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Fuzzy Parameterized Intuitionistic Fuzzy Soft Expert Set Theory and its Application in Decision Making

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Abstract: In this study, we propose the theory of fuzzy parameterized intuitionistic fuzzy soft expert set theory and define some related concepts pertaining to this notion as well as the basic operations on this concept, namely the complement, union, intersection AND and OR. The basic properties and relevant laws pertaining to this concept such as the De Morgan's laws are proved. Lastly, a generalized algorithm is introduced and applied to the proposed concept of fuzzy parameterized intuitionistic fuzzy soft expert sets in a hypothetical decision making problem.

Key words: Fuzzy parameterized intuitionistic fuzzy soft expert set, fuzzy parameterized soft expert set, soft expert set, fuzzy soft expert set, soft set

INTRODUCTION

Fuzzy set theory by Zadeh (1965), rough set theory by Pawlak (1982), probability theory and interval mathematics are theories that are traditionally used to deal with uncertainty and vagueness. However, all of these theories have their inherent weaknesses and disadvantages as pointed out by Molodtsov (1999). Molodtsov (1999) subsequently introduced the theory of soft sets as a general mathematical tool used to deal with uncertainty and vagueness. After Molodtsov's work on soft sets, Maji *et al.* (2001a, b) generalized Molodtsov's soft sets to establish the notion of fuzzy soft sets and also presented an application of fuzzy soft sets in a decision making problem. Majumdar and Samanta (2010) defined the notion of generalized fuzzy soft sets where the degree is attached with the parameterization of fuzzy sets while defining a fuzzy soft set. Alkhazaleh *et al.* (2011a) introduced the theory of soft expert sets, fuzzy soft expert sets and possibility fuzzy soft sets (Majumdar and Samanta, 2010). Cagman *et al.* (2010, 2011) then introduced the notion of fuzzy parameterized fuzzy soft sets and fuzzy parameterized soft sets as well as their related properties. Alkhazaleh *et al.* (2011b) then introduced the concept of fuzzy parameterized interval-valued fuzzy soft sets as a generalization by Cagman *et al.* (2010) and gave an application of this concept in a decision making problem. Bashir and Salleh (2012) then introduced the notion of fuzzy parameterized soft expert sets while Hazaymeh *et al.* (2012) introduced the notion of fuzzy parameterized fuzzy soft expert sets as a generalization by Bashir and Salleh (2012). In this study, we establish the notion of fuzzy parameterized intuitionistic fuzzy soft expert sets which utilizes the concept of intuitionistic fuzzy sets which is superior to fuzzy sets and thus can better reflect the imprecision, uncertainties and vagueness of the data and the associated problem parameters. This results in a significantly better and improved generalization of intuitionistic fuzzy soft sets which would in turn produce more accurate results, especially in problem solving contexts. The basic operations of this concept, namely the complement, union, intersection, "AND" and "OR" are established and examples are given to illustrate these operations. Finally, a generalized algorithm is given and applied to the fuzzy parameterized intuitionistic fuzzy soft expert set model to solve a decision making problem.

PRELIMINARIES

In this study, we recall some definitions and properties pertaining to soft sets, intuitionistic fuzzy sets,

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intuitionistic fuzzy soft sets, soft expert sets, fuzzy parameterized fuzzy soft sets and its generalizations.

Definition 1: A pair (F, A) is called a soft set over U where F is a mapping given by F: $A \rightarrow P(U)$. In other words, a soft set over U is a parameterized family of subsets of the universe U. For $\varepsilon \in A$, $F(\varepsilon)$ may be considered as the set of ε-elements of the soft set (F, A) oras the ε-approximate elements of the soft set (Molodtsov, 1999).

Definition 2: An intuitionistic fuzzy set A defined over a universe of discourse U is an object in the following form (Maji *et al*., 2001a):

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in U\}$$

where, the function μ_A: $U \rightarrow [0, 1]$ and ν_A: $U \rightarrow [0, 1]$ are the membership function and non-membership function respectively of every element $x \in U$ to set A and $0 \leq \mu_A(x)+\nu_A(x) \leq 1$ for every $x \in U$. In the event that $0 \leq \mu_A(x)+\nu_A(x)<1$, there is a degree of uncertainty that exists for element x with respect to set A. This degree of uncertainty, denoted as $\pi_A(x)$ is defined as $\pi_A(x) = 1-\mu_A(x)-\nu_A(x)$. In general, a high degree of uncertainty implies that there are a lot things that are unknown about element x with respect to set A. From now on, let A and B be intuitionistic fuzzy sets defined over a universal set U and are as defined:

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in U\}$$

$$B = \{\langle x, \mu_B(x), \nu_B(x)\rangle : x \in U\}$$

Definition 3: The subset and equality of two intuitionistic fuzzy sets A and B are as defined (Atanassov, 1986):

- $A \subset B \leftrightarrow \mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in U$
- $A = B \leftrightarrow A \subset B$ and $B \subset A$

Definition 4: The complement, union and intersection of two intuitionistic fuzzy sets A and B are as defined (Atanassov, 1986):

- $\bar{A} = \{\langle x, \nu_A(x), \mu_A(x)\rangle : x \in U\}$
- $A \cup B = \{\langle x, \max(\mu_A(x), \mu_B(x)), \min(\nu_A(x), \nu_B(x))\rangle : x \in U\}$
- $A \cap B = \{\langle x, \max(\mu_A(x), \mu_B(x)), \min(\nu_A(x), \nu_B(x))\rangle : x \in U\}$

Definition 5: The support of set A is a classical set that consists of all elements of U with non-zero membership values in A (Atanassov, 1986):

$$\text{Supp } A = \{x \in U: \mu_A(x) \neq 0\}$$

Definition 6: An intuitionistic fuzzy set A is said to be a null intuitionistic fuzzy set if Supp A = 0 that is $\mu_A(x)$ for all $x \in U$ although, $\nu_A(x) \neq 0$ for any $x \in U$ (Atanassov, 1986).

Definition 7: An intuitionistic fuzzy set A is said to be an absolute intuitionistic fuzzy set if Supp $A \neq 0$ that is $\mu_A(x) \neq 0$ for all $x \in U$ although, $\nu(x) = 0$ for any $x \in U$ (Atanassov, 1986).

Definition 8: Consider U and E as a universe set and a set of parameters, respectively. Let P(U) denote the set of all intuitionistic fuzzy sets of U. Let $A \subseteq E$. A pair (F, E) is an intuitionistic fuzzy soft set over U where F is a mapping given by F: $A \rightarrow P(U)$ (Maji *et al*., 2001b).

From now on let U be a universe, E be a set of parameters, X be a set of experts (agents), O be a set of opinions, $Z = E \times X \times O$ and $A \subseteq Z$.

Definition 9: A pair (F, A) is called a soft expert set over U where F is a mapping given by Alkhazaleh and Salleh (2014):

$$F: A \rightarrow P(U)$$

where, P(U) denotes the power set of U.

Definition 10: The complement of a soft expert set (F, A) is denoted by $(F, A)^c$ and is defined by $(F, A)^c = (F^c, \sim A)$ where F^c: $\sim A \rightarrow P(U)$ is a mapping given by $F^c(a) = U - F(\sim a)$, for all $a \in \sim A$ (Alkhazaleh and Salleh, 2011).

Definition 11: Let U be an initial universe, E the set of all parameters and X a fuzzy set over E with membership function (Cagman *et al*., 2010):

$$\mu_X: E \rightarrow [0, 1]$$

and let τ_X be a fuzzy set over U for all $x \in E$. Then, a fuzzy parameterized fuzzy soft set (fpfs-set) ρ_X over U is a set defined by a function $\tau_X(x)$ representing a mapping τ_X: $E \rightarrow F(U)$ such that $\tau_X(x) = \varnothing$ if $\mu_X(x = 0)$. Here, τ_X is called a fuzzy approximate function of the fpfs-set ρ_X and the value $\tau_X(x)$ is a set called x-element of the fpfs-set for all $x \in E$. Thus, a fpfs-set ρ_X over U can be represented by the set of ordered pairs:

$$\rho_X = \left\{\left(\frac{\mu_X(x)}{x}, \tau_X(x)\right) : x \in E, \tau_X(x) \in F(U), \mu_X(x) \in [0, 1]\right\}$$

Definition 12: A pair $(F, A)_D$ is called a Fuzzy Parameterized Soft Expert Set (FPSES) over U where F is a mapping given by F_D: $A \to P(U)$, D is a fuzzy subset of the set of parameters E and P(U) denotes the power set of U (Bashir and Salleh, 2012).

Definition 13: Let F be a mapping given by F_D: $A \to I^U$ where I^U denotes the power set of U. A pair $(F, A)_D$ is called a Fuzzy Parameterized Fuzzy Soft Expert Set (FPFSES) over U (Hazaymeh *et al.*, 2012).

FUZZY PARAMETERIZED INTUITIONISTIC FUZZY SOFT EXPERT SETS

In this study, the notion of fuzzy parameterized intuitionistic fuzzy soft expert sets and several related concepts are established. The properties of this concept are then studied and discussed.

From now on, let U be universal set of elements, E be a set of parameters, I^E be the set of fuzzy subsets of E, X be a set of experts (agents), Q be a set of opinions, $Z = E \times X \times Q$ and $A \subseteq Z$.

Definition 14: Let $U = \{u_1, u_2, u_3, ..., u_n\}$ be a universal set of elements, $E = \{e_1, e_2, e_3, ..., e_m\}$ be a universal set of parameters, I^E denote the set of fuzzy subsets of E, $X = \{x_1, x_2, x_3, ..., x_i\}$ be a set of experts (agents) and Q = {1 = agree, 0 = disagree} be a set of opinions. Let $Z = \{D \times X \times Q\}$ and $A \subseteq Z$ where $D \subset I^E$. Then, the cartesian product $Z = D \times X \times Q$ is defined as follows:

$$Z = \{(d, x, q): d \in D, x \in X, q \in Q\}$$

For the sake of simplicity, in this study, it is assumed that the set of opinions only consist of two values, namely agree and disagree. However, it is possible to include other options for the set of opinions including more specific opinions.

Definition 15: Let F be a mapping given by F_D: $A \to I^U$ where I^U denotes the power set of U. Then, a pair $(F, A)_D$ is called a Fuzzy Parameterized Intuitionistic Fuzzy Soft Expert Set (FPIFSES for the sake of simplicity) over U.

Example 1: Consider a house selection problem where a potential house buyer is considering a set of houses to be purchased. Suppose that there exist three houses u_1, u_2, u_3 to be considered for purchasing. This set of houses are denoted by $U = \{u_1, u_2, u_3\}$. Suppose that three parameters e_1 (beautiful), e_2 (cheap), e_3 (good location) are taken into consideration in this situation and the set of all these parameters are given by E = {beautiful, cheap, good location}. Let $X = \{x_1, x_2\}$ be a set of experts and $D = \{0.9/e_1, 0.5/e_2, 0.3/e_3\}$ be a fuzzy subset of I^E. The two experts are the requested to make a decision on the houses with respect to the parameters mentioned above and the following information is obtained:

$$F\left(\frac{0.9}{e_1}, x_1, 1\right) = \left\{\frac{u_1}{\langle 0.8, 0.1\rangle}, \frac{u_2}{\langle 0.4, 0.5\rangle}, \frac{u_3}{\langle 0.6, 0.4\rangle}\right\}, \quad F\left(\frac{0.9}{e_1}, x_2, 1\right) = \left\{\frac{u_1}{\langle 0.6, 0.3\rangle}, \frac{u_2}{\langle 0.1, 0.9\rangle}, \frac{u_3}{\langle 0.9, 0.05\rangle}\right\}$$

$$F\left(\frac{0.5}{e_2}, x_1, 1\right) = \left\{\frac{u_1}{\langle 0.4, 0.6\rangle}, \frac{u_2}{\langle 0.5, 0.5\rangle}, \frac{u_3}{\langle 0.9, 0.1\rangle}\right\}, \quad F\left(\frac{0.5}{e_2}, x_2, 1\right) = \left\{\frac{u_1}{\langle 0.1\rangle}, \frac{u_2}{\langle 0.3, 0.4\rangle}, \frac{u_3}{\langle 0.2, 0.75\rangle}\right\}$$

$$F\left(\frac{0.3}{e_3}, x_1, 1\right) = \left\{\frac{u_1}{\langle 0.2, 0.8\rangle}, \frac{u_2}{\langle 0.3, 0.7\rangle}, \frac{u_3}{\langle 0.9, 0\rangle}\right\}, \quad F\left(\frac{0.3}{e_3}, x_2, 1\right) = \left\{\frac{u_1}{\langle 0, 0.82\rangle}, \frac{u_2}{\langle 0.3, 0.7\rangle}, \frac{u_3}{\langle 0.8, 0.2\rangle}\right\}$$

$$F\left(\frac{0.9}{e_1}, x_1, 0\right) = \left\{\frac{u_1}{\langle 0.8, 0.1\rangle}, \frac{u_2}{\langle 1, 0\rangle}, \frac{u_3}{\langle 0.3, 0.55\rangle}\right\}, \quad F\left(\frac{0.9}{e_1}, x_2, 0\right) = \left\{\frac{u_1}{\langle 0.3, 0.5\rangle}, \frac{u_2}{\langle 0.7, 0.3\rangle}, \frac{u_3}{\langle 1, 0\rangle}\right\}$$

$$F\left(\frac{0.5}{e_2}, x_1, 0\right) = \left\{\frac{u_1}{\langle 1, 0\rangle}, \frac{u_2}{\langle 1, 0\rangle}, \frac{u_3}{\langle 1, 0\rangle}\right\}, \quad F\left(\frac{0.5}{e_2}, x_2, 0\right) = \left\{\frac{u_1}{\langle 0.9, 0\rangle}, \frac{u_2}{\langle 0.4, 0.4\rangle}, \frac{u_3}{\langle 0.1, 0.7\rangle}\right\}$$

$$F\left(\frac{0.3}{e_3}, x_1, 0\right) = \left\{\frac{u_1}{\langle 0.3, 0.6\rangle}, \frac{u_2}{\langle 0.25, 0.75\rangle}, \frac{u_3}{\langle 0, 0.7\rangle}\right\}, \quad F\left(\frac{0.3}{e_3}, x_2, 0\right) = \left\{\frac{u_1}{\langle 0.8, 0\rangle}, \frac{u_2}{\langle 0.7, 0.3\rangle}, \frac{u_3}{\langle 0.4, 0.2\rangle}\right\}$$

Then, we can view the FPIFSES $(F, A)_D$ as consisting of the following collection of approximations:

$$(F, A)_D = \left\{\begin{array}{l} \left(\left(\frac{0.9}{e_1}, x_1, 1\right) = \left\{\frac{u_1}{\langle 0.8, 0.1\rangle}, \frac{u_2}{\langle 0.4, 0.5\rangle}, \frac{u_3}{\langle 0.6, 0.4\rangle}\right\}\right), \left(\left(\frac{0.9}{e_1}, x_2, 1\right) = \left\{\frac{u_1}{\langle 0.6, 0.3\rangle}, \frac{u_2}{\langle 0.1, 0.9\rangle}, \frac{u_3}{\langle 0.9, 0.05\rangle}\right\}\right) \\ \left(\left(\frac{0.5}{e_2}, x_1, 1\right) = \left\{\frac{u_1}{\langle 0.4, 0.6\rangle}, \frac{u_2}{\langle 0.5, 0.5\rangle}, \frac{u_3}{\langle 0.9, 0.1\rangle}\right\}\right), \left(\left(\frac{0.5}{e_2}, x_2, 1\right) = \left\{\frac{u_1}{\langle 0, 1\rangle}, \frac{u_2}{\langle 0.3, 0.4\rangle}, \frac{u_3}{\langle 0.2, 0.75\rangle}\right\}\right) \\ \left(\left(\frac{0.3}{e_3}, x_1, 1\right) = \left\{\frac{u_1}{\langle 0.2, 0.8\rangle}, \frac{u_2}{\langle 0.3, 0.7\rangle}, \frac{u_3}{\langle 0.9, 0\rangle}\right\}\right), \left(\left(\frac{0.3}{e_3}, x_2, 1\right) = \left\{\frac{u_1}{\langle 0, 0.82\rangle}, \frac{u_2}{\langle 0.3, 0.7\rangle}, \frac{u_3}{\langle 0.8, 0.2\rangle}\right\}\right) \end{array}\right.$$

$$(F,A)_D = \left\{\begin{array}{l} \left(\left(\frac{0.9}{e_1}, x_1, 0\right) = \left\{\frac{u_1}{\langle 0.8, 0.1\rangle}, \frac{u_2}{\langle 1, 0\rangle}, \frac{u_3}{\langle 0.3, 0.55\rangle}\right\}\right), \left(\left(\frac{0.9}{e_1}, x_2, 0\right) = \left\{\frac{u_1}{\langle 0.3, 0.5\rangle}, \frac{u_2}{\langle 0.7, 0.3\rangle}, \frac{u_3}{\langle 1, 0\rangle}\right\}\right) \\ \left(\left(\frac{0.5}{e_2}, x_1, 0\right) = \left\{\frac{u_1}{\langle 1, 0\rangle}, \frac{u_2}{\langle 1, 0\rangle}, \frac{u_3}{\langle 1, 0\rangle}\right\}\right), \left(\left(\frac{0.5}{e_2}, x_2, 0\right) = \left\{\frac{u_1}{\langle 0.9, 0\rangle}, \frac{u_2}{\langle 0.4, 0.4\rangle}, \frac{u_3}{\langle 0.1, 0.7\rangle}\right\}\right) \\ \left(\left(\frac{0.3}{e_3}, x_1, 0\right) = \left\{\frac{u_1}{\langle 0.3, 0.6\rangle}, \frac{u_2}{\langle 0.25, 0.75\rangle}, \frac{u_3}{\langle 0, 0.7\rangle}\right\}\right), \left(\left(\frac{0.3}{e_3}, x_2, 0\right) = \left\{\frac{u_1}{\langle 0.8, 0\rangle}, \frac{u_2}{\langle 0.7, 0.3\rangle}, \frac{u_3}{\langle 0.4, 0.2\rangle}\right\}\right) \end{array}\right\}$$

Then $(F, A)_D$ is a fuzzy parameterized intuitionistic fuzzy soft expert set over U.

Definition 16: Let $(F, A)_D$ and $(G, B)_K$ be FPIFSESs over a universe U. Then $(F, A)_D$ is said to be a Fuzzy Parameterized Intuitionistic Fuzzy Soft Expert subset (FPIFSE subset) of $(G, B)_K$ if the following conditions are satisfied:

(i) $A \subseteq B$.
(ii) For all $\varepsilon \epsilon A$, $F_D(\varepsilon)$ is an intuitionistic fuzzy subset of $G_K(\varepsilon)$.

This relationship is denoted as $(F, A)_D \subseteq (G, B)_K$. In this case, $(G, B)_K$ is called a Fuzzy Parameterized Intuitionistic Fuzzy Soft Expert superset (FPIFSE superset) of $(F, A)_D$.

Definition 17: Let $(F, A)_D$ and $(G, B)_K$ be FPIFSESs over a universe U. Then $(F, A)_D$ and $(G, B)_K$ are said to be equal if $(F, A)_D$ is a FPIFSE sebset of $(G, B)_K$ and $(G, B)_K$ is a FPIFSE subset of $(F, A)_D$.

Definition 18: Let $(F, A)_D$ be a FPIFSES over a universe U. An agree-Fuzzy Paramaterized Intuitionistic Fuzzy Soft Expert Set (agree-FPIFSES) over U, denoted as $(F, A)_{D_1}$ is a fuzzy parameterized intuitionistic fuzzy soft expert subset of $(F, A)_D$ which is defined as:

$$(F, A)_{D_1} = \{F_D(\alpha): \alpha \in D \times X \times \{1\}\}$$

Definition 19: Consider the FPIFSES given in example 1. Then, the agree-FPIFSES $(F, A)_{D_1}$ over U is:

$$(F, A)_{D_1} = \left\{\begin{array}{l} \left(\left(\frac{0.9}{e_1}, x_1, 1\right) = \left\{\frac{u_1}{\langle 0.8, 0.1\rangle}, \frac{u_2}{\langle 0.4, 0.5\rangle}, \frac{u_3}{\langle 0.6, 0.4\rangle}\right\}\right), \left(\left(\frac{0.9}{e_1}, x_2, 1\right) = \left\{\frac{u_1}{\langle 0.6, 0.3\rangle}, \frac{u_2}{\langle 0.1, 0.9\rangle}, \frac{u_3}{\langle 0.9, 0.05\rangle}\right\}\right) \\ \left(\left(\frac{0.5}{e_2}, x_1, 1\right) = \left\{\frac{u_1}{\langle 0.4, 0.6\rangle}, \frac{u_2}{\langle 0.5, 0.5\rangle}, \frac{u_3}{\langle 0.9, 0.1\rangle}\right\}\right), \left(\left(\frac{0.5}{e_2}, x_2, 1\right) = \left\{\frac{u_1}{\langle 0, 1\rangle}, \frac{u_2}{\langle 0.3, 0.4\rangle}, \frac{u_3}{\langle 0.2, 0.75\rangle}\right\}\right) \\ \left(\left(\frac{0.3}{e_3}, x_1, 1\right) = \left\{\frac{u_1}{\langle 0.2, 0.8\rangle}, \frac{u_2}{\langle 0.3, 0.7\rangle}, \frac{u_3}{\langle 0.9, 0\rangle}\right\}\right), \left(\left(\frac{0.3}{e_3}, x_2, 1\right) = \left\{\frac{u_1}{\langle 0, 0.82\rangle}, \frac{u_2}{\langle 0.3, 0.7\rangle}, \frac{u_3}{\langle 0.8, 0.2\rangle}\right\}\right) \end{array}\right\}$$

Definition 20: Let $(F, A)_D$ be a FPIFSES over a universe U. A disagree-Fuzzy Parameterized Intuitionistic Fuzzy Soft Expert set (disagree-FPIFSES) over U, denoted as $(F, A)_{D_0}$ is a fuzzy parameterized intuitionistic fuzzy soft expert subset of $(F, A)_D$ which is defined as:

$$(F, A)_{D_0} = \{F_D(\alpha): \alpha \in D \times X \times \{1\}\}$$

Definition 21: Consider the FPIFSES given in example 1. Then, the disagree-FPIFSES $(F, A)_{D_0}$ over U is:

$$(F, A)_{D_0} = \left\{\begin{array}{l} \left(\left(\frac{0.9}{e_1}, x_1, 0\right) = \left\{\frac{u_1}{\langle 0.8, 0.1\rangle}, \frac{u_2}{\langle 1, 0\rangle}, \frac{u_3}{\langle 0.3, 0.55\rangle}\right\}\right), \left(\left(\frac{0.9}{e_1}, x_2, 0\right) = \left\{\frac{u_1}{\langle 0.3, 0.5\rangle}, \frac{u_2}{\langle 0.7, 0.3\rangle}, \frac{u_3}{\langle 1, 0\rangle}\right\}\right) \\ \left(\left(\frac{0.5}{e_2}, x_1, 0\right) = \left\{\frac{u_1}{\langle 1, 0\rangle}, \frac{u_2}{\langle 1, 0\rangle}, \frac{u_3}{\langle 1, 0\rangle}\right\}\right), \left(\left(\frac{0.5}{e_2}, x_2, 0\right) = \left\{\frac{u_1}{\langle 0.9, 0\rangle}, \frac{u_2}{\langle 0.4, 0.4\rangle}, \frac{u_3}{\langle 0.1, 0.7\rangle}\right\}\right) \\ \left(\left(\frac{0.3}{e_3}, x_1, 0\right) = \left\{\frac{u_1}{\langle 0.3, 0.6\rangle}, \frac{u_2}{\langle 0.25, 0.75\rangle}, \frac{u_3}{\langle 0, 0.7\rangle}\right\}\right), \left(\left(\frac{0.3}{e_3}, x_2, 0\right) = \left\{\frac{u_1}{\langle 0.8, 0\rangle}, \frac{u_2}{\langle 0.7, 0.3\rangle}, \frac{u_3}{\langle 0.4, 0.2\rangle}\right\}\right) \end{array}\right\}$$

BASIC OPERATIONS ON FUZZY PARAMETERIZED INTUITIONISTIC FUZZY SOFT EXPERT SETS

In this study, we introduce some basic operations on FPIFSES, namely the complement "AND", "OR", union and intersection of FPIFSES and proceed to study some of the properties related to these operations.

Definition 22: Let $(F, A)_D$ be a FPIFSES over U. Then, the complement of $(F, A)_D$, denoted by $(F, A)_D{}^C$ is defined by $(F, A)_D{}^C = (F^C, \sim A)_D$ where $F_D{}^C: \sim A \to I^U)$ is a mapping given by:

$$F_D{}^C(\alpha) = c(F_D(\alpha)), \quad \forall \alpha \in \sim A$$

where, C is an intuitionistic fuzzy complement and $\sim A \subset \{D^C \times X \times Q\}$.

Example 2: Consider the FPIFSES $(F, A)_D$ over a universe U as given in example 1. By using the intuitionistic fuzzy complement, we obtain:

$$(F, A)_D^C = \left\{\begin{array}{l}
\left(\left(\frac{0.1}{e_1}, x_1, 1\right) = \left\{\frac{u_1}{\langle 0.1, 0.8\rangle}, \frac{u_2}{\langle 0.5, 0.4\rangle}, \frac{u_3}{\langle 0.4, 0.6\rangle}\right\}\right), \left(\left(\frac{0.1}{e_1}, x_2, 1\right) = \left\{\frac{u_1}{\langle 0.3, 0.6\rangle}, \frac{u_2}{\langle 0.9, 0.1\rangle}, \frac{u_3}{\langle 0.05, 0.9\rangle}\right\}\right) \\
\left(\left(\frac{0.5}{e_2}, x_1, 1\right) = \left\{\frac{u_1}{\langle 0.6, 0.4\rangle}, \frac{u_2}{\langle 0.5, 0.5\rangle}, \frac{u_3}{\langle 0.1, 0.9\rangle}\right\}\right), \left(\left(\frac{0.5}{e_2}, x_2, 1\right) = \left\{\frac{u_1}{\langle 1, 0\rangle}, \frac{u_2}{\langle 0.4, 0.3\rangle}, \frac{u_3}{\langle 0.75, 0.2\rangle}\right\}\right) \\
\left(\left(\frac{0.7}{e_3}, x_1, 1\right) = \left\{\frac{u_1}{\langle 0.8, 0.2\rangle}, \frac{u_2}{\langle 0.7, 0.3\rangle}, \frac{u_3}{\langle 0, 0.9\rangle}\right\}\right), \left(\left(\frac{0.3}{e_3}, x_2, 1\right) = \left\{\frac{u_1}{\langle 0.82, 0\rangle}, \frac{u_2}{\langle 0.7, 0.3\rangle}, \frac{u_3}{\langle 0.2, 0.8\rangle}\right\}\right) \\
\left(\left(\frac{0.1}{e_1}, x_1, 0\right) = \left\{\frac{u_1}{\langle 0.1, 0.8\rangle}, \frac{u_2}{\langle 0, 1\rangle}, \frac{u_3}{\langle 0.55, 0.3\rangle}\right\}\right), \left(\left(\frac{0.1}{e_1}, x_2, 0\right) = \left\{\frac{u_1}{\langle 0.5, 0.3\rangle}, \frac{u_2}{\langle 0.3, 0.7\rangle}, \frac{u_3}{\langle 0, 1\rangle}\right\}\right) \\
\left(\left(\frac{0.5}{e_2}, x_1, 0\right) = \left\{\frac{u_1}{\langle 1, 0\rangle}, \frac{u_2}{\langle 0, 1\rangle}, \frac{u_3}{\langle 0, 1\rangle}\right\}\right), \left(\left(\frac{0.5}{e_2}, x_2, 0\right) = \left\{\frac{u_1}{\langle 0, 0.9, 0.3\rangle}, \frac{u_2}{\langle 0.4, 0.4\rangle}, \frac{u_3}{\langle 0.7, 0.1\rangle}\right\}\right) \\
\left(\left(\frac{0.7}{e_3}, x_1, 0\right) = \left\{\frac{u_1}{\langle 0.6, 0.3\rangle}, \frac{u_2}{\langle 0.75, 0.25\rangle}, \frac{u_3}{\langle 0.7, 0\rangle}\right\}\right), \left(\left(\frac{0.3}{e_3}, x_2, 0\right) = \left\{\frac{u_1}{\langle 0, 0.8\rangle}, \frac{u_2}{\langle 0.3, 0.7\rangle}, \frac{u_3}{\langle 0.2, 0.4\rangle}\right\}\right)
\end{array}\right\}$$

Proposition 1: Let $(F, A)_D$ be a FPIFSES over a universe U. Then, the following properties hold true:

(i) $((F, A)_D{}^C)^C = (F, A)_D$
(ii) $\left((F, A)_{D_1}{}^C\right)^C = (F, A)_{D_0}$
(iii) $\left((F, A)_{D_0}{}^C\right)^C = (F, A)_{D_1}$

Proof:
(i) Let $(F, A)_D$ be a FPIFSES over a universe U. Then by definition 22, $F_D{}^C: \sim A \to I^U$ is a mapping given by $F_D{}^C(\alpha) = c(F_D(\alpha))$ for all $\alpha \in A$ and $\sim A \subset \{D^C \times X \times Q\}$. Now $(F_D{}^C)^C: \sim(\sim A) \to I^U$ is a mapping given by $(F_D{}^C)^C(\alpha) = c(F_D{}^C(\sim(\sim\alpha)))$ for all $\alpha \in \sim(\sim A)$ and $\sim(\sim A) \subset \{(D^C)^C \times X \times Q\}$ and since $(D^C)^C = D$, it is proven that $((F, A)_D{}^C)^C = (F, A)_D$.
(ii) The proof is straightforward and is therefore omitted.
(iii) The proof is straightforward and is therefore omitted.

Definition 23: Let $(F, A)_D$ and $(G, B)_K$ be FPIFSESs over a universe U. Then, the union of $(F, A)_D$ and $(G, B)_K$ is a FPIFSES $(H, C)_R$ such that $C = R \times X \times Q$ where $R = D \cup K$ and for all $\alpha \in C$:

$$H_R(\alpha) = F_D(\alpha) \cup G_K(\alpha)$$

where, $\cup$ is the intuitionistic fuzzy union. This relationship is denoted by $(F, A)_D \,\tilde{\cup}\, (G, B)_K$.

Example 3: Let $(F, A)_D$ and $(G, B)_K$ be two FPIFSESs over the universe U defined as given:

$$(F, A)_D = \left\{\begin{array}{l} \left(\left(\frac{0.9}{e_1}, x_1, 1\right) = \left\{\frac{u_1}{\langle 0.8, 0.1\rangle}, \frac{u_2}{\langle 0.4, 0.5\rangle}, \frac{u_3}{\langle 0.6, 0.4\rangle}\right\}\right), \left(\left(\frac{0.9}{e_1}, x_2, 1\right) = \left\{\frac{u_1}{\langle 0.6, 0.3\rangle}, \frac{u_2}{\langle 0.1, 0.9\rangle}, \frac{u_3}{\langle 0.9, 0.05\rangle}\right\}\right) \\ \left(\left(\frac{0.5}{e_2}, x_2, 1\right) = \left\{\frac{u_1}{\langle 0, 1\rangle}, \frac{u_2}{\langle 0.3, 0.4\rangle}, \frac{u_3}{\langle 0.2, 0.75\rangle}\right\}\right), \left(\left(\frac{0.3}{e_3}, x_1, 1\right) = \left\{\frac{u_1}{\langle 0.2, 0.8\rangle}, \frac{u_2}{\langle 0.3, 0.7\rangle}, \frac{u_3}{\langle 0.9, 0\rangle}\right\}\right) \\ \left(\left(\frac{0.9}{e_1}, x_1, 0\right) = \left\{\frac{u_1}{\langle 0.8, 0.1\rangle}, \frac{u_2}{\langle 1, 0\rangle}, \frac{u_3}{\langle 0.3, 0.55\rangle}\right\}\right), \left(\left(\frac{0.5}{e_2}, x_1, 0\right) = \left\{\frac{u_1}{\langle 0, 1\rangle}, \frac{u_2}{\langle 1, 0\rangle}, \frac{u_3}{\langle 1, 0\rangle}\right\}\right) \\ \left(\left(\frac{0.5}{e_2}, x_2, 0\right) = \left\{\frac{u_1}{\langle 0.9, 0\rangle}, \frac{u_2}{\langle 0.4, 0.4\rangle}, \frac{u_3}{\langle 0.1, 0.7\rangle}\right\}\right), \left(\left(\frac{0.3}{e_3}, x_1, 0\right) = \left\{\frac{u_1}{\langle 0.3, 0.6\rangle}, \frac{u_2}{\langle 0.25, 0.75\rangle}, \frac{u_3}{\langle 0, 0.7\rangle}\right\}\right) \\ \left(\left(\frac{0.3}{e_3}, x_2, 0\right) = \left\{\frac{u_1}{\langle 0.8, 0\rangle}, \frac{u_2}{\langle 0.7, 0.3\rangle}, \frac{u_3}{\langle 0.4, 0.2\rangle}\right\}\right) \end{array}\right\}$$

$$(G, B)_K = \left\{\begin{array}{l} \left(\left(\frac{0.5}{e_1}, x_1, 1\right) = \left\{\frac{u_1}{\langle 0.3, 0.7\rangle}, \frac{u_2}{\langle 0.8, 0\rangle}, \frac{u_3}{\langle 0, 1\rangle}\right\}\right), \left(\left(\frac{0.5}{e_1}, x_2, 1\right) = \left\{\frac{u_1}{\langle 0, 0.5\rangle}, \frac{u_2}{\langle 0.5, 0.3\rangle}, \frac{u_3}{\langle 0.4, 0.6\rangle}\right\}\right) \\ \left(\left(\frac{0.7}{e_2}, x_2, 1\right) = \left\{\frac{u_1}{\langle 0.4, 0.3\rangle}, \frac{u_2}{\langle 0.8, 0.2\rangle}, \frac{u_3}{\langle 0.9, 0.1\rangle}\right\}\right), \left(\left(\frac{0.1}{e_3}, x_1, 1\right) = \left\{\frac{u_1}{\langle 0.7, 0.3\rangle}, \frac{u_2}{\langle 0.15, 0.8\rangle}, \frac{u_3}{\langle 0.35, 0.65\rangle}\right\}\right) \\ \left(\left(\frac{0.5}{e_1}, x_2, 0\right) = \left\{\frac{u_1}{\langle 0.2, 0.6\rangle}, \frac{u_2}{\langle 0.4, 0.4\rangle}, \frac{u_3}{\langle 0.9, 0\rangle}\right\}\right), \left(\left(\frac{0.7}{e_2}, x_1, 0\right) = \left\{\frac{u_1}{\langle 0.1, 0.6\rangle}, \frac{u_2}{\langle 0.3, 0.45\rangle}, \frac{u_3}{\langle 0.85, 0.05\rangle}\right\}\right) \\ \left(\left(\frac{0.1}{e_3}, x_1, 0\right) = \left\{\frac{u_1}{\langle 0.01, 0.9\rangle}, \frac{u_2}{\langle 0.3, 0.7\rangle}, \frac{u_3}{\langle 0.5, 0.45\rangle}\right\}\right) \end{array}\right\}$$

Then by using the intuitionistic fuzzy union, we have $(F, A)_D \,\tilde{\cup}\, (G, B)_K = (H, C)_R$ where $(H, C)_R$ is a FPIFSES defined as:

$$(H, C)_R = \left\{\begin{array}{l} \left(\left(\frac{0.9}{e_1}, x_1, 1\right) = \left\{\frac{u_1}{\langle 0.8, 0.1\rangle}, \frac{u_2}{\langle 0.8, 0\rangle}, \frac{u_3}{\langle 0.6, 0.4\rangle}\right\}\right), \left(\left(\frac{0.9}{e_1}, x_2, 1\right) = \left\{\frac{u_1}{\langle 0.6, 0.3\rangle}, \frac{u_2}{\langle 0.5, 0.3\rangle}, \frac{u_3}{\langle 0.9, 0.05\rangle}\right\}\right) \\ \left(\left(\frac{0.7}{e_2}, x_2, 1\right) = \left\{\frac{u_1}{\langle 0.4, 0.3\rangle}, \frac{u_2}{\langle 0.8, 0.2\rangle}, \frac{u_3}{\langle 0.9, 0.1\rangle}\right\}\right), \left(\left(\frac{0.3}{e_3}, x_1, 1\right) = \left\{\frac{u_1}{\langle 0.7, 0.3\rangle}, \frac{u_2}{\langle 0.3, 0.7\rangle}, \frac{u_3}{\langle 0.9, 0\rangle}\right\}\right) \\ \left(\left(\frac{0.9}{e_1}, x_1, 0\right) = \left\{\frac{u_1}{\langle 0.8, 0.1\rangle}, \frac{u_2}{\langle 1, 0\rangle}, \frac{u_3}{\langle 0.3, 0.55\rangle}\right\}\right), \left(\left(\frac{0.5}{e_1}, x_2, 0\right) = \left\{\frac{u_1}{\langle 0.2, 0.6\rangle}, \frac{u_2}{\langle 0.4, 0.4\rangle}, \frac{u_3}{\langle 0.9, 0\rangle}\right\}\right) \\ \left(\left(\frac{0.7}{e_2}, x_1, 0\right) = \left\{\frac{u_1}{\langle 0.1, 0.6\rangle}, \frac{u_2}{\langle 1, 0\rangle}, \frac{u_3}{\langle 1, 0\rangle}\right\}\right), \left(\left(\frac{0.5}{e_2}, x_2, 0\right) = \left\{\frac{u_1}{\langle 0.9, 0\rangle}, \frac{u_2}{\langle 0.4, 0.4\rangle}, \frac{u_3}{\langle 0.1, 0.7\rangle}\right\}\right) \\ \left(\left(\frac{0.3}{e_3}, x_1, 0\right) = \left\{\frac{u_1}{\langle 0.3, 0.6\rangle}, \frac{u_2}{\langle 0.3, 0.7\rangle}, \frac{u_3}{\langle 0.5, 0.45\rangle}\right\}\right), \left(\left(\frac{0.3}{e_3}, x_2, 0\right) = \left\{\frac{u_1}{\langle 0.8, 0\rangle}, \frac{u_2}{\langle 0.7, 0.3\rangle}, \frac{u_3}{\langle 0.4, 0.2\rangle}\right\}\right) \end{array}\right\}$$

Proposition 2: Let $(F, A)_D$, $(G, B)_K$ and $(H, C)_R$ be any three FPIFSESs over a universe U. Then, the following results hold true:

(i) $(F, A)_D \,\tilde{\cup}\, (G, B)_K = (G, B)_K \,\tilde{\cup}\, (F, A)_D$ (communitative law)
(ii) $(F, A)_D \,\tilde{\cup}\, ((G, B)_K \,\tilde{\cup}\, (H, C)_R) = ((F, A)_D \,\tilde{\cup}\, (G, B)_K) \,\tilde{\cup}\, (H, C)_R$ (associative law)
(iii) $(F, A)_D \,\tilde{\cup}\, (F, A)_D \subseteq (F, A)_D$

Proof:

(i) Let $(F, A)_D \tilde{\cup} (G, B)_K = (H, C)_R$. Then by definition 23 for all $\alpha \in C$, we have $R = D \cup K$ and $H_R(\alpha) = F_D(\alpha) \cup G_K(\alpha)$. However, since the union of fuzzy sets and intuitionistic fuzzy sets are commutative then $R = D \cup K = K \cup D$ and $H_R(\alpha) = F_D(\alpha) \cup G_K(\alpha) = G_K(\alpha) = G_K(\alpha) \cup F_D(\alpha)$. Therefore $(H, C)_R = (G, B)_K \tilde{\cup} (F, A)_D$. Thus, the union of two FPIFSES are commutative, i.e., $(F, A)_D \tilde{\cup} (G, B)_K = (G, B)_K \tilde{\cup} (F, A)_D$.

(ii) The proof is similar to the proof of part (i) and is therefore omitted.

(iii) The proof is straightforward and is therefore omitted.

Definition 24: Let $(F, A)_D$ and $(G, B)_K$ be FPIFSESs over a universe U. Then, the intersection of $(F, A)_D$ and $(G, B)_K$ is a FPIFSES $(H, C)_R$ such that $C = R \times X \times Q$ where $R = D \cap K$ and for all $\alpha \in C$:

$$H_R(\alpha) = F_D(\alpha) \cap G_K(\alpha)$$

where, $\cap$ is the intuitionistic fuzzy intersection. This relationship is denoted by $(F, A)_D \tilde{\cap} (G, B)_K$.

Example 4: Let $(F, A)_D$ and $(G, B)_K$ be two FPIFSESs over a universe U as defined in example 3. Then by using the intuitionistic fuzzy intersection, we obtain $(F, A)_D \tilde{\cap} (G, B)_K = (H, C)_R$ where $(H, C)_R$ is a FPIFSES defined as:

$$(H,C)_R = \left\{ \begin{array}{l}
\left(\left(\frac{0.5}{e_1}, x_1, 1 \right) = \left\{ \frac{u_1}{\langle 0.3, 0.7 \rangle}, \frac{u_2}{\langle 0.4, 0.5 \rangle}, \frac{u_3}{\langle 0, 1 \rangle} \right\} \right), \left(\left(\frac{0.5}{e_1}, x_2, 1 \right) = \left\{ \frac{u_1}{\langle 0, 0.5 \rangle}, \frac{u_2}{\langle 0.1, 0.9 \rangle}, \frac{u_3}{\langle 0.4, 0.6 \rangle} \right\} \right) \\
\left(\left(\frac{0.5}{e_2}, x_2, 1 \right) = \left\{ \frac{u_1}{\langle 0, 1 \rangle}, \frac{u_2}{\langle 0.3, 0.7 \rangle}, \frac{u_3}{\langle 0.2, 0.75 \rangle} \right\} \right), \left(\left(\frac{0.1}{e_3}, x_1, 1 \right) = \left\{ \frac{u_1}{\langle 0.2, 0.8 \rangle}, \frac{u_2}{\langle 0.15, 0.8 \rangle}, \frac{u_3}{\langle 0.35, 0.65 \rangle} \right\} \right) \\
\left(\left(\frac{0.9}{e_1}, x_1, 0 \right) = \left\{ \frac{u_1}{\langle 0.8, 0.1 \rangle}, \frac{u_2}{\langle 1, 0 \rangle}, \frac{u_3}{\langle 0.3, 0.55 \rangle} \right\} \right), \left(\left(\frac{0.5}{e_1}, x_2, 0 \right) = \left\{ \frac{u_1}{\langle 0.2, 0.6 \rangle}, \frac{u_2}{\langle 0.4, 0.4 \rangle}, \frac{u_3}{\langle 0.9, 0 \rangle} \right\} \right) \\
\left(\left(\frac{0.5}{e_2}, x_1, 0 \right) = \left\{ \frac{u_1}{\langle 0, 1 \rangle}, \frac{u_2}{\langle 0.3, 0.45 \rangle}, \frac{u_3}{\langle 0.85, 0.05 \rangle} \right\} \right), \left(\left(\frac{0.5}{e_2}, x_2, 0 \right) = \left\{ \frac{u_1}{\langle 0.9, 0 \rangle}, \frac{u_2}{\langle 0.4, 0.4 \rangle}, \frac{u_3}{\langle 0.1, 0.7 \rangle} \right\} \right) \\
\left(\left(\frac{0.1}{e_3}, x_1, 0 \right) = \left\{ \frac{u_1}{\langle 0.01, 0.9 \rangle}, \frac{u_2}{\langle 0.25, 0.75 \rangle}, \frac{u_3}{\langle 0, 0.7 \rangle} \right\} \right), \left(\left(\frac{0.3}{e_3}, x_2, 0 \right) = \left\{ \frac{u_1}{\langle 0.8, 0 \rangle}, \frac{u_2}{\langle 0.7, 0.3 \rangle}, \frac{u_3}{\langle 0.4, 0.2 \rangle} \right\} \right)
\end{array} \right\}$$

Proposition 3: Let $(F, A)_D$, $(G, B)_K$ and $(H, C)_R$ be any three FPIFSES over a universe U. Then, the following results hold true:

(i) $(F, A)_D \tilde{\cap} (G, B)_K = (G, B)_K \tilde{\cap} (F, A)_D$ (communitative law)

(ii) $(F, A)_D \tilde{\cap} ((G, B)_K \tilde{\cap} (H, C)_R) = ((F, A)_D \tilde{\cap} (G, B)_K) \tilde{\cap} (H, C)_R$ (associative law)

(iii) $(F, A)_D \tilde{\cap} (F, A)_D \subseteq (F, A)_D$

Proof:

(i) The proof is similar to that of example 3 (i) and is therefore omitted.

(ii) The proof is similar to the proof of part (i) and is therefore omitted.

(iii) The proof is straightforward and is therefore omitted.

Proposition 4: Let $(F, A)_D$, $(G, B)_K$ and $(H, C)_R$ be any three FPIFSES over a universe U. Then, the following results hold true:

(i) $(F, A)_D \tilde{\cup} ((G, B)_K \tilde{\cap} (H, C)_R) = ((F, A)_D \tilde{\cup} (G, B)_K) \tilde{\cap} ((F, A)_D \tilde{\cup} (H, C)_R)$

(ii) $(F, A)_D \tilde{\cap} ((G, B)_K \tilde{\cup} (H, C)_R) = ((F, A)_D \tilde{\cap} (G, B)_K) \tilde{\cup} ((F, A)_D \tilde{\cap} (H, C)_R)$

Proof: The proof is straightforward by definitions 23 and proposition 2 and is therefore omitted.

Proposition 5: Let $(F, A)_D$ and $(G, B)_K$ be any two FPIFSES over a universe U. Then, the following De Morgan's laws hold true:

(i) $((F, A)_D \tilde{\cup} (G, B)_K)^C = (F, A)_D{}^C \tilde{\cap} (G, B)_K{}^C$
(ii) $((F, A)_D \tilde{\cap} (G, B)_K)^C = (F, A)_D{}^C \tilde{\cup} (G, B)_K{}^C$

Proof:
(i) Let $(F, A)_D$ and $(G, B)_K$ be FPIFSESs over a universe U. Then, for all $\alpha \in \sim A$ where $\sim A \subset \{D^C \times X \times Q\}$, it follows that:

$$\begin{aligned}(F, A)_D^C \tilde{\cap} (G, B)_K^C &= F_D^C(\alpha) \cap G_K^C(\alpha)\\ &= C(F_D(\sim\alpha)) \cap C(G_K(\alpha))\\ &= C((F_D(\sim\alpha)) \cap G_K(\sim\alpha)))\\ &= C(F_D(\alpha) \cup G_K(\alpha))\\ &= ((F, A)_D \tilde{\cup} (G, B)_K)^C\end{aligned}$$

(ii) The proof is similar to the proof of part (i) and is therefore omitted.

Definition 25: Let $(F, A)_D$ and $(G, B)_K$ be FPIFSESs over a universe U. Then "$(F, A)_D$ and $(G, B)_K$", denoted by $(F, A)_D \tilde{\wedge} (G, B)_K$ is a FPIFSES defined as:

$$(F, A)_D \tilde{\wedge} (G, B)_K = (H, A \times B)_R$$

such that $H_R(\alpha, \beta) = F_D(\alpha) \cap G_K(\beta)$ for all $(\alpha, \beta) \in A \times B$ where $R = D \times K$ and $\cap$ is the intuitionistic fuzzy intersection.

Example 5: Let $(F, A)_D$ and $(G, B)_K$ be two FPIFSESs over a universe U defined as given below:

$$(F, A)_D = \left\{ \begin{array}{l} \left(\left(\frac{0.9}{e_1}, x_1, 1\right) = \left\{\frac{u_1}{\langle 0.8, 0.1\rangle}, \frac{u_2}{\langle 0.4, 0.5\rangle}, \frac{u_3}{\langle 0.6, 0.4\rangle}\right\}\right), \left(\left(\frac{0.5}{e_2}, x_2, 1\right) = \left\{\frac{u_1}{\langle 0, 1\rangle}, \frac{u_2}{\langle 0.3, 0.4\rangle}, \frac{u_3}{\langle 0.2, 0.75\rangle}\right\}\right) \\ \left(\left(\frac{0.3}{e_3}, x_1, 1\right) = \left\{\frac{u_1}{\langle 0.2, 0.8\rangle}, \frac{u_2}{\langle 0.3, 0.7\rangle}, \frac{u_3}{\langle 0.9, 0\rangle}\right\}\right), \left(\left(\frac{0.5}{e_2}, x_1, 0\right) = \left\{\frac{u_1}{\langle 0, 1\rangle}, \frac{u_2}{\langle 1, 0\rangle}, \frac{u_3}{\langle 1, 0\rangle}\right\}\right) \\ \left(\left(\frac{0.3}{e_3}, x_2, 0\right) = \left\{\frac{u_1}{\langle 0.8, 0\rangle}, \frac{u_2}{\langle 0.7, 0.3\rangle}, \frac{u_3}{\langle 0.4, 0.2\rangle}\right\}\right) \end{array} \right\}$$

$$(G, B)_K = \left\{ \begin{array}{l} \left(\left(\frac{0.5}{e_1}, x_2, 1\right) = \left\{\frac{u_1}{\langle 0, 0.5\rangle}, \frac{u_2}{\langle 0.5, 0.3\rangle}, \frac{u_3}{\langle 0.4, 0.6\rangle}\right\}\right), \left(\left(\frac{0.5}{e_1}, x_2, 0\right) = \left\{\frac{u_1}{\langle 0.2, 0.6\rangle}, \frac{u_2}{\langle 0.4, 0.4\rangle}, \frac{u_3}{\langle 0.9, 0\rangle}\right\}\right) \\ \left(\left(\frac{0.1}{e_3}, x_1, 0\right) = \left\{\frac{u_1}{\langle 0.01, 0.9\rangle}, \frac{u_2}{\langle 0.3, 0.7\rangle}, \frac{u_3}{\langle 0.5, 0.45\rangle}\right\}\right) \end{array} \right\}$$

Then by using the intuitionistic fuzzy intersection, we obtain $(F, A)_D \tilde{\wedge} (G, B)_K = (H, C)_R$ where $C = A \times B$ and $(H, C)_R$ is a FPIFSES defined as:

$$
(H, C)_R = \left\{
\begin{array}{l}
\left(\left(\frac{0.9}{e_1}, x_1, 1\right), \left(\frac{0.5}{e_1}, x_2, 1\right), \left\{\frac{u_1}{\langle 0, 0.5\rangle}, \frac{u_2}{\langle 0.4, 0.5\rangle}, \frac{u_3}{\langle 0.4, 0.6\rangle}\right\}\right) \\
\left(\left(\frac{0.9}{e_1}, x_1, 1\right), \left(\frac{0.5}{e_1}, x_2, 0\right), \left\{\frac{u_1}{\langle 0.2, 0.6\rangle}, \frac{u_2}{\langle 0.4, 0.4\rangle}, \frac{u_3}{\langle 0.6, 0.4\rangle}\right\}\right) \\
\left(\left(\frac{0.9}{e_1}, x_1, 1\right), \left(\frac{0.1}{e_3}, x_1, 0\right), \left\{\frac{u_1}{\langle 0.01, 0.9\rangle}, \frac{u_2}{\langle 0.3, 0.7\rangle}, \frac{u_3}{\langle 0.5, 0.45\rangle}\right\}\right) \\
\left(\left(\frac{0.5}{e_2}, x_2, 1\right), \left(\frac{0.5}{e_1}, x_2, 1\right), \left\{\frac{u_1}{\langle 0, 1\rangle}, \frac{u_2}{\langle 0.3, 0.4\rangle}, \frac{u_3}{\langle 0.2, 0.75\rangle}\right\}\right) \\
\left(\left(\frac{0.5}{e_2}, x_2, 1\right), \left(\frac{0.5}{e_1}, x_2, 0\right), \left\{\frac{u_1}{\langle 0, 1\rangle}, \frac{u_2}{\langle 0.3, 0.4\rangle}, \frac{u_3}{\langle 0.2, 0.75\rangle}\right\}\right) \\
\left(\left(\frac{0.5}{e_2}, x_2, 1\right), \left(\frac{0.1}{e_3}, x_1, 0\right), \left\{\frac{u_1}{\langle 0, 1\rangle}, \frac{u_2}{\langle 0.3, 0.7\rangle}, \frac{u_3}{\langle 0.2, 0.75\rangle}\right\}\right) \\
\left(\left(\frac{0.3}{e_3}, x_1, 1\right), \left(\frac{0.5}{e_1}, x_2, 1\right), \left\{\frac{u_1}{\langle 0, 0.5\rangle}, \frac{u_2}{\langle 0.3, 0.7\rangle}, \frac{u_3}{\langle 0.4, 0.6\rangle}\right\}\right) \\
\left(\left(\frac{0.3}{e_3}, x_1, 1\right), \left(\frac{0.5}{e_1}, x_2, 0\right), \left\{\frac{u_1}{\langle 0.2, 0.8\rangle}, \frac{u_2}{\langle 0.3, 0.7\rangle}, \frac{u_3}{\langle 0.9, 0\rangle}\right\}\right) \\
\left(\left(\frac{0.3}{e_3}, x_1, 1\right), \left(\frac{0.1}{e_3}, x_1, 0\right), \left\{\frac{u_1}{\langle 0.01, 0.9\rangle}, \frac{u_2}{\langle 0.3, 0.7\rangle}, \frac{u_3}{\langle 0.5, 0.45\rangle}\right\}\right) \\
\left(\left(\frac{0.5}{e_2}, x_1, 0\right), \left(\frac{0.5}{e_1}, x_2, 1\right), \left\{\frac{u_1}{\langle 0, 1\rangle}, \frac{u_2}{\langle 0.5, 0.3\rangle}, \frac{u_3}{\langle 0.4, 0.6\rangle}\right\}\right) \\
\left(\left(\frac{0.5}{e_2}, x_1, 0\right), \left(\frac{0.5}{e_1}, x_2, 0\right), \left\{\frac{u_1}{\langle 0, 1\rangle}, \frac{u_2}{\langle 0.4, 0.4\rangle}, \frac{u_3}{\langle 0.9, 0\rangle}\right\}\right) \\
\left(\left(\frac{0.5}{e_2}, x_1, 0\right), \left(\frac{0.1}{e_3}, x_1, 0\right), \left\{\frac{u_1}{\langle 0, 1\rangle}, \frac{u_2}{\langle 0.3, 0.7\rangle}, \frac{u_3}{\langle 0.5, 0.45\rangle}\right\}\right) \\
\left(\left(\frac{0.3}{e_3}, x_2, 0\right), \left(\frac{0.5}{e_1}, x_2, 1\right), \left\{\frac{u_1}{\langle 0, 0.5\rangle}, \frac{u_2}{\langle 0.5, 0.3\rangle}, \frac{u_3}{\langle 0.4, 0.6\rangle}\right\}\right) \\
\left(\left(\frac{0.3}{e_3}, x_2, 0\right), \left(\frac{0.5}{e_1}, x_2, 0\right), \left\{\frac{u_1}{\langle 0.2, 0.6\rangle}, \frac{u_2}{\langle 0.4, 0.4\rangle}, \frac{u_3}{\langle 0.4, 0.2\rangle}\right\}\right) \\
\left(\left(\frac{0.3}{e_3}, x_2, 0\right), \left(\frac{0.1}{e_3}, x_1, 0\right), \left\{\frac{u_1}{\langle 0.01, 0.9\rangle}, \frac{u_2}{\langle 0.3, 0.7\rangle}, \frac{u_3}{\langle 0.4, 0.45\rangle}\right\}\right)
\end{array}
\right\}
$$

Definition 26: Let $(F, A)_D$ and $(G, B)_K$ be FPIFSESs over a universe U. Then "$(F, A)_D$ OR $(G, B)_K$", denoted by $(F, A)_D \tilde{\vee} (G, B)_K$ is a FPIFSES defined as:

$$(F, A)_D \tilde{\vee} (G, B)_K = (H, A \times B)_R$$

such that $H_R(\alpha, \beta) = F_D(\alpha) \cup G_K(\beta)$, for all $(\alpha, \beta) \in A \times B$, where $R = D \times K$ and $\cup$ is the intuitionistic fuzzy union.

Proposition 6: Let $(F, A)_D$, $(G, B)_K$ and $(H, C)_R$ be any three FPIFSESs over a universe U. Then, the following properties hold true:

(i) $(F, A)_D \tilde{\wedge} ((G, B)_K \tilde{\wedge} (H, C)_R) = ((F, A)_D \tilde{\wedge} (G, B)_K \tilde{\wedge} (H, C)_R$

(ii) $(F, A)_D \tilde{\vee} ((G, B)_K \tilde{\vee} (H, C)_R) = ((F, A)_D \tilde{\vee} (G, B)_K \tilde{\vee} (H, C)_R$

(iii) $(F, A)_D \tilde{\vee} ((G, B)_K \tilde{\wedge} (H, C)_R) = ((F, A)_D \tilde{\vee} (G, B)_K) \tilde{\wedge} (F, A)_D \tilde{\vee} (H, C)_R)$

(iv) $(F, A)_D \tilde{\wedge} ((G, B)_K \tilde{\vee} (H, C)_R) = ((F, A)_D \tilde{\wedge} (G, B)_K) \tilde{\vee} ((F, A)_D \tilde{\wedge} (H, C)_R)$

Proof: The proofs are straightforward by proposition 6 and definitions 26 and is therefore omitted. The "AND" and "OR" operations are not commutative since, generally $A\times B\neq B\times A$.

Proposition 7: Let $(F, A)_D$ and $(G, B)_K$ be any two FPIFSESs over a universe U. Then, the following results hold true:

(i) $((F, A)_D \tilde{\wedge} (G, B)_K)^C = (F, A)_D{}^C \tilde{\vee} (G, B)_K{}^C$

(ii) $((F, A)_D \tilde{\vee} (G, B)_K)^C = (F, A)_D{}^C \tilde{\wedge} (G, B)_K{}^C$

Proof:

(i) Suppose that $(F, A)_D$ and $(G, B)_K$ be FPIFSESs over a universe U. Then by proposition 6 and definition 26, it follows that:

$$\begin{aligned}((F, A)_D \tilde{\wedge} (G, B)_K)^C &= (F_D(\alpha) \cap G_K(\beta))^C \\ &= F_D^C(\alpha) \cup G_K^C(\beta) \\ &= C(F_D(\sim\alpha)) \cup C(G_K(\sim\beta)) \\ &= (F, A)_D^C \tilde{\vee} (G, B)_K^C\end{aligned}$$

(ii) The proof is similar to that of part (i) and is therefore omitted

APPLICATION OF FUZZY PARAMETERIZED INTUITIONISTIC FUZZY SOFT EXPERT SETS IN A DECISION MAKING PROBLEM

In this study, we introduce a generalized algorithm which will be applied to the FPIFSES Model introduced in study and used to solve a hypothetical decision making problem.

Suppose that company Y is looking to hire a person to fill in the vacancy for a position in their company. Out of all the people who applied for the position, three candidates were shortlisted and these three candidates form the universe of elements, $U = \{u_1, u_2, u_3\}$. The hiring committee consists of the hiring manager, head of department and the HR director of the company and this committee is represented by the set $X = \{x_1, x_2, x_3\}$ (a set of experts) while the set $Q = \{1 = \text{agree}, 0 = \text{disagree}\}$ represents the set of opinions of the hiring committee members. The hiring committee considers a set of parameters, $E = \{e_1, e_2, e_3, e_4\}$ where the parameters e_i $(i = 1, 2, 3, 4)$ represent the characteristics or qualities that the candidates are assessed on namely "relevant job experience", "excellentacademic qualifications in the relevant field", "attitude and level of professionalism" and "technical knowledge", respectively. After interviewing all the three candidates and going through their certificates and other supporting documents, the hiring committee constructs the fuzzy set:

$$D = \left\{\frac{0.9}{e_1}, \frac{0.3}{e_2}, \frac{0.8}{e_3}, \frac{0.5}{e_4}\right\}$$

and subsequently use it to form the following FPIFSES:

$$(F, Z)_D = \left\{\begin{array}{l}
\left(\left(\frac{0.9}{e_1}, x_1, 1\right), \left\{\frac{u_1}{\langle 0.8, 0.1\rangle}, \frac{u_2}{\langle 0.4, 0.5\rangle}, \frac{u_3}{\langle 0.6, 0.4\rangle}\right\}\right), \left(\left(\frac{0.9}{e_1}, x_2, 1\right), \left\{\frac{u_1}{\langle 0.6, 0.3\rangle}, \frac{u_2}{\langle 0.1, 0.9\rangle}, \frac{u_3}{\langle 0.9, 0.05\rangle}\right\}\right) \\
\left(\left(\frac{0.9}{e_1}, x_3, 1\right), \left\{\frac{u_1}{\langle 0.4, 0.6\rangle}, \frac{u_2}{\langle 0.5, 0.5\rangle}, \frac{u_3}{\langle 0.9, 0.1\rangle}\right\}\right), \left(\left(\frac{0.3}{e_2}, x_1, 1\right), \left\{\frac{u_1}{\langle 0, 1\rangle}, \frac{u_2}{\langle 0.3, 0.4\rangle}, \frac{u_3}{\langle 0.2, 0.75\rangle}\right\}\right) \\
\left(\left(\frac{0.3}{e_2}, x_2, 1\right), \left\{\frac{u_1}{\langle 0.2, 0.8\rangle}, \frac{u_2}{\langle 0.3, 0.7\rangle}, \frac{u_3}{\langle 0.9, 0\rangle}\right\}\right), \left(\left(\frac{0.3}{e_2}, x_3, 1\right), \left\{\frac{u_1}{\langle 0, 0.82\rangle}, \frac{u_2}{\langle 0.3, 0.7\rangle}, \frac{u_3}{\langle 0.8, 0.2\rangle}\right\}\right) \\
\left(\left(\frac{0.8}{e_3}, x_1, 1\right), \left\{\frac{u_1}{\langle 0.8, 0.1\rangle}, \frac{u_2}{\langle 1, 0\rangle}, \frac{u_3}{\langle 0.3, 0.55\rangle}\right\}\right), \left(\left(\frac{0.8}{e_3}, x_2, 1\right), \left\{\frac{u_1}{\langle 0.3, 0.5\rangle}, \frac{u_2}{\langle 0.7, 0.3\rangle}, \frac{u_3}{\langle 1, 0\rangle}\right\}\right) \\
\left(\left(\frac{0.8}{e_3}, x_3, 1\right), \left\{\frac{u_1}{\langle 0, 1\rangle}, \frac{u_2}{\langle 1, 0\rangle}, \frac{u_3}{\langle 1, 0\rangle}\right\}\right), \left(\left(\frac{0.5}{e_4}, x_1, 1\right), \left\{\frac{u_1}{\langle 0.9, 0\rangle}, \frac{u_2}{\langle 0.4, 0.4\rangle}, \frac{u_3}{\langle 0.1, 0.7\rangle}\right\}\right) \\
\left(\left(\frac{0.5}{e_4}, x_2, 1\right), \left\{\frac{u_1}{\langle 0.3, 0.6\rangle}, \frac{u_2}{\langle 0.25, 0.75\rangle}, \frac{u_3}{\langle 0, 0.7\rangle}\right\}\right), \left(\left(\frac{0.5}{e_4}, x_3, 1\right), \left\{\frac{u_1}{\langle 0.8, 0\rangle}, \frac{u_2}{\langle 0.7, 0.3\rangle}, \frac{u_3}{\langle 0.4, 0.2\rangle}\right\}\right) \\
\left(\left(\frac{0.9}{e_1}, x_1, 0\right), \left\{\frac{u_1}{\langle 0.25, 0.7\rangle}, \frac{u_2}{\langle 0, 0.1\rangle}, \frac{u_3}{\langle 0.3, 0.7\rangle}\right\}\right), \left(\left(\frac{0.9}{e_1}, x_2, 0\right), \left\{\frac{u_1}{\langle 0, 1\rangle}, \frac{u_2}{\langle 0.5, 0.5\rangle}, \frac{u_3}{\langle 0.05, 0.9\rangle}\right\}\right)
\end{array}\right.$$

$$(F, Z)_D = \left\{ \begin{array}{l} \left(\left(\frac{0.9}{e_1}, x_3, 0\right), \left\{\frac{u_1}{\langle 0.4, 0.4\rangle}, \frac{u_2}{\langle 1, 0\rangle}, \frac{u_3}{\langle 0.8, 0.2\rangle}\right\}\right), \left(\left(\frac{0.3}{e_2}, x_1, 0\right), \left\{\frac{u_1}{\langle 1, 0\rangle}, \frac{u_2}{\langle 0.7, 0.2\rangle}, \frac{u_3}{\langle 0.9, 0\rangle}\right\}\right) \\ \left(\left(\frac{0.3}{e_2}, x_2, 0\right), \left\{\frac{u_1}{\langle 0.6, 0.3\rangle}, \frac{u_2}{\langle 0.7, 0.25\rangle}, \frac{u_3}{\langle 0, 1\rangle}\right\}\right), \left(\left(\frac{0.3}{e_2}, x_3, 0\right), \left\{\frac{u_1}{\langle 0.7, 0.1\rangle}, \frac{u_2}{\langle 0.9, 0.1\rangle}, \frac{u_3}{\langle 0, 0.99\rangle}\right\}\right) \\ \left(\left(\frac{0.8}{e_3}, x_1, 0\right), \left\{\frac{u_1}{\langle 0.15, 0.85\rangle}, \frac{u_2}{\langle 0, 1\rangle}, \frac{u_3}{\langle 0.6, 0.4\rangle}\right\}\right), \left(\left(\frac{0.8}{e_3}, x_2, 0\right), \left\{\frac{u_1}{\langle 0.5, 0.5\rangle}, \frac{u_2}{\langle 1, 0\rangle}, \frac{u_3}{\langle 0, 0.95\rangle}\right\}\right) \\ \left(\left(\frac{0.8}{e_3}, x_3, 0\right), \left\{\frac{u_1}{\langle 1, 0\rangle}, \frac{u_2}{\langle 0.5, 0.2\rangle}, \frac{u_3}{\langle 0.01, 0.9\rangle}\right\}\right), \left(\left(\frac{0.5}{e_4}, x_1, 0\right), \left\{\frac{u_1}{\langle 0.1, 0.8\rangle}, \frac{u_2}{\langle 0.5, 0.5\rangle}, \frac{u_3}{\langle 0.5, 0.1\rangle}\right\}\right) \\ \left(\left(\frac{0.5}{e_4}, x_2, 0\right), \left\{\frac{u_1}{\langle 0.9, 0\rangle}, \frac{u_2}{\langle 0.7, 0.3\rangle}, \frac{u_3}{\langle 0.8, 0.2\rangle}\right\}\right), \left(\left(\frac{0.5}{e_4}, x_3, 0\right), \left\{\frac{u_1}{\langle 0.2, 0.8\rangle}, \frac{u_2}{\langle 0.45, 0.55\rangle}, \frac{u_3}{\langle 0.98, 0.01\rangle}\right\}\right) \end{array} \right\}$$

Next the FPIFSES $(F, Z)_D$ is used together with a generalized algorithm to solve the decision making problem stated at the beginning of this section. The algorithm given below is employed by the hiring committee to determine the best or most suitable candidate to be hired for the position. This algorithm is a generalization of the algorithm introduced by Alkhazaleh and Salleh (2011) which is used in the context of the FPIFSES Model that is introduced in this study. The generalized algorithm is as follows:

Algorithm:

1. Input the FPIFSES $(F, Z)_D$
2. Find the values of $c_i = \mu_{F_D(z_i)}(\mu_i) - v_{F_D(z_i)}(u_i)$ for each element $u_i \in U$ where $\mu_{F_D(z_i)}(u_i)$ and $v_{F_D(z_i)}(u_i)$ are the membership function and non-membership function of each of the elements $u_i \in U$, respectively
3. Find the highest numerical grade for the agree-FPIFSES and disagree-FPIFSES
4. Compute the score of each element $u_i \in U$ by taking the sum of the products of c_i of each element with the corresponding membership function of the fuzzy set D, $\mu_D(e_i)$ for the agree-FPIFSES and disagree-FPIFSES, denoted by A_i and B_i, respectively. The formula for A_j and B_j are as given:

$$A_j = \sum_{x \in X} \sum_{i=1}^{4} (c_{ij})(\mu_D(e_i)),\ B_j = \sum_{x \in X} \sum_{i=1}^{4} (c_{ij})(\mu_D(e_i))$$

5. Find the values of the score $r_j = A_j - B_j$ for each element $u_j \in U$
6. Determine the value of the highest score, $s = \max_{u_j \in U}\{r_j\}$. Then, the decision is to choose element u_j as the optimal or best solution to theproblem. If there are more than one element with the highest r_j score then any one of those elements can be chosen as the optimal solution

Then, we can conclude that the optimal choice for the hiring committee is to hire candidate u_j to fill the vacant position. Table 1 gives the values of $c_i = \mu_{F_D(z_i)}(\mu_i) - v_{F_D(z_i)}(\mu_i)$ for each element $u_i \in U$. Table 2 and 3 give the numerical grade for the elements in the agree-FPIFSES and disagree-FPIFSES, respectively.

Let A_i and D_i represent the score of the numerical grade for each of the elements in the agree-FPIFSES and disagree-FPIFSES, respectively. These values are given in Table 4. From the computation done in Table 1-4, we obtain $s = \max_{u_j \in U}\{r_j\} = r_3$. Therefore, the hiring committee should hire candidate u_3 to fill in the vacant position.

Table 1: Values of $c_i = \mu_{F_D}(z_i)^{(u_i)} - v_{F_D}(z_i)^{(u_i)}$ for all $u_i \in U$

U	u_1	u_2	u_3	U	u_1	u_2	u_3
$((0.9/e_1), x_1, 1)$	0.70	-0.10	0.20	$((0.9/e_1), x_1, 0)$	-0.45	-0.10	-0.40
$((0.9/e_1), x_2, 1)$	0.30	-0.80	0.85	$((0.9/e_1), x_2, 0)$	-1.00	0.00	-0.85
$((0.9/e_1), x_3, 1)$	-0.20	0.00	0.80	$((0.9/e_1), x_3, 0)$	0.00	1.00	0.60
$((0.3/e_2), x_1, 1)$	-1.00	-0.10	-0.55	$((0.3/e_2), x_1, 0)$	1.00	0.50	0.90
$((0.3/e_2), x_2, 1)$	-0.60	-0.40	0.90	$((0.3/e_2), x_2, 0)$	0.30	0.45	-1.00
$((0.3/e_2), x_3, 1)$	-0.82	-0.40	0.60	$((0.3/e_2), x_3, 0)$	0.60	0.80	-0.99
$((0.8/e_3), x_1, 1)$	0.70	1.00	-0.25	$((0.8/e_3), x_1, 0)$	-0.70	-1.00	0.20
$((0.8/e_3), x_2, 1)$	-0.20	0.40	1.00	$((0.8/e_3), x_2, 0)$	0.00	1.00	-0.95
$((0.8/e_3), x_3, 1)$	-1.00	1.00	1.00	$((0.8/e_3), x_3, 0)$	1.00	0.30	-0.89
$((0.5/e_4), x_1, 1)$	0.90	0.00	-0.60	$((0.5/e_4), x_1, 0)$	-0.70	0.00	0.40
$((0.5/e_4), x_2, 1)$	-0.30	-0.50	-0.70	$((0.5/e_4), x_2, 0)$	0.90	0.40	0.60
$((0.5/e_4), x_3, 1)$	0.80	0.40	0.20	$((0.5/e_4), x_3, 0)$	0.60	-0.10	0.97

Table 2: Agree-FPIFSES

U	u_1	u_2	u_3
$((0.9/e_1), x_1)$	0.700	-0.10	0.20
$((0.9/e_1), x_2)$	0.300	-0.80	0.85
$((0.9/e_1), x_3)$	-0.200	0.00	0.80
$((0.3/e_2), x_1)$	-1.000	-0.10	-0.55
$((0.3/e_2), x_2)$	-0.600	-0.40	0.90
$((0.3/e_2), x_3)$	-0.820	-0.40	0.60
$((0.8/e_3), x_1)$	0.700	1.00	-0.25
$((0.8/e_3), x_2)$	-0.200	0.40	1.00
$((0.8/e_3), x_3)$	-1.000	1.00	1.00
$((0.5/e_4), x_1)$	0.900	0.00	-0.60
$((0.5/e_4), x_2)$	-0.300	-0.50	-0.70
$((0.5/e_4), x_3)$	0.800	0.40	0.20
$A_j = \sum_{x \in X}\sum_{i=1}^{4}(c_{ij})(\mu_D(e_i))$	0.294	0.79	2.80

Table 3: Disagree-FPIFSES

U	u_1	u_2	u_3
$((0.9/e_1), x_1)$	-0.450	-0.100	-0.400
$((0.9/e_1), x_2)$	-1.000	0.000	-0.850
$((0.9/e_1), x_3)$	0.000	1.000	0.600
$((0.3/e_2), x_1)$	1.000	0.500	0.900
$((0.3/e_2), x_2)$	0.300	0.450	-1.000
$((0.3/e_2), x_3)$	0.600	0.800	-0.990
$((0.8/e_3), x_1)$	-0.700	-1.000	0.200
$((0.8/e_3), x_2)$	0.000	1.000	-0.950
$((0.8/e_3), x_3)$	1.000	0.300	-0.890
$((0.5/e_4), x_1)$	-0.700	0.000	0.400
$((0.5/e_4), x_2)$	0.900	0.400	0.600
$((0.5/e_4), x_3)$	-0.600	-0.100	0.970
$B_j = \sum_{x \in X}\sum_{i=1}^{4}(c_{ij})(\mu_D(e_i))$	-0.695	1.725	-1.239

Table 4: The score $r_j = A_j - B_j$

A_j	B_j	r_j
$A_1 = 0.294$	$B_1 = -0.695$	$r_1 = 0.989$
$A_2 = 0.79$	$B_2 = 1.725$	$r_2 = -0.935$
$A_3 = 2.80$	$B_3 = -1.239$	$r_3 = 4.039$

CONCLUSION

In this study, the concept of fuzzy parameterized intuitionistic fuzzy soft expert set was established. The basic operations on FPIFSESs, namely the complement, union, intersection and and OR operations were defined. Subsequently, the basic properties of pertaining to these operations such as the De Morgan's laws and other relevant laws pertaining to the concept of FPIFSES are proved. Finally, a generalized algorithm is introduced and applied to the FPIFSES model to solve a hypothetical decision making problem.

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REFERENCES

Alkhazaleh, S. and A.R. Salleh, 2011. Soft expert sets. Adv. Decis. Sci. 10.1155/2011/757868.

Alkhazaleh, S. and A.R. Salleh, 2014. Fuzzy soft expert set and its application. Applied Math., 5: 1349-1368.

Alkhazaleh, S., A.R. Salleh and N. Hassan, 2011a. Fuzzy parameterized interval-valued fuzzy soft set. Applied Math. Sci., 5: 3335-3346.

Alkhazaleh, S., A.R. Salleh and N. Hassan, 2011b. Possibility fuzzy soft set. Adv. Decis. Sci. 10.1155/2011/479756.

Atanassov, K.T., 1986. Intuitionistic fuzzy sets. Fuzzy Sets Syst., 20: 87-96.

Bashir, M. and A.R. Salleh, 2012. Fuzzy parameterized soft expert set. Abstr. Applied Anal. 10.1155/2012/258361.

Cagman, N., F. Citak and S. Enginoglu, 2010. Fuzzy parameterized fuzzy soft set theory and its applications. Turk. J. Fuzzy Syst., 1: 21-35.

Cagman, N., F. Citak and S. Enginoglu, 2011. Fuzzy parameterized soft set theory and its applications. Ann. Fuzzy Math. Inform., 2: 219-226.

Hazaymeh, A., I.B. Abdullah, Z. Balkhi and R. Ibrahim, 2012. Fuzzy parameterized fuzzy soft expert set. Applied Math. Sci., 6: 5547-5546.

Maji, P.K., R. Biswas and A.R. Roy, 2001a. Fuzzy soft sets. J. Fuzzy Math., 3: 589-602.

Maji, P.K., R. Biswas and A.R. Roy, 2001b. Intuitionistic fuzzy soft sets. J. Fuzzy Math., 9: 677-692.

Majumdar, P. and S.K. Samanta, 2010. Generalised fuzzy soft sets. Comput. Math. Applic., 59: 1425-1432.

Molodtsov, D., 1999. Soft set theory-First results. Comput. Math. Applic., 37: 19-31.

Pawlak, Z., 1982. Rough sets. Int. J. Comput. Inform. Sci., 11: 341-356.

Zadeh, L.A., 1965. Fuzzy sets. Inform. Control, 8: 338-353.