sh}\left(\sqrt{z}\right)/\sqrt{z};$$

$$E_{\alpha,\beta}(z)=\frac{1}{\Gamma(\beta)}+zE_{\alpha,\beta+\alpha}(z), E_{\alpha,\beta}(z);$$

$$=\beta E_{\alpha,\beta}(z)+z\alpha\frac{dE_{\alpha,\beta+1}(z)}{dz};$$

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$$\left(\frac{d}{dz}\right)^m \left(z^{\beta-1}E_{\alpha,\beta}\left(z^\alpha\right)\right) = z^{\beta-m-1}E_{\alpha,\beta-m}\left(z^\alpha\right), m \geq 1;$$

$$E_{\alpha,\beta}(z) = \frac{1}{m}\sum_{k=0}^{m-1} E_{\alpha/m,\beta}\left(z^{1/m}\exp(2\pi ki/m)\right),$$

$$\int_0^z z^{\beta-1}E_{\alpha,\beta}\left(\lambda t^\alpha\right)t^{\beta-1}dt = zE_{\alpha,\beta+1}\left(\lambda z^\alpha\right)$$

Asymptotic expansions of the Mittag-Leffler type function also hold true at higher values $|z|$: when $|\arg(z)| \leq \alpha_1\pi$, $\alpha_1 \in (\alpha/2, \alpha)$, $\alpha \in (0, 2)$, $n \in N$ we obtain:

$$E_{\alpha,\beta}(z) = \frac{1}{\alpha} z^{\frac{1-\beta}{\alpha}} \exp\left(z^{\frac{1}{\alpha}}\right)$$

$$-\sum_{k=1}^{n} \frac{z^{-k}}{\Gamma(\beta-\alpha k)} + O\left(\frac{1}{|z|^{n+1}}\right)$$

б) whe $\alpha_1\pi \leq |\arg(z)| \leq \pi, \alpha_1 \in (\alpha/2, \alpha), \alpha \in (0,2), n \in N$ n, we obtain:

$$E_{\alpha,\beta}(z) = -\sum_{k=1}^{n} \frac{z^{-k}}{\Gamma(\beta-\alpha k)} + O\left(\frac{1}{|z|^{n+1}}\right)$$

if $\alpha \geq 2, n \in N$, we obtain:

$$E_{\alpha,\beta}(z) = -\sum_{k=1}^{n} \frac{z^{-k}}{\Gamma(\beta-\alpha k)} + O\left(\frac{1}{|z|^{n+1}}\right)$$

summation with respect to the first item is taken over those m where the following condition is met:

$$|\arg(z) + 2\pi m| \leq \varepsilon + \alpha\pi/2, \varepsilon > 0$$

Proofs concerning asymptotic expansions of the Mittag-Leffler type function is to be found in study (Dzherbashyan, 1966).

Integral representation of the Mittag-Leffler type function: We assume that $\beta>0$. Let us consider the Hankel contour (loop) $\gamma(\varepsilon, \delta)$ (Fig.1) which comprises three parts as follows: ray $S_\delta = \{\arg(\xi) = -\delta, |\xi| \geq \varepsilon\}$, ray $S_{-\delta} = \{\arg(\xi) = \delta, |\xi| \geq \varepsilon\}$ and an arc of a circle $C_\delta(0,\varepsilon) = \{-\delta \leq \arg(\xi) \leq \delta, |\xi| = \varepsilon\}$. In its turn, the contour $\gamma(\varepsilon, \delta)$ divides ξ into two infinite subdomains: $G^{(-)}(\varepsilon, \delta)$ and $G^{(+)}(\varepsilon, \delta)$. Study (Dzherbashyan, 1966) shows that if the conditions $0 < \alpha < 2\alpha\pi < \delta \leq \min\{\pi, \pi\alpha\}$ are met, the following integral representations occur:

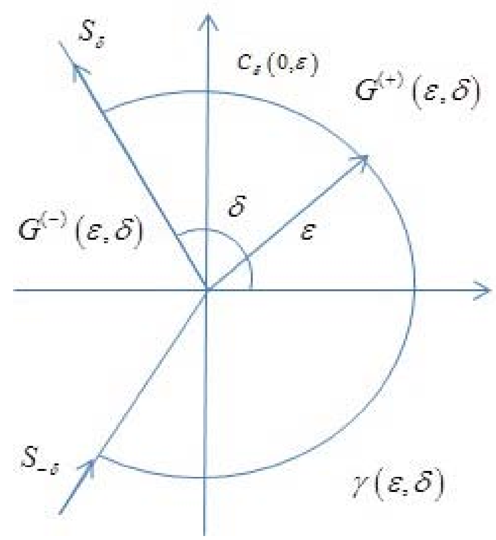

Fig. 1: Hankel contour

$$E_{\alpha,\beta}(z) = \frac{1}{2\alpha\pi i}\int_{\gamma(\varepsilon,\delta)} \frac{\exp\left(\xi^{1/\alpha}\right)\xi^{\frac{1-\beta}{\alpha}}d\xi}{\xi - z}, z \in G^{(-)}(\varepsilon,\delta) \quad (4)$$

$$E_{\alpha,\beta}(z) = \frac{1}{\alpha} z^{\frac{1-\beta}{\alpha}} \exp\left(z^{\frac{1}{\alpha}}\right) +$$

$$\frac{1}{2\alpha\pi i}\int_{\gamma(\varepsilon,\delta)} \frac{\exp\left(\xi^{1/\alpha}\right)\xi^{\frac{1-\beta}{\alpha}}d\xi}{\xi - z}, z \in G^{(+)}(\varepsilon,\delta)$$

An integral in ratio Eq. 3 is computed over each part of the contour $\gamma(\varepsilon, \delta)$ with consideration $\xi = \varepsilon\exp(i\delta)$ for and based on the findings of study (Gorenflo *et al*., 2002):

$$E_{\alpha,\beta}(z) = \int_\varepsilon^\infty K(\alpha,\beta,\delta,r,z)\,dr +$$

$$\int_{-\alpha\pi}^{\alpha\pi} P(\alpha,\beta,\varepsilon,\phi,z)d\phi, z \in G^{(-)}(\varepsilon,\delta),$$

$$E_{\alpha,\beta}(z) = \frac{1}{\alpha} z^{\frac{1-\beta}{\alpha}} \exp\left(z^{1/\alpha}\right) + \int_\varepsilon^\infty K(\alpha,\beta,\delta,r,z)\,dr + \quad (5)$$

$$\int_{-\alpha\pi}^{\alpha\pi} P(\alpha,\beta,\varepsilon,\phi,z)d\phi, z \in G^{(-)}(\varepsilon,\delta)$$

$$K(\alpha,\beta,\delta,r,z) = \frac{1}{\alpha\pi} r^{\frac{1-\beta}{\alpha}} \exp\left(r^{\frac{1}{\alpha}}\cos(\delta/\alpha)\right)$$

$$\frac{\left[r\sin(\varphi-\delta) - z\sin(\varphi)\right]}{r^2 - 2rz\cos(\delta) + z^2},$$

$$P(\alpha,\beta,\varepsilon,\phi,z)=\frac{\varepsilon^{1+(1-\beta)/\alpha}}{2\alpha\pi}\exp\left(\varepsilon^{\frac{1}{\alpha}}\cos(\phi/\alpha)\right)\frac{(\cos(\omega)+i\sin(\omega))}{\varepsilon\exp(i\phi)-z},$$

$$\varphi=r^{\frac{1}{\alpha}}\sin(\delta/\alpha)+\delta(1+(1-\beta)/\alpha),$$

$$\omega=\varepsilon^{\frac{1}{\alpha}}\sin(\phi/\alpha)+\phi(1+(1-\beta)/\alpha)$$

Integral representation Eq. 4 comprises two parts (integrals), the first integral is monotone while the second one is oscillation. Now, we consider a case where 0<α<1. Then we obtain that $\delta=\min\{\pi,\alpha\pi\}=\alpha\pi$. Therefore, we have:

$$K(\alpha,\beta,\alpha\pi,r,z)=\bar{K}(\alpha,\beta,r,z)=\frac{r^{\frac{1-\beta}{\alpha}}}{\alpha\pi}\frac{r\sin(\pi(1-\beta))-z\sin(\pi(1-\beta+\alpha))}{r^2-2rz\cos(\alpha\pi)+z^2}$$

Three cases are possible here: $|\arg(z)|=\alpha\pi$, $|\arg(z)|>\alpha\pi$, $|\arg(z)|<\alpha\pi$ study of Gorenflo *et al.* (2002) establishes the theorems for these cases. Let us provide the results. In the first case, the Mittag-Leffler type function can be computed using the following ratio:

$$E_{\alpha,\beta}(z)=\int_{\varepsilon}^{\infty}\bar{K}(\alpha,\beta,r,z)dr+\int_{-\alpha\pi}^{\alpha\pi}P(\alpha,\beta,\varepsilon,\phi,z)d\phi,\varepsilon\ge|z| \quad (6)$$

In the second case:

$$E_{\alpha,\beta}(z)=\begin{cases}\int_{\varepsilon}^{\infty}\bar{K}(\alpha,\beta,r,z)dr,\beta<1+\alpha,\\ -\frac{\sin(\alpha\pi)}{\alpha\pi}\int_{\varepsilon}^{\infty}\frac{\exp(-r^{1/\alpha})dr}{r^2-2rz\cos(\alpha\pi)+z^2}-\frac{1}{z},\ \beta=1+\alpha,\\ \int_{\varepsilon}^{\infty}\bar{K}(\alpha,\beta,r,z)dr+\int_{-\alpha\pi}^{\alpha\pi}P(\alpha,\beta,\varepsilon,\phi,z)d\phi+\frac{z^{\frac{1-\beta}{\alpha}}\exp(z^{1/\alpha})}{\alpha},\varepsilon>0,\beta>0\end{cases} \quad (7)$$

In the third case:

$$E_{\alpha,\beta}(z)=\begin{cases}\int_{\varepsilon}^{\infty}\bar{K}(\alpha,\beta,r,z)dr+\frac{z^{\frac{1-\beta}{\alpha}}\exp\left(z^{\frac{1}{\alpha}}\right)}{\alpha},\beta<1+\alpha,\\ -\frac{\sin(\alpha\pi)}{\alpha\pi}\int_{\varepsilon}^{\infty}\frac{\exp\left(-r^{\frac{1}{\alpha}}\right)dr}{r^2-2rz\cos(\alpha\pi)+z^2}+\frac{z^{\frac{1-\beta}{\alpha}}\exp\left(z^{\frac{1}{\alpha}}\right)}{\alpha},\beta=1+\alpha,\\ \int_{\varepsilon}^{\infty}\bar{K}(\alpha,\beta,r,z)dr+\int_{-\alpha\pi}^{\alpha\pi}P(\alpha,\beta,\varepsilon,\phi,z)d\phi+\frac{z^{\frac{1-\beta}{\alpha}}\exp\left(z^{\frac{1}{\alpha}}\right)}{\alpha},\varepsilon>0,\beta>0\end{cases} \quad (9)$$

Algorithm for computing the Mittag-Leffler type function: In integral representations Eq. 5-7 an improper integral is of interest. It can be presented as a definite integral. According to study of Gorenflo *et al.* (2002) let us, introduce a fixed constant q = 0.9 and consider the following three cases: $|z|\le q, 0<\alpha$, $|z|>q, 0<\alpha\le 1$, $|z|>q, 1<\alpha$ In the first case according to Eq. 2:

$$E_{\alpha,\beta}(z)=\sum_{k=0}^{k_0}\frac{z^k}{\Gamma(\beta+\alpha k)}+\eta(z),|\eta(z)|<\rho,$$

$$k_0=\max\left\{\lfloor(1-\beta)/\alpha\rfloor+1,\lfloor\ln(\rho(1-|z|))/\ln(|z|)\rfloor\right\}$$

here ρ is the accuracy of computation, $\lfloor\ \rfloor$ is the integer part of a number. In the second case:

$$\int_a^{\infty}\bar{K}(\alpha,\beta,r,z)dr=\int_a^{r_0}\bar{K}(\alpha,\beta,r,z)dr+\eta(z),|\eta(z)|<\rho,a\in(0,\varepsilon),$$

$$r_0=\max\left\{1,2|z|,(-\ln(\pi\rho/6))^{\alpha}\right\}$$

In the third case with account of property 5 where $m=\lfloor\alpha\rfloor+1$ and $0<\alpha/m\le 1$, the Mittag-Leffler type function is computed by this equation:

$$E_{\alpha,\beta}(z)=\frac{1}{m}\sum_{k=0}^{m-1}E_{\alpha/m,\beta}\left(z^{1/m}\exp(2\pi ik/m)\right)$$

for cases:

$$|z|^{\frac{1}{m}}\le q, |z|^{\frac{1}{m}}>q$$

Computation of the Mittag-Leffler type function using the mathematical software MAPLE: Suppose that $\rho = 10^{-4}$. Taking into account the techniques described above and the Mittag-Leffler type function properties, let us generate the following procedure ML1 in the MAPLE programming language:

MAPLE programming language:

```
>ML1:= proc (alpha, beta, x)
 local k0, phi, r, rho, r0;
r0:=evalf(max(1, 2*abs(x), trunc(-
 ln(0.0001*Pi/6))^alpha)); if x=0 then
 1/GAMMA(beta)
elif alpha=1 and beta=1 then evalf(exp(x))
 elif alpha=1 and beta=2 then evalf((exp(x)-1)/x)
 elif alpha=2 and beta=2 then evalf(sinh(sqrt(x))/sqrt(x))
 elif alpha = 2 and beta = 1 then evalf(cosh(x^(1/2)))
 elif 0<alpha and alpha < 1 then
 if abs(x)<0.9 then
 k0:=max(trunc((1-beta)/alpha)+1, trunc(ln(0.0001*(1-abs(x)))/ln(abs(x))));
 sum(x^k/GAMMA(beta+alpha*k), k=0 .. k0)
 elif abs(x)<trunc(10+5*alpha) then
 if alpha*evalf(Pi)<abs(evalf(argument(x))) and
0.0001< abs(evalf(abs(argument(x)))-
 alpha*evalf(Pi)) then if beta<1+alpha then
evalf(Int(r^((1-beta)/alpha)*exp(-r^(1/alpha))*(r*sin(Pi*(1-beta))-
x*sin(Pi*(1-beta+alpha)))/(alpha*Pi*(r^2-2*x*r*cos(alpha*Pi)+x^2)),
r=0..r0)) else evalf(Int(r^((1-beta)/alpha)*exp(-r^(1/alpha))*(r*sin(Pi*(1-
beta))-x*sin(Pi*(1-beta+alpha)))/(alpha*Pi*(r^2-2*x*r*cos(alpha*Pi)+x^2)),
r = 1..r0))+ evalf(Int(0.5*exp(cos(phi/alpha)*
exp(I*(phi*(1+(1-beta)/alpha)+sin(phi/alpha))))/(alpha*Pi*(exp(I*phi)-x)),
phi = alpha*evalf(Pi).-alpha*evalf(Pi)))
 end if
 elif abs(x)<trunc(10+5*alpha) and abs(argument(x))<alpha*
evalf(Pi) and 0.0001<abs(abs(argument(x))-alpha*evalf(Pi)) then
 if beta<1+alpha then evalf(Int(r^((1-beta)/alpha)*exp
(r^(1/alpha))*(r*sin(Pi*(1-beta))-x*sin(Pi*(1-beta+alpha)))
/(alpha*Pi*(r^2-2*x*r*cos(alpha*Pi)+x^2)),
r = 0..r0))+x^((1-beta)/alpha)*exp(x^(1/alpha))/alpha
 else evalf(Int(r^((1-beta)/alpha)*exp(-r^(1/alpha))*(r*sin(Pi*
(1-beta))-x*sin(Pi*(1-beta+alpha)))/
(alpha*Pi*(r^2-2*x*r*cos(alpha*Pi)+x^2)),
r=0.5*abs(x)..r0)+simplify
(evalf(Int(0.5*(0.5*abs(x))^(1+(1-beta)/alpha)*exp((0.5*abs(x))^
(1/alpha)*cos(phi/alpha))*(cos(phi*(1+(1-beta)/alpha)+
(0.5*abs(x))^(1/alpha)*sin(phi/alpha))+I*
sin(phi*(1+(1-beta)/alpha)+(0.5*abs(x))^(1/alpha)*sin(phi/alpha)))/
(alpha* Pi*(0.5*abs(x)*exp(I*phi)-x)), phi=alpha*Pi.. -alpha*Pi)))+
x^((1-beta)/alpha)*exp(x^(1/alpha))/alpha
end if else evalf(Int(r^((1-beta)/alpha)*
exp(-r^(1/alpha))*(r*sin(Pi*(1-beta))-
x*sin(Pi*(1-beta+alpha)))/(alpha*Pi*(r^2-2*x*r*cos(alpha*Pi)+x^2)),
r = 0.5*abs(x)+0.5..r0))+simplify(evalf(Int(0.5*(0.5*abs(x)+0.5)
^(1+(1-beta)/alpha)
*exp((0.5*abs(x)+0.5)^(1/alpha)*cos(phi/alpha))*(cos
(phi*(1+(1-beta)/alpha)+(0.5*abs(x)+0.5)^(1/alpha)*sin(phi/alpha))+I*sin
(phi*(1+(1-beta)/alpha)+(0.5*abs(x)+0.5)^(1/alpha)*sin(phi/alpha)))/
(alpha*Pi*((0.5*abs(x)+0.5)*exp(I*phi)- x)), phi=alpha*Pi..alpha*Pi)))+
x^((1-beta)/alpha)*exp(x^(1/alpha))/alpha
 end if else k0:=trunc(-ln(0.0001)/ln(abs(x)));
if abs(evalf(argument(x)))<0.75*alpha*evalf(Pi) then x^((1-beta)/alpha)*
exp(x^(1/alpha))/alpha-(sum(x^(-k)/GAMMA(beta-alpha*k),k=0..k0))
else simplify(evalf(-(sum(x^(-k)/GAMMA(beta-alpha*k), k=0..k0))))
end if
end if
end if
end proc:
```

In cases where $1<\alpha<2$ or m = 2, let us generate an ML2 procedure:

```
>ML2:=proc(a,b,z)
if z= 0 then 1/GAMMA(b) elif a = 2 and b =1 then cosh(sqrt(z));
elif a=2 and b=2 then evalf(sinh(sqrt(z))/sqrt(z)) elif 1<a and a<2 then
0.5*(ML1(a/2,b,z^0.5)+ML1(a/2,b,-z^0.5))
fi;
end proc:
```

RESULTS AND DISCUSSION

Computation results of the Mittag-Leffler type function:
Example 1: First, we compute the Mittag-Leffler type function by definition Eq. 2:

- >a1: = 0.7; a1: = 0.7
- >b1: = 1.3; b1: = 1.3
- >z: = 0.7; z: = 0.7
- > t: = time(); # computation time; t = 3391.446
- > E: = sum(z^k/GAMMA(b1+a1*k), k = 0..infinity); E = 2.029026906-0.5438226316 10^{-9}I
- >TIME: =time()-t; TIME = 20.233

Let us compute the same function using ML1:

- >t1: = time(); t1 = 3431.959
- >ML1 (0.7, 1.3, 0.7); 2.294030341
- >TIME = time()-t1; TIME = 0

The output in this example shows that the techniques considered in the study make it possible to compute the Mittag-Leffler type function with singularities.

Example 2: Suppose $1<\alpha<2$:

- >a1: = 1.3; al = 1.3
- >b1: = 1.3; bl = 1.3
- >z: = 0.7; z = 0.7
- >t:=time(); t: = 3432.022
- >E: = sum(z^k/GAMMA(b1+a1*k),k = 0..infinity); E = 1.530959770-0.2083825586 10^{-9}I
- >TIME = time()-t; TIME = 464.508
- >ML2 (1.3, 1.3, 0.7); 1.707760854
- >TIME = time()-t; TIME = 0

As can be seen from the foregoing example, the ML2 procedure does not only accurately compute the Mittag-Leffler type function, but also does so much faster than by definition (Eq. 2). Let us consider the following example.

Example 3:

- >a1: = 0.6; a1: = 0.6
- >b1: = 2; b1: = 2
- >z: = 0.01; z: = 0.01
- >t: = time(); t: = 33391.102

- >E: = sum(z^k/GAMMA(b1+a1*k), k = 0 infinity); E = 1.007036573
- >TIME = time()-t; TIME = 0.328
- >t1:=time(); t1 = 33391.430
- >ML1(0.6, 2, 0.01); 1.007036098
- >TIME=time()-t1; TIME = 0

The example here illustrates that the numerical algorithm for the Mittag-Leffler type function is consistent with computations according to Eq. 2.

CONCLUSION

The study has reviewed the computational algorithm for the special Mittag-Leffler type function with parameters $0<\alpha<2$ and $\beta>0$. The computation results are in good agreement with definition of the function according to Eq. 2. Notably, the speed of computing the Mittag-Leffler type function is greater than that by definition. This is because we have correctly truncated the infinite series with an error of ρ. The study could be considered as a tutorial and recommended for computing the Mittag-Leffler type function in the MAPLE system. For more detailed information on the computation algorithm for this function, the reader is referred to study of Diethelm *et al.* (2005). Study of Gorenflo *et al.* (2002) proposes an algorithm to compute a two-parameter function of the Mittag-Leffler type.

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